XX IMEKO World Congress
Metrology for Green Growth
September 9-14, 2012, Busan, Republic of Korea

EXPERIMENTAL COMPARISON OF MAXIMUM LIKELIHOOD AND LS FITTING FOR ADC TESTING

Ján Šaliga, Linus Michaeli, Ján Buša, Jozef Lipták, [*]István Kollár, [*]Tamás Virosztek

Technical University of Košice, Letná 9, 04120 Košice, Slovakia
Email: {jan.saliga, linus.michaeli, jan.busa, jozef.liptak}@tuke.sk
[*]Budapest University of Technology and Economics, Budapest, Hungary, Email: kollar@mit.bme.hu, virro.vik@gmail.com

Abstract: ADC test methods require the best possible reconstruction of the input signal to the ADC under test from the acquired, therefore erroneous ADC output data. The commonly used LS fit and the recently introduced maximum likelihood estimation are competing methods. This paper presents an experimental comparative study of these estimation methods with the goal to investigate the behaviour of both methods and to determine their limits. An alternative algorithm for the calculation of the maximum likelihood fit is also examined. Some practical recommendations for the choice of optimal method for various conditions of ADC testing are also given as a conclusion of the comparative study.

Keywords: ADC test, maximum likelihood estimation, least squares method, LS method, four-parameter fit, signal recovery, estimation of signal parameters, differential evolution.

1. INTRODUCTION

Standardized dynamic test methods for analog-to-digital converters (ADCs) ([1] – [3]) are based on comparison of acquired ADC output codes with the ADC input stimulus. The stimulus is not exactly known and cannot be measured with the necessary accuracy, therefore it must be reconstructed from the erroneous ADC output codes acquired during testing. Any inaccuracy in the estimation of ADC stimulus parameters leads to inaccuracy in determination of ADC parameters measured by the dynamic test.

The most common way how to recover input signal and estimate its parameters is least squares (LS) fitting. According to the theory the LS fitting gives the best estimation under the condition that the observation (quantization) noise is additive to the input, is independent, white, and normally distributed with zero mean ([4], [5]). This all is clearly not true for ADC testing and therefore the LS fit is usually worse than ML estimation would be.

The general, systematic "best" way of estimation is fitting based on the maximum likelihood (ML) method. This idea was introduced in [6] for sine wave and later it was generalized also for exponential stimulus ([7]). The ML estimation is optimal in a certain sense, but has disadvantages like its computation complexity and possible local minima.

Although both fitting procedures are known, until now no deeper comparative study on limitations of these methods has been performed. The main novelty of this paper is just this comparative research. Moreover, we also examined the differential evolution based optimization method for ML fit [9]. In contrary of the previous works ([6]-[9]) we performed calculations and signal processing in LabVIEW instead of Matlab – this does not change general statements about the problem, but paves the way to alternative implementations.

2. FITTING METHODS

The general setup for dynamic ADC testing is shown in Fig. 1. To perform simulation experiments we developed a few software modules including non-ideal ADC model with optional test specific INL. The modules enable simulating real ADC test according to Fig. 1.

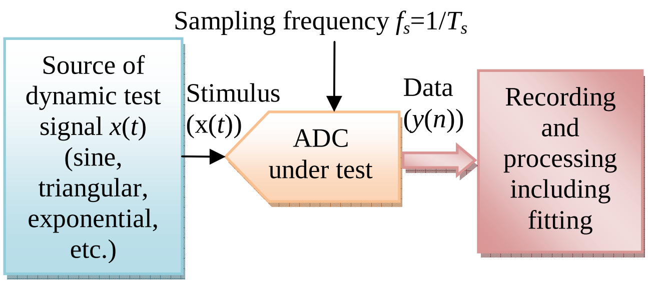

Fig. 1. General setup for dynamic ADC testing

2.1. Least Squares (LS) fit

LS fitting is well known and very often used procedure for recovering distorted and noisy signals in testing and measurements. Estimation of parameters of ADC input signal is obtained from minimization of the cost function CF_{LS}:

$$\min_{\mathbf{a}}\left(\mathrm{CF}_{\mathrm{LS}}(\mathbf{a})\right)=\min\sum_{n=0}^{N-1}\left(y(n)-\mathrm{f}(\mathbf{a},n)\right)^2 \qquad (1)$$

where f($\mathbf{a}$,t) is the time model of the testing signal, $\mathbf{a}$ is a vector of its unknown parameters, and $y(n)$ are digitized samples of testing signal applied to the input of ADC under test, taken at sampling instances nT_s (T_s is the sampling period). Solution of (1) leads to a system of equations which is nonlinear in the cases of the four-parameter fit of a sine wave or for dual slope exponential stimulus. Solution of such a nonlinear system requires application of an appropriate numerical method, and this increases the complexity of the LS method.

2.2. Maximum likelihood fit

Maximum likelihood method comes from estimation theory and looks for the most probable ADC input signal. The ML fitting is based on maximization of a likelihood function $L(\mathbf{a})$:

$$\max_{\mathbf{a},\,\mathbf{q},\,\sigma}(L(\mathbf{a})) = \max_{\mathbf{a},\,\mathbf{q},\,\sigma}\prod_{n=0}^{N-1} P\left(y(n) = Y_{\mathbf{a},\,\mathbf{q},\,\sigma}(n)\right), \qquad (2)$$

where $\mathbf{q}$ is the vector of ADC transition code levels, σ is the standard deviation of the Gaussian noise of the input electronics, assuming here that the noise samples are independent. $P(y(n) = Y(n))$ is the probability that the n-th ADC output sample $y(n)$ is equal to the expected output value $Y(n)$, which represents a possible output code. To simplify the maximization $L(\mathbf{a})$, minimization of the sum of negative logarithms of probabilities P(.) is preferred.

$$\begin{aligned}&\arg\max_{\mathbf{a},\,\mathbf{q},\,\sigma}(L(\mathbf{a})) \approx \arg\min_{\mathbf{a},\,\mathbf{q},\,\sigma}(-\ln(L(\mathbf{a}))) = \\ &= \arg\min_{\mathbf{a},\,\mathbf{q},\,\sigma}\left(-\sum_{n=0}^{N-1}\ln P\left(y(n) = Y_{\mathbf{a},\,\mathbf{q},\,\sigma}(n)\right)\right)\end{aligned} \qquad (3)$$

Searching the extreme value can be a complex task and it can be performed only by an appropriate numerical method.

2.3. Implementation of fitting and comparison

Experiments below were primarily performed in software developed in LabVIEW. Properties, advantages and disadvantages of LabVIEW are generally well known. We choose LabVIEW not only because of our previous experience but also because we wanted to compare our results with previous experiments performed in Matlab ([6] – [7]).

LS fit for a sine wave was implemented according to standards ([1] – [3]) in the form of 3- and 4-parameter fits. ML fit implementation was rather complex. In contrast to minimization methods based on gradients used in [6]-[7], the LabVIEW built-in function `GlobalOptimization.vi` ([8]) has been utilized. This function is based on the Differential Evolution method (DE), which is a kind of genetic algorithm, being one of the general methods used to solve the global optimization problem. Although finding the global optimum is not guaranteed with this method either, obtaining it is more likely with proper settings.

DE methods approach the (often global) optimum by mutating and improving the candidate parameters from the initial ones. Fig. 2 illustrates how the algorithm works. GlobalOptimalization.vi uses (if available) multi-core processing, which speeds up the calculation of the best parameters.

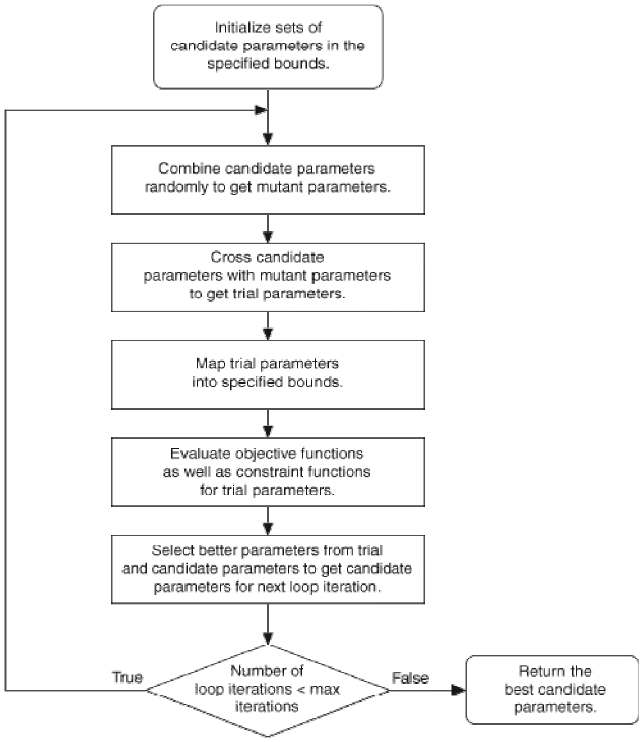

Fig. 2. Algorithm of DE method

For comparison of accuracy of signal recovering achieved by LS and LM methods we utilized two parameters:

1. Estimation error defined as RMS value of the difference between estimated signal and the exact input signal.
2. Difference between SINADs calculated from LS fit, or from ML fit and the exact value calculated from exactly known ADC stimulus.

The RMS value was chosen because it is the basic descriptor of the error which has influence on nearly all final ADC dynamic parameters such as ENOB and SINAD. Moreover, a simple comparison of differences between, e.g., sine wave amplitudes is not well readable from the point view of its influence on the final error in calculation of ADC parameters. The estimation error D_{method} (RMS value of the difference between the recovered signal by a given method and the known exact one) was calculated according to the formula:

$$D_{\text{method}} = \sqrt{\frac{1}{N}\sum_{n=1}^{N-1}\left(y_{\text{method}}(n) - y_{\text{exact}}(n)\right)^2}\ , \qquad (4)$$

where $y_{\text{method}}(n)$ are the samples calculated from signal recovered by the given method and $y_{\text{exact}}(n)$ are the equivalent samples of the exact ADC stimulus.

Comparison of SINAD calculated from LS and ML fits to SINAD calculated from the exact value of stimulus was utilized only in tests, where the stimulating sine wave did not overload ADC full scale as it is required by standards. SINAD in dB was calculated according to the standard defined formula:

$$SINAD_{\text{method}} = 20\log\frac{A_{\text{method}}}{D_{\text{method}}}, \qquad (5)$$

where A_{method} is RMS value of sine wave recovered by a given method or from exactly known ADC stimulus. Note that the influence of D is not significant in SINAD if it is much smaller than standard deviation of the quantization noise, 0.3 LSB, but more significantly corresponds to ENOB.

3. EXPERIMENTAL RESULTS

The first experiments were performed on linear ideal ADC with user specifiable resolution (up to 16 bits), and standard deviation of noise equal to 0.2 LSB, which represented ADC code alternating behaviour. One would expect that ideal (linear) ADC characteristic causes similar LS and ML errors. Records with optional number of samples were utilized for fitting by a chosen method. Fig. 3 shows a zoomed segment of record of digitized stimulus from the ideal 8-bits ADC together with exact stimulus, and recovered signals. Stimulus did not overload ADC full scale. The differences between recovered signals and exact stimulus are hardly visible.

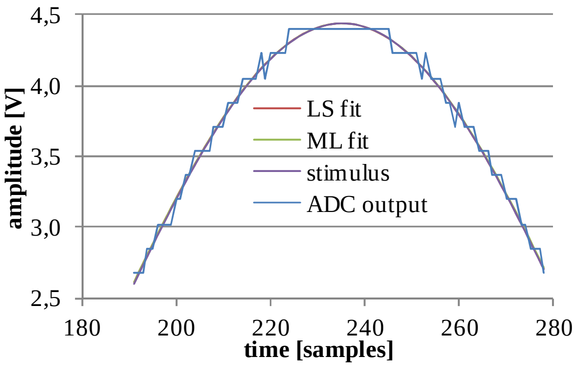

Fig. 3. Comparison of LS and ML fit, digitized and exact stimulus (zoomed time segment)

Fig. 4 shows the same signals with the condition that input signal slightly overloads ADC full scale. For both fits all samples in record were used including samples taken in time instances when signal overloaded ADC input range (overloading samples). In addition the same pre-processed record with the overloading samples excluded was fitted by LS method. In accordance with expectations, ML fit gives much better estimation of ADC stimulus than LS fit of the whole record because of its nature to approach also the extreme ADC overloading samples. If LS fit is applied on a pre-processed record where overloading samples are excluded from calculation the results obtained by LS and ML are nearly the same (Fig. 4).

Fig. 5 shows the dependence acquired for an 8-bit ADC and 500 coherently taken samples with standard deviation of noise σ = 0.2 LSB for varying ADC overloading. If the input signal covers ADC range up to 100 percent, both methods give nearly the same results. RMS value of the difference of LS fit from exact stimulus quickly increases if input stimulus even only slightly overloads ADC input range and if LS fit is applied on whole record. Very similar results were achieved for other ADC resolutions, σ, and numbers of samples in record.

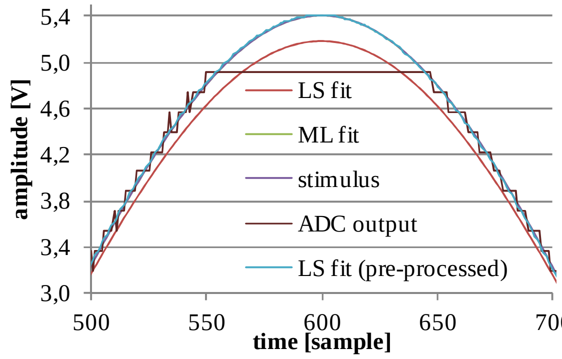

Fig. 4. Comparison of LS and ML fit, digitized and original stimulus (zoomed time segment) if stimulus overloads full range of the ADC. ML fit utilized all samples in record, LS fit utilized once all samples and next the record with excluded overloading samples.

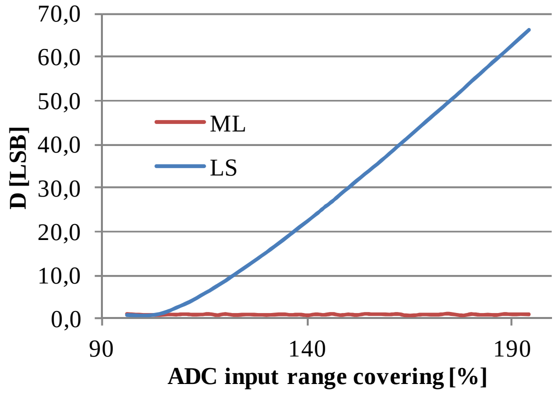

Fig. 5. LS and ML fits for overloaded ADC (100% is the ADC full range). Both fits utilize all samples in record

Fig. 6 shows the same circumstances but LS fit was applied on pre-processed records with excluded overloading samples. At these circumstances, the estimation errors for both fits are small and comparable. The only difference between fits is that the ML fit gave more “stable” error what indicates its smaller sensitivity to quantization effects.

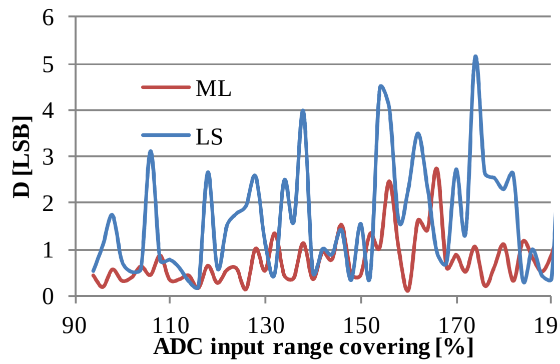

Fig. 6. LS vs. ML fit for overloaded ADC (100% is equal to ADC full scale). LS fit uses pre-processed record.

The resulting practical conclusions coming from the above performed experiments is that

- the dominating error factor in "blind" LS fitting is ADC overloading. ML fit is much better for any, even very small, ADC overloading and it does not require to preprocess the record and eliminate overloading samples. LS fit can give nearly the same results but requires excluding the overloaded samples from the record,

- even with pre-processed data used for LS fit, the error of the ML fit is somewhat smaller than that of the LS fit (Fig. 6).

The next study was focused on the accuracy of recovered stimulus for different resolutions of ADC. The test conditions were that ADC under test was not overloaded, $\sigma = 1$ LSB and the length of record $N = 2^{b+1}$, where b is the ADC resolution in bits. The achieved results – estimation errors (Eq. 4) are shown in Figs. 7 and 8.

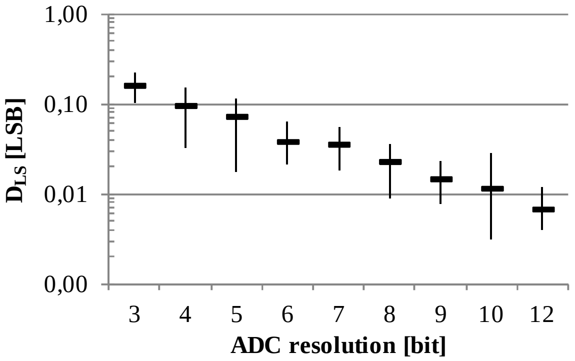

Fig. 7. Dependence of estimation error on number of ADC bits for LS fit

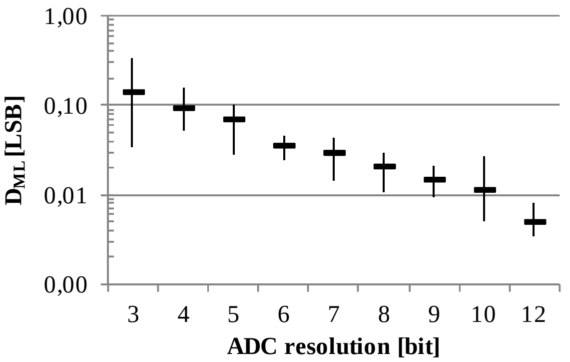

Fig. 8. Dependence of estimation error on number of ADC bits for ML fit

The achieved results show that both methods give very similar and very good estimations. ML fit gives a little smaller estimation error (D) but the differences are in order of the uncertainties of results in repeated experiments (see the variances in the figures).

The next experiment was focused on dependence of estimation error D on number of samples in the record with coherent sampling. Fig. 9 and Fig. 10 show this dependence for 8-bits ADC and $\sigma = 0.5$ LSB.

Both errors are very small and decrease with increasing number of samples. Again as in the previous experiments, the variance of the results is in the same order as uncertainties for repeated experiments. Fig. 11 presents the difference of errors' mean value between LS fit and ML fit.

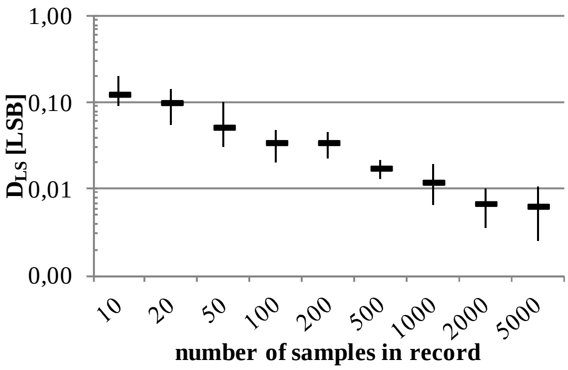

Fig. 9. Dependence of estimation error on number of samples for 8 bit ADC for LS fit.

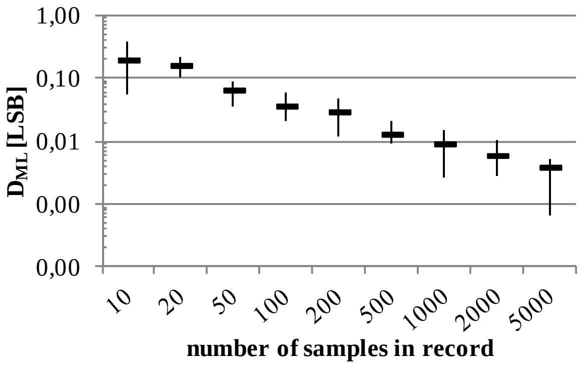

Fig. 10. Dependence of estimation error on number of samples for 8 bit ADC for ML fit.

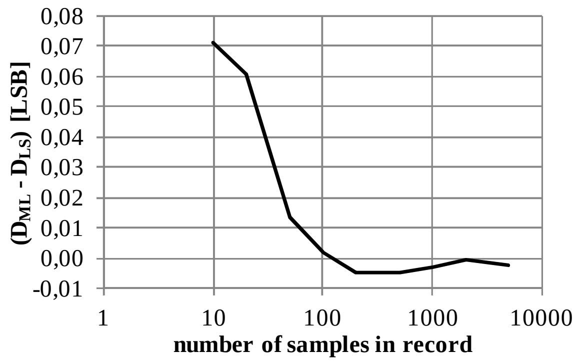

Fig. 11. Difference of error of ML and LS fits in dependence on number of samples in record.

The following experiments were focused on influence of the ADC noise that causes alternating of ADC output codes on estimation error. The results achieved for 8 bits ADC and 400 samples in record are shown in Fig. 12 and Fig. 13.

ML fit is better for low noise and with increasing noise both fits give nearly the same estimation error. This can be explained by noticing that the input noise acts like dither, thus quantization noise is becoming more and more independent on the signal for larger nose amplitude. Uncertainties of results for repeated experiments are also comparable with mean values of the error as it was in previous cases.

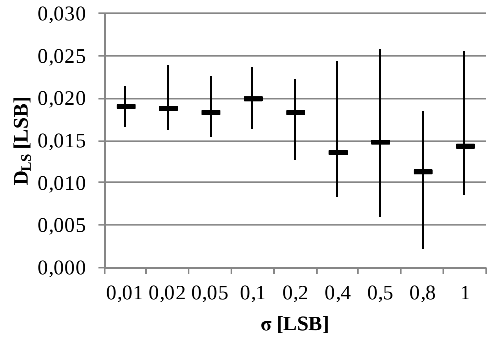

Fig. 12. Dependence of LS fit error on noise level.

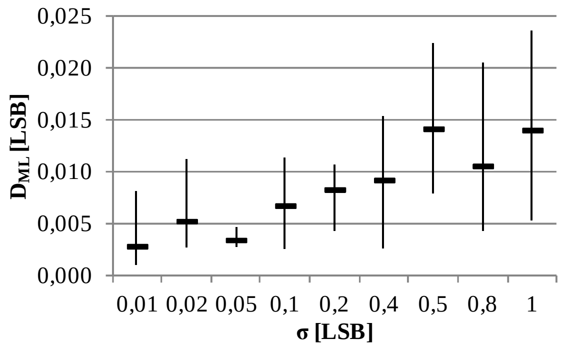

Fig. 13. Dependence of ML fit error on noise level.

The main goal of this comparative study was to qualify the effects of different fitting methods on precision of calculation of SINAD. To qualify the error, $SINAD_{REF}$ was calculated from precisely known stimulating sine wave and quantisation noise achieved as a difference of record and precisely known stimulating sine wave by Eq. 5. Accordingly, $SINAD_{LS}$ and $SINAD_{ML}$ were calculated from sine wave recovered by LS and ML fit, respectively.

Fig. 14 and Fig. 15 show difference of $SINAD_{LS}$ and $SINAD_{ML}$ from $SINAD_{REF}$, respectively, as well as variance of repeated testing as a dependence on ADC noise σ. The results were achieved for linear 8 bit ADC and the record length N=400 samples taken by coherent sampling.

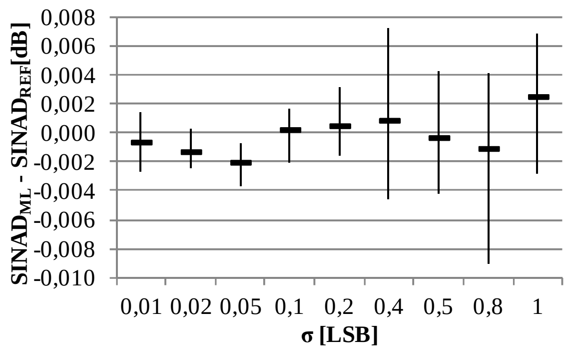

Fig. 14. Error of SINAD calculated from ML fit for different variance ADC noises. The SINAD error is larger for larger sigma partly because SINAD is measured with variance

The errors of SINAD calculated from both fits are very small and very similar. Increasing noise causes increasing of variance in results but this effect is also very similar for both methods.

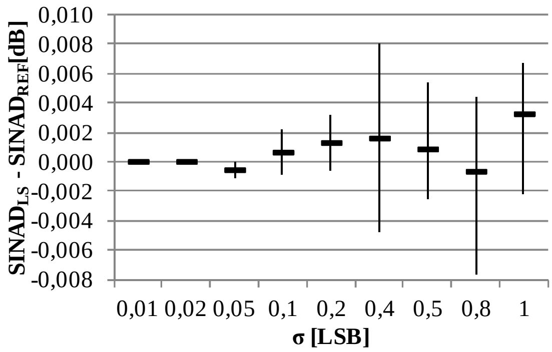

Fig. 15. Error of SINAD calculated from LS fit for different variance ADC noises.

Fig. 16 and Fig. 17 show again results achieved for the same uniform ADC and for different numbers of samples in a coherent record.

Errors for both fits are again very similar as it was in previous experiments. According to expectations the error decreases with increasing number of samples. This decreasing seems to be a bit faster for ML but this effect is very small.

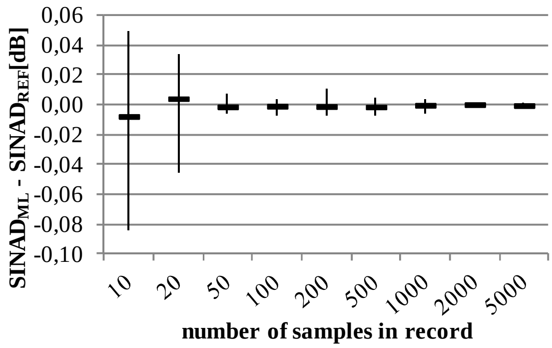

Fig. 16. Error of SINAD calculated from ML fit for linear ADC and different number of samples in coherent record.

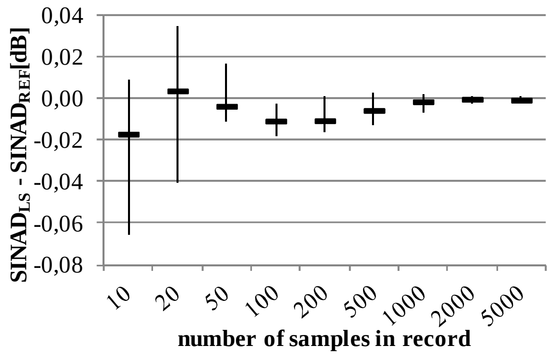

Fig. 17. Error of SINAD calculated from LS fit for linear ADC and different number of samples in coherent record.

Tests in the second group of evaluations were focused on effects on ADC nonlinearity at recovering the testing signals by LS and ML fits. To approximate as much as possible a real ADC in simulations, the INL measured on the real ADC in USB 6009 by National instruments was implemented in the simulation software. The real INL was simplified by rounding the lowest bits, in order to be equivalent to INL of a 8 bits ADC as it is shown in Fig 18. Approximate maximum likelihood estimates of the transition levels were determined from the sample record via the histogram method [1], applied to the available 1024 samples, with the standard correction for the sine wave. In our view, this is the maximum information that can be extracted from the record for the T(k) values. During the fit, these estimates were kept constant, and the other 5 parameters were optimized.

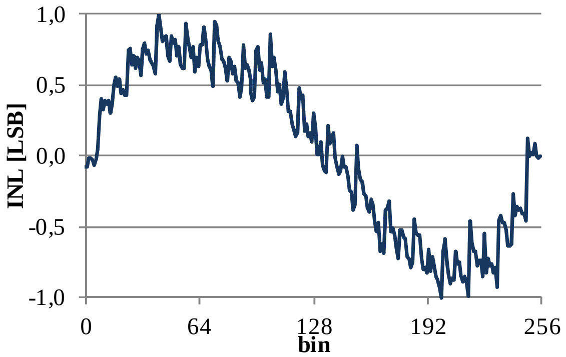

Fig. 18. Simplified INL of real ADC implemented in following simulation

Fig. 19 - 20 shows results of repetitive test achieved for different noise. ML fits is less sensitive on noise and the difference of calculated SINAD from reference one even decreases with increasing noise. On the other hand the variance of results from ML fit is higher than that from LS fit for high σ. Anyway the differences are small in order of 1-2 dB.

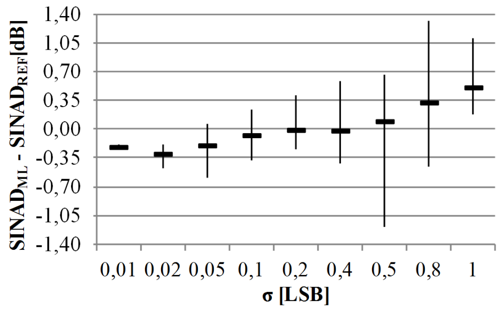

Fig. 19. Error of SINAD calculated from ML fit for different variance ADC noises.

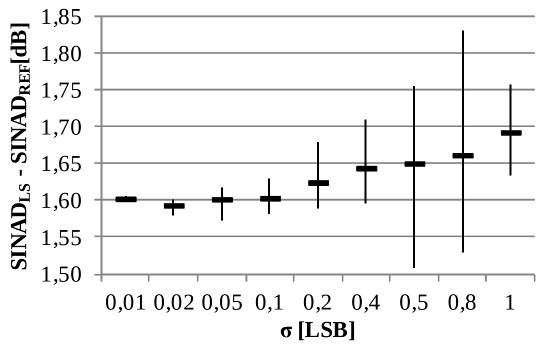

Fig. 20. Error of SINAD calculated from LS fit for different variance ADC noises.

4. CONCLUSIONS

In this paper a comparative experimental study of LS and ML fits for ADC testing was carried out. Two main aspects were investigated: (i) effectiveness and precision of sinewave fit from record if ADC input range is overloaded and (ii) influence of fitting method on SINAD testing. The achieved results show that both methods give similar results for both studied conditions and applications. For overloading ADC ML fit may be directly applied for whole record while using LS fit the record must be first pre-processed by excluding the overloading samples. ML is not dramatically better, but it is better.

For SINAD testing where ADC input is not overloaded by the test signal, both fit methods give reasonable results. The differences are in the order of 1-2 dB for LS, and 0.1 - 0.3 dB for ML.

Using the numerical minimization method based on the DE algorithm, the process of ML fit can be simply implemented in LabVIEW using built-in function `GlobalOptimization.vi`. For 1024 samples, the run time of the DE method was about 10 seconds on a standard PC, and a few seconds for the gradient-based method.

ACKNOWLEDGEMENT

The work is a part of the project supported by the Science Grant Agency of the Slovak Republic (No. 1/0555/11).

This work was supported by the Agency of the Ministry of Education of the Slovak Republic for the Structural Funds of the EU under the project Development of Centre of Information and Communication Technologies for Knowledge Systems (project number: 26220120030).

5. REFERENCES

[1] IEEE Standard 1241-2011, “Standard for Terminology and Test Methods for Analog-to-Digital Converters”, The Institute of Electrical and Electronics Engineers.

[2] IEEE Standard 1057-2007, “Standard for Digitizing Waveform Recorders”, The Institute of Electrical and Electronics Engineers.

[3] D. Dallet and J. M. da Silva: “Dynamic Characterisation of Analogue-to-Digital converters.” Springer 2005, ISBN-13 978-0-387-25902-4

[4] J. J. Blair: “Sine-fitting software for IEEE standards 1057 and 1241”, in Proc. of the 16th IEEE Instrumentation and Measurement Technology Conference, IMTC/99, Venice, Italy, May 1999, vol. 3, pp. 1504-1506.

[5] I. Kollár and J. J. Blair: “Improved Determination of the Best Fitting Sine Wave in ADC Testing”, IEEE Trans. on Instrumentation and Measurement, Vol. 54 No. 5, pp. 1978-1983, 2005.

[6] L. Balogh, B. Fodor, A. Sárhegyi, I. Kollár: “Maximum Likelihood Estimation of ADC Parameters from Sine Wave Test Data”, In: Proc. of 15th IMEKO TC4 Symposium on Novelties in Electrical Measurements and Instrumentation: 12th Workshop on ADC Modelling and Testing. Iasi, Romania, 2007. (IMEKO), pp. 85-90.

[7] L. Balogh, I. Kollár, L. Michaeli, J. Šaliga, J. Lipták: „Full Information from Measured ADC Test Data using Maximum Likelihood Estimation”. MEASUREMENT 45:(2) pp. 164-169. (2012)

[8] LabVIEW 2011 built-in help/guide

[9] Differential Evolution home page http://www.icsi.berkeley.edu/~storn/code.html