A Fast Filter-Bank for Adaptive Fourier Analysis

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***Abstract*—Adaptive Fourier analyzers have been developed for measuring periodic signals with unknown or changing fundamental frequency. Typical applications are vibration measurements and active noise control related to rotating machinery and calibration equipment that can avoid the changes of the line frequency by adaptation. Higher frequency applications have limitations since the computational complexity of these analyzers are relatively high as the number of the harmonic components to be measured (or suppressed) is usually above 50.**

In this paper, based on the concept of transformed domain signal processing, a fast filter-bank structure is proposed to reduce the above computational complexity. The first step of the suggested solution is the application of the filter-bank version of the fast Fourier transform or any other fast transformations that convert input data into the transformed domain. These fast transform structures operate as single-input multiple-output filter-banks, however, they can not be adapted since their efficiency is due to their special symmetry. As a second step, the adaptation of the filter-bank is performed at the transform structure's output by adapting a simple linear combiner to the fundamental frequency of the periodic signal to be processed.

***Index Terms*—Adaptive Fourier analysis, digital signal processing, fast filter-banks, fast recursive transformations, transform-domain measurements.**

I. Introduction

IN CERTAIN applications, like vibration analysis and active noise control, periodic signals with unknown or changing fundamental frequency are to be measured. The traditional Fourier transform structures do not offer very good performance unless the sampling frequency and the fundamental frequency of the signal are synchronized. Without synchronization, the picket-fence effect and the leakage can not be avoided. There are two known alternatives to match the signal and the Fourier analyzer. The first technique [1], [2] applies (re)sampling controlled by the estimate of the fundamental frequency. If the signal to be measured is already digitized, first an interpolation is to be performed, which is followed by resampling. The resampled signal can be processed using the fast Fourier transformation (FFT) algorithm without the side-effects of picket-fence and leakage. The second technique is the application of a special filter-bank, the so-called adaptive Fourier analyzer (AFA) [3] and [4]. This filter-bank is tunable to match signal components and the measuring channels. The price to be paid for this solution is that fast transformation algorithms, like the FFT, cannot be directly utilized.

The purpose of this paper is to propose a third alternative. The technique to be introduced is based on the concept of transform domain adaptive signal processing. As a first step the signal transformation is performed using a filter-bank version of one of the fast transformations, like the FFT. Unfortunately, however, none of these transformers can be adapted since their computational efficiency is due to their special symmetry. Therefore, the second step is tuning or adaptation which is performed at the transformer's output by setting the weights of simple linear combiners as a function of the estimated fundamental frequency of the periodic input signal in hand. Explicit formulas are given to calculate the weights. To avoid the burden of these calculations a proper tabulation of the weights is also suggested. This latter considerably reduces the overall computational complexity and thus, implementations with higher sampling frequency can also be considered.

In Section II the concept of the AFA is summarized rather briefly. Since the technique to adapt the fundamental frequency [3], [4] can be directly utilized also in the case of the proposed method, in the subsequent sections we consider the fundamental frequency as an input variable. In Section III, the details of the transform domain adaptive Fourier analysis are presented. These include the basic design equations of the analyzer in hand. Section IV is devoted to an illustrative example where the AFA versions are compared and the strength of the proposed method is shown.

II. The Adaptive Fourier Analyzer

The AFA [3] and [4] is an adaptive filter-bank structure strongly related to the recursive discrete Fourier transformation (DFT). This is suitable for the calculation of the Fourier coefficients and/or components in a sliding-window mode with computational complexity proportional to the window size for every input sample [5]. The classical version of the recursive DFT based on the Lagrange interpolation has been replaced by an observer structure [6] and resulted in a common framework for signal processing algorithms.

The AFA described in [3] and [4] applies the general recursive transform structure suggested in [6]. For the case of Fourier analysis, this structure can be characterized by a set of parallel complex demodulator/modulator pairs with a two-channel discrete integrator between them, together with a common feedback resulting in a recursive DFT structure (see Fig. 1). The two-channel integrators process the real and the imaginary parts of the demodulated signal, respectively. After convergence, this structure produces Fourier coefficients

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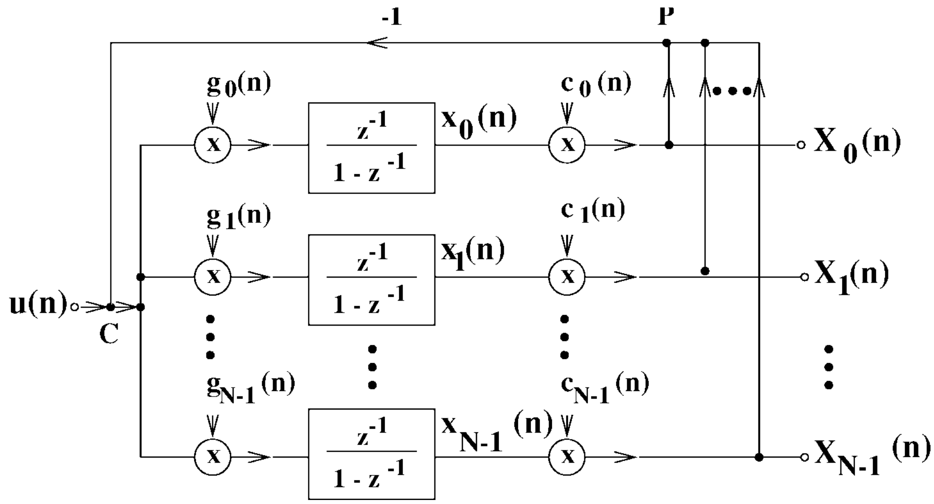

Fig. 1. Recursive DFT structure. $g_m(n) = (1/N)\, e^{-j(2\pi/N)mn}$; $c_m(n) = e^{j(2\pi/N)mn}$; $m = 0, 1, \cdots, N-1$.

$(x_m(n),\, m = 0, 1, \cdots, N-1)$ at the output of the integrators, and Fourier components $(X_m(n),\, m = 0, 1, \cdots, N-1)$ at the output of the complex modulators.

In [3] and [4], an adaptive procedure has been added to the recursive DFT for the analysis of periodic signals with unknown frequency. This adaptation procedure "locks" the fundamental frequency component of the periodic signals like a PLL and tunes the recursive DFT channels accordingly.

The actual estimate of the fundamental frequency can be calculated by (see [3])

$$f_1(n+1) = f_1(n) + f_s \frac{1}{2\pi N(n)} \ \text{arc}\left(\frac{x_1(n+1)}{x_1(n)}\right) \qquad (1)$$

where $f_1(n)$ and f_s denote the fundamental frequency at time instant n and the sampling frequency, respectively. Variable $x_1(n)$ is the output of the corresponding discrete integrator, while arc$(x_1(n+1)/x_1(n))$ stands for the one-step phase variation of the fundamental frequency. $N(n)$ denotes the actual number of the channels, which should meet the condition

$$[N(n) - 1]f_1(n) < f_s < [N(n) + 1]f_1(n). \qquad (2)$$

This results in applying, always, as many DFT channels as can be accommodated within the frequency range up to one half of the sampling frequency. This means that the proposed method implements a structurally adaptive system with an order depending on the actual ratio of the fundamental to the sampling frequencies.

The actual phase estimate of the fundamental component is given by

$$\varphi_1(n+1) = \varphi_1(n) + 2\pi \frac{f_1(n)}{f_s} \qquad (3)$$

which replaces $2\pi n/N$ in the demodulator/modulator pairs (see Fig. 1). It is important to note that simply changing the phase of the complex exponentials in Fig. 1 modifies not only the filter-channel positions but also introduces system poles and therefore infinite impulse response (IIR) behavior unless the phases happen to be integer multiples of $2\pi/N$. The presence of poles out of the origin results in transients which

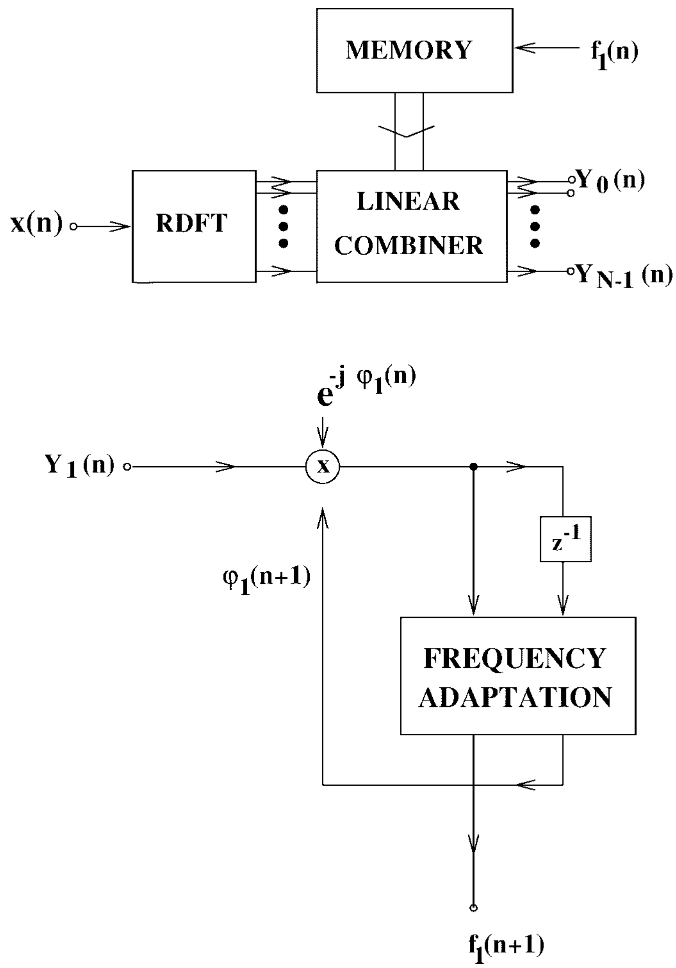

Fig. 2. Block diagram of the adaptive Fourier analyzer.

disturb the adaptation procedure. Under such circumstances the convergence properties are extremely hard to investigate and no proof is presently available concerning global convergence.

In [7] the concept of a so-called block-adaptive Fourier analyzer (BAFA) is introduced which avoids the disturbance due to the transients, and therefore the investigation of the convergence properties becomes much easier. Since for finite

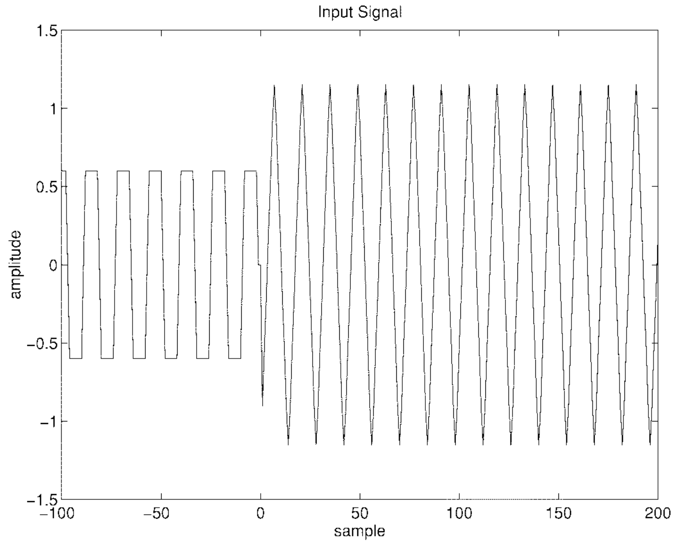

Fig. 3. Input signal modified at zero sampling time. Shape: from trapezoidal to triangular, amplitude change: appr. 100%, frequency change: appr. 10%.

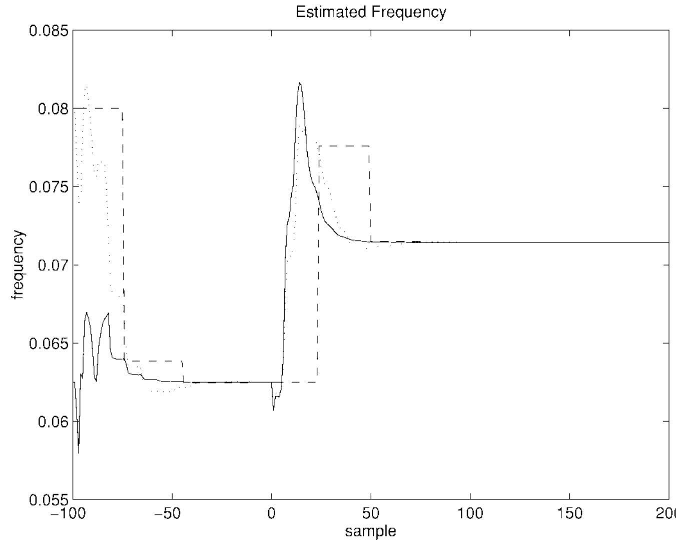

Fig. 4. Estimated frequency for the new AFA (continuous), for the previous AFA (dotted), and for the BAFA (dashed) structure.

impulse response (FIR) filters, the transients of channels will die out in a maximum of N steps, it seems to be reasonable to start filter operation with a complete block (N samples) of input data without changing/adapting the filter coefficients. After filtering the first N samples, the transients due to the parameter changes will disappear. As a second step, based on further P $(P \geq 1)$ output samples, the phase measurements are to be performed. These phase values are used to derive the estimate of the fundamental frequency, similarly to (1), where P equals 1. The third step is the parameter adaptation, i.e., a new setting of the filter parameters. The block-adaptive analyzer applies the very same filter-bank structure (see Fig. 1) except the weights of the demodulators (or modulators) are set somewhat differently to provide FIR behavior. The design of such a parameter set is straightforward since explicit formulas are available (see, e.g., [6]). For the case of the BAFA the conditions of global convergence are also derived [7]. The original algorithm and its modifications have been successfully utilized in low frequency applications (see, e.g., [8]).

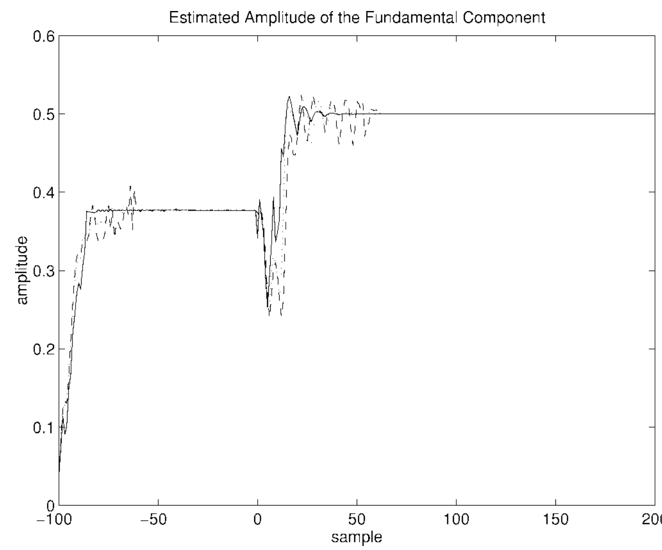

Fig. 5. Estimated amplitude of the fundamental component for the new AFA (continuous), for the previous AFA (dotted), and for the BAFA (dashed) structure.

III. The Fast Filter-Bank for Adaptive Fourier Analysis

The fast filter-bank structure suggested in [9] and further developed in [10] combined with the adaptive "frequency sampling" method [11] can be utilized for fast adaptive Fourier analysis, as well. The size of the fast DFT filter-bank is determined by the smallest possible value of the fundamental frequency to be detected. To such a transform structure, a set of adaptive linear combiners is connected (see Fig. 2). After convergence at the output of the linear combiners the separated signal components will appear. The adaptation consists of two different types of calculations. The first is the estimation of the fundamental frequency. This can be done following the algorithms described in [3], [4], or [7]. According to these algorithms the actual value of the basic harmonic's angular frequency is calculated recursively from the previous estimate by adding a correcting term proportional with the phase variation of the corresponding Fourier coefficient. The second step is the calculation of the coefficients for the linear combiners. The filters to be implemented can be characterized by the following transfer function:

$$H_m(z) = K_m \prod_{n=0,\, n\neq m}^{M-1} (1 - z_n\, z^{-1}) \tag{4}$$

where $m = 0, 1, \cdots, N-1$, $z_0 = 1$, $z_{M/2} = -1$, $z_n = e^{jn2\pi f_1/f_s}$, $n = 1, 2, \cdots M-1$, $n \neq M/2$. The K_m serves as normalizing factor, which can be derived by evaluating (4) at $z = z_m$. M is the actual "order" of the linear combiners in such a way that $M(f_1/f_s) < 1$. The weights for the linear combiners are calculated by "sampling" (4) in the frequency domain at the locations dictated by the Nth roots of unity. This latter requires only the evaluation of some formulas derived from (4). However, these calculations become very intensive if N is large. The "on-line" calculation of the weights can be avoided by a proper tabulation for the frequency range to be covered. For nontabulated frequencies, simple (possibly linear) interpolation techniques can be applied. It is important to note that these AFA filters are FIR filters producing minimum response time.

The frequency-sampling technique can be utilized in the case of other discrete transformations, as well. The fast recursive Walsh–Hadamard transformation (WHT) [13] can be also a good candidate for the transformer part in this new AFA. This change affects the weights of the adaptive linear combiners. However, the modification required is easy to calculate and finally tabulate, if needed.

Another very important aspect is that in the transform domain the sampling rate can be reduced and the only linear combiners to be evaluated are those which are of interest to the actual measurement.

The whole system can be operated both in sliding-window and block-oriented modes. For this latter the transform structure's outputs are calculated only in every Nth time instant, i.e., only once for each block. This is always acceptable if the system is "tuned" by an external frequency signal. In the adaptive case the phase calculation of the basic harmonic is critical, however for slowly varying signals slower adaptation is also acceptable.

The original form of the AFA ([3] and [4]), due to its feedback structure, requires a complete evaluation in every time instant. The necessary computational power is determined by the maximum value of M, which corresponds to the order of the system to be implemented. In our proposition the AFA is replaced by a fast, and recursive transform structure $(N \geq M)$ with computational complexity proportional to that of the FFT, and by a set of linear combiners. The weights of the linear combiners can be calculated on-line or tabulated for the frequency range to tracked. This new system can be advantageous if not all the outputs are needed in every time instant, if minimum response time is required, and if both the true DFT and the AFA outputs are to be calculated.

The transposed version of the proposed adaptive Fourier analyzer can be utilized for synthesizing periodic signals with controlled fundamental frequency and applied similarly as is suggested in [12].

IV. Illustrative Example

The strength of the new adaptive Fourier analysis is illustrated in Figs. 3–5 compared to other AFA versions. The shape, magnitude, and frequency of the input signal are modified at time instant zero (see Fig. 3). Fig. 4 shows the fundamental frequency estimates of the different structures, while in Fig. 5 the amplitude estimates can be followed.

V. Conclusions

In this paper a new transformed domain adaptive Fourier analyzer has been described. The frequency adaptation algorithm is the same as that of the previous AFA proposed in [3] and [4], but the signal processing part has a more efficient formulation. The first part of this formulation is the utilization of the fast recursive Fourier transformation, while the second one is the implementation of adaptive frequency-sampling filters. The frequency of the basic harmonic component is either calculated by recursive adaptation or can be tuned by an external signal. The weights of the frequency-sampling filters are given by explicit formulas and thus, can be calculated on-line or can be tabulated for the frequency range to tracked. This new system may be especially advantageous if not all the outputs are needed in every time instant, if minimum response time is required, and if both the true DFT and the AFA outputs are to be calculated. The proposed method can significantly contribute to the frequency range extension of this technique.

References

[1] R. B. Randall, *Frequency Analysis.* Brüel & Kjaer, 1987.
[2] R. W. Potter, "Tracking and resampling method and apparatus for monitoring the performance of rotating machines," U.S. Patent 4912661, 1990.
[3] F. Nagy, "Measurement of signal parameters using nonlinear observer theory," *IEEE Trans. Instrum. Meas.*, vol. 41, pp. 152–155, Feb. 1992.
[4] ——, "Application of nonlinear filter and observer theory in adaptive signal processing," in *Proc. IEEE Winter Workshop Nonlinear Digital Signal Processing*, Tampere, Finland, Jan. 17–20, 1993, pp. 6.2-3.1–6.2.-3.6.
[5] L. R. Rabiner and B. Gold, *Theory and Application of Digital Signal Processing.* Englewood Cliffs, NJ: Prentice-Hall, 1975, p. 47.
[6] G. Péceli, "A common structure for recursive discrete transforms," *IEEE Trans. Circuits Syst.*, vol. 33, pp. 1035–1036, Oct. 1986.
[7] G. Simon, and G. Péceli, "Convergence properties of an adaptive Fourier analyzer," *IEEE Trans. Circuits Systems—II: Analog and Digital Signal Processing*, submitted for publication.
[8] L. Sujbert and G. Péceli, "Periodic noise cancellation using resonator based controller," in *Proc. 1997 Int. Symp. Active Control Sound Vibration, ACTIVE'97*, Budapest, Hungary, Aug. 1997, pp. 905–916.
[9] A. R. Várkonyi-Kóczy, "A recursive fast Fourier transformation algorithm," *IEEE Trans. Circuits Syst.—II: Analog and Digital Signal Processing*, vol. 42, pp. 614–616, Sept. 1995.
[10] ——, "Efficient polyphase DFT filter banks with fading memory," *IEEE Trans. Circuits Syst.—II: Analog and Digital Signal Processing*, vol. 44, pp. 670–673, Aug. 1997.
[11] R. R. Bitmead and B. D. O. Anderson, "Adaptive frequency sampling filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 29, no. 3, pp. 684–693, 1981.
[12] A. R. Várkonyi-Kóczy, "Synchronized multi-sine measurements via DSP methods," *IEEE Trans. Instrum. Meas.*, vol. 46, pp. 929–932, Aug. 1997.
[13] N. Ahmed and K. R. Rao, *Orthogonal Transforms for Digital Signal Processing.* New York: Springer-Verlag, 1975.

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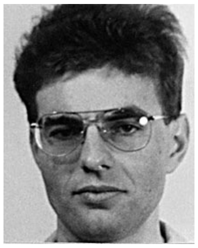

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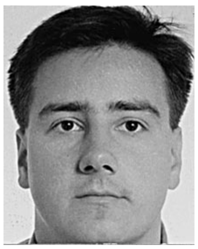

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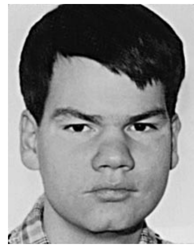

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