An Improved Adaptive Fourier Analyzer

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Abstract

The adaptive Fourier analyzer (AFA) is a structurally adaptive system for exact measurement of band-limited periodic signals of arbitrary fundamental frequency. It is an extension of the resonator based observers which calculate the recursive discrete Fourier transform. The AFA adapts the resonator frequencies to coincide with those of the input signal, avoiding the picket-fence effect and leakage. However, the lower the frequency of the input signal, the higher the computational demand required by the structure. Due to this fact, in practical cases, there is a lower limit for the fundamental frequency. This paper describes a new AFA, which sets the sampling frequency adaptively, using the actual estimator of the fundamental frequency. The input signal is first filtered by a decimation filter bank, and its appropriate output is fed to a conventional AFA. The paper investigates the transients of the system as well as shows some examples.

1. Introduction

Traditional Fourier analysis via the discrete Fourier transform (DFT) is distorted due to the picket-fence effect and leakage. The exact spectrum can be estimated only if synchronized measurements are possible. A successful solution of the problem is the interpolation and the resampling of the measurement record using the estimated fundamental frequency [7], [6]. After resampling, a conventional DFT is utilized for the estimation of the spectrum.

The adaptive Fourier analyzer (AFA) [2] is an alternative solution of the problem. It is an extension of the resonator based observers developed earlier to calculate the recursive discrete Fourier transform (RDFT) [1], [5]. In these observers the resonators work in a common feedback loop providing zero steady-state feedback error at the resonator frequencies. The AFA adapts the resonator frequencies to coincide with those of the input signal. The adaptation procedure is very similar to the locking of a phased locked loop.

The AFA has been known for some time and it is proved to be a fast and robust system. It was successfully utilized e.g. in high-precision vector-voltmeters [2] or in active noise control systems [9]. In the last years different algorithms were developed based on the original AFA. [4] describes the modification of the algorithm which is able to analyze sweeping periodic signals. Although the system has excellent features, the exact convergence analysis is not present. [8] describes a slight modification of the structure which allows to investigate its convergence properties. [10] introduces a fast filter bank for efficient calculation of the algorithm.

Although the AFA can theoretically lock on periodic signals of arbitrary fundamental frequency, there is a practical limit for the lowest fundamental frequency. As the fundamental frequency changes, the number of the represented harmonics changes, as well. Since the AFA has resonators at each harmonic frequency, the number of resonators depends on the estimated fundamental frequency. The lower the frequency of the input signal, the higher the number of the resonators and therefore the computational demand required by the structure. Assuming 40 kHz sampling frequency and 40 MIPS (or MFLOPS) digital signal processor (DSP), the maximal number of harmonics is about 100, hence the lowest fundamental frequency is about 200 Hz.

On the other hand, the input signal with a frequency near to the lowest possible fundamental frequency is highly oversampled. Therefore it seems to be a reasonable idea to decimate the input signal, if its frequency is too low. In this way the required number of resonators can be reduced, according to the decimation factor. Consequently, the lowest possible fundamental frequency is reduced, as well. If the frequency of the input signal gets higher or lower, the decimation factor can be changed accordingly. This procedure requires a decimation filter bank, the appropriate output of which is used as input of the AFA. Our paper introduces such an adaptive Fourier analyzer algorithm, and investigates its behavior.

Section 2. recalls the original AFA algorithm, and section 3. introduces the improved analyzer. Section 4. deals with the decimation filter design, while section 5. shows some examples. The paper is closed with a short conclusion.

2. Adaptive Fourier Analyzer

The theoretical background of the AFA is the resonator based observer. It was designed to follow the state variables of the so-called conceptual signal model [2], [5]. The signal model is described as follows:

$$y_n = \mathbf{c}_n^T \mathbf{x}_n \quad (1)$$

$$\mathbf{c}_n = [c_{n,k}] = e^{j2\pi f_1 kn},\ k = -L \dots L \quad (2)$$

$$Lf_1 < 0.5 < (L+1)f_1 \quad (3)$$

where $\mathbf{x}_n$ is the state vector of the signal model at time step n, y_n is its output (the input of the observer), $\mathbf{c}_n$ represents the basis of the Fourier expansion, and f_1 is the fundamental frequency relative to the sampling frequency. The corresponding observer is (Fig. 1):

$$\hat{\mathbf{x}}_{n+1} = \hat{\mathbf{x}}_n + \mathbf{g}_n(y_n - \mathbf{c}_n^T \hat{\mathbf{x}}_n); \quad \mathbf{g}_n = [g_{n,k}] = r_k \bar{c}_{n,k} \quad (4)$$

where $\hat{\mathbf{x}}_n$ is the estimated state vector, $\{r_k;\ k = 1..N;\ N = 2L+1\}$ are free parameters to set the poles of the system, and the overbar denotes the complex conjugate operator. Due to the complex exponentials, the channels of the observer can be considered as time-invariant systems with a pole on the unit circle $\{z_k;\ k = 1..N\}$ (Fig. 1). This is why they are called resonators. If the resonator poles are arranged uniformly on the unit circle, and $\{r_k = 1/N;\ k = 1..N\}$, the observer has finite impulse response [5]. In practical applications [2] where the fundamental frequency changes, the resonators cannot be placed uniformly, and the above setting of parameters r_k does not provide finite impulse response. But, if (2), and (3) hold, the system is fairly fast. If the estimated frequency does not coincide with that of the input signal y_n, the complex state variables will rotate, and the speed of this rotation at each resonator is proportional to the corresponding frequency difference. This is the basic idea for the frequency adaptation in the AFA [2]. The exact formula is the following:

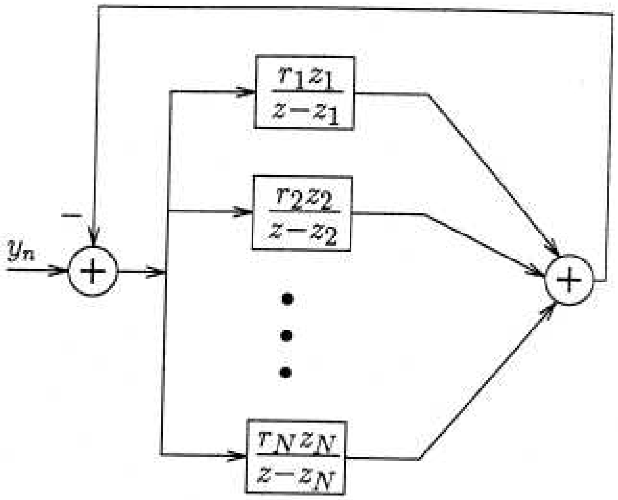

Figure 1. The structure of the original AFA

$$f_{1,n+1} = f_{1,n} + \frac{1}{2\pi N}\text{angle}(\hat{x}_{1,n+1}, \hat{x}_{1,n}) \quad (5)$$

where $\hat{x}_{1,i}$ is the estimated state variable belonging to the positive fundamental frequency, and "angle" gives the angle between two complex numbers.

The algorithm and the update procedure of the variables is decribed step by step in [3]. Assuming real input signals, the AFA requires about 10 DSP instructions for each harmonic component.

3. The Improved AFA

The computational demand mentioned at the end of the previous section determines the maximal number of harmonics at a given sampling frequency. Thus the lowest frequency of the input signal, which the AFA can lock on is also determined. However, the input signal with a frequency near to the lowest possible fundamental frequency is highly oversampled. Due to this fact, it is possible to decimate the input signal in such cases. In this way the required number of resonators can be reduced, according to the decimation factor. If the frequency of the input signal gets higher again, the original sampling frequency can be

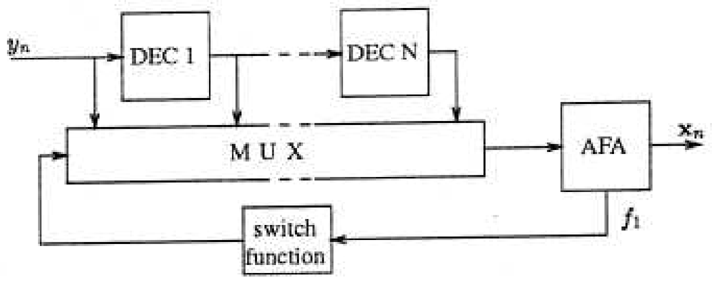

Figure 2. The structure of the improved AFA

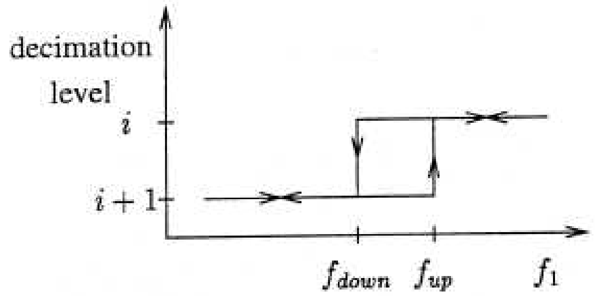

Figure 3. The switch function

used. If the frequency of the input signal gets even lower, a higher decimation factor can be used. The lowest possible fundamental frequency depends only on the highest decimation factor. This procedure requires a decimation filter bank, the appropriate output of which is used as input of the AFA. The output selection is controlled by the estimated fundamental frequency. The proposed structure can be seen on Fig. 2. The switch function depicted in the figure has an essential role. A straightforward function can be seen in Fig. 3. In order to avoid fast up/down switches between two levels, the switch function contains a hysteresis. Both f_{down} and f_{up} are higher than the lowest possible fundamental frequency at level i, and the lag in the hysteresis depends on the signal to noise ratio (SNR) of the signal to be analyzed. Smaller SNR requires larger lag and vice versa.

The structure described above is already able to lock on periodic signals of arbitrary fundamental frequency also in practice. However, some important design and implementation questions are open. The next section deals with such problems.

4. Filter Bank Design

The filter bank of the improved AFA can be designed in different ways. The adequate method depends on the measurement problem to be solved. This section introduces some simple solutions.

The computational demand and the dynamic properties of the AFA depend on the number of resonators (N). These can be very different if N varies in a wide range. To avoid such problems, it is reasonable to keep N of the same order. To achieve this, half-band decimation filters can be used, so the ratio between the maximal and the minimal N is 2. As it is usual in decimation filter banks, the same filter can be used at each level.

The result of the adaptive Fourier analysis is the estimated fundamental frequency (f_1) and the complex amplitudes ($\mathbf{x}_n$). The measurement of $\mathbf{x}_n$ is distorted by the decimation filters. Their frequency response has to be compensated, if it is required by the measurement problem. In order to do this, the frequency response of the decimation filters has to be known and it has to be evaluated using the actual harmonic frequencies.

The magnitude response of the filters is close to the unity in the pass band, and its compensation can be ignored in some applications. However, the phase shift of the filters is different at each decimation level. As a consequence the state variables forced to set to a new value at each sampling rate change, which can cause long transients in the measurement. To avoid such problems, different technics can be used. In the following two possibilities are sketched.

The general solution is the update of the state variables, according to the phase shift caused by the decimation filters. This is a theoretically good solution, but its computational complexity strongly depends on the structure of the decimation filter. Complicated infinite impulse response (IIR) filters make difficulties, but linear phase FIR filters can be calculated easily.

A simple solution is the utilization of linear phase FIR filters as decimation filters and extra delays at lower decimation level, according to the maximal delay at the highest

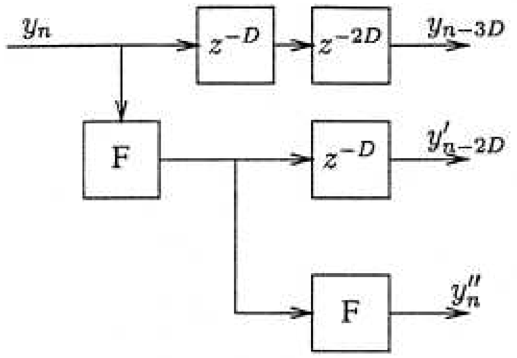

Figure 4. The improved AFA with compensator delays at lower decimation levels. Example for decimation factor of 4.

decimation level. An example with a decimation factor of 4 can be seen in Fig. 4. The method works well, if:

$$D = \frac{M-1}{2} \quad (6)$$

where M is the length of the half-band decimation filter "F". If the original sampling frequency is f_s, it is $f_s/2$ and $f_s/4$ on the second and the third level, respectively. One can calculate that a signal appearing at all levels is delayed by the same amount.

This simple structure works without transients at sampling frequency change. The penalty to be payed is the delay in the signal path.

5. Examples

In this section two simulation examples are presented. Both simulations deal with the same measurement problem. The task is the Fourier analysis of a band limited triangular signal of changing fundamental frequency. The amplitude of its first harmonic is set to unity. The sampling frequency $f_s = 1$ kHz, the signal changes in the range of 38..42 Hz. The signal is analyzed in both cases by the improved AFA, with switch frequencies of 39 and 41 Hz. One decimation filter is applied which allows to reduce the sampling frequency to 500 Hz. The decimation filter is a Chebishev type linear phase filter specified as follows:

passband	$0..f_s/5.12$	0.1 dB ripple
stopband	$f_s/4..f_s/2$	80 dB suppression

This specification requires 63 taps. The examples show the behavior of the improved AFA without and with transient elimination. The transient elimination was done by a delay of 31 samples (see (6)) in the path where no decimation is required. The simulation results can be followed in Fig. 5.

The diagrams show the time records of the input signal frequency, the estimated frequency, the feedback error and the estimated amplitude belonging to the fundamental frequency, respectively. The frequency of the input signal is constant or changes linearly, as it is depicted in the uppermost plot. During about the first 100 samples the structures lock on the input signal of constant frequency of 42 Hz. (Note the zero feedback error and the exact frequency and amplitude estimators.) At time step 200 the frequency starts to decrease. Like a phased locked loop, AFA can follow this change with a constant error, as the nonzero feedback error shows it. At a frequency of 39 Hz both structures switch to the decimated input, which cause undesirable transients in the structure without transient elimination (Fig. 5.a), while no effect can be seen in the structure with transient elimination (Fig. 5.b). At a constant frequency of 38 Hz both structures are locked. However, the second structure has shorter settling time, due to the eliminated transients. At time step 600 the frequency starts to increase. The phenomena are similar to those of the decreasing case, but the switching accours only at frequency of 41 Hz, because of the hysteresis. The superiority of the transient elimination is clearly visible on the simulation results.

6. Conclusion

The paper presented a new adaptive Fourier analyzer structure, which can be used for measurement of band-limited periodic signals of arbitrary fundamental frequency. The original AFA suffers from the high computational burden if the frequency of the input signal is too low. The improved AFA sets the sampling frequency adaptively, using the actual estimator of the fundamental frequency. The input signal is first filtered by a decimation filter bank, and its appropriate output is fed to a conventional AFA. The paper sketched some possibilities of filter bank design, emphasizing the importance of the elimination of the transients which can be caused by the sampling frequency exchange. Simulational results illustrated the capability of the structure.

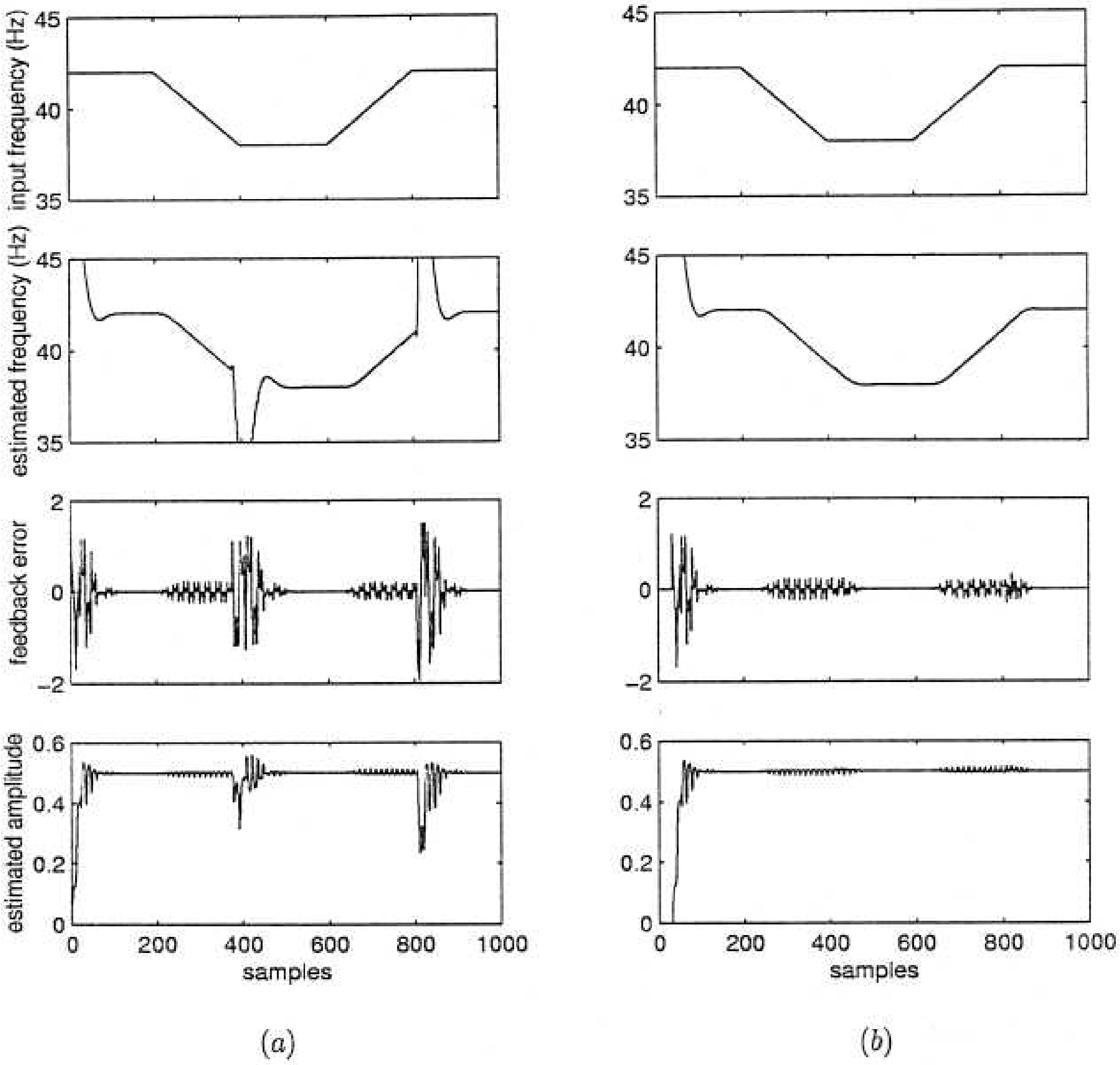

Figure 5. Simulation results without (a) and with (b) transient elimination.

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