Filter design for GOCE gravity gradients

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The GOCE satellite observes gravity gradients with unprecedented accuracy and resolution. The GOCE observations are reliable within a well-defined measurement bandwidth. In this study different finite and infinite impulse response filters have been designed to obtain the demanded pass. Exhaustive time and frequency domain investigations prove that the proposed infinite impulse response filter can be a real competitor of the existing solution of the filtering problem.

Keywords: gravity gradiometry, GOCE, MBW, band pass filtering

1. Introduction

Several applications in geoscience deal with global scale phenomena, which are described conveniently in an Earth-fixed global reference frame. The horizontal and vertical directions in such a system are described with respect to an equipotential surface. One surface among these equipotential surfaces is called the geoid, which defines the niveau level surface in global sense. Generally, adequate determination of the geoid in global scale is of prime importance for geosciences.

Appropriate tools of that are gravity satellites, which are providing us gradually improving accuracy with increasing spatial resolution in the most recent decades. The most up to date gravity satellite is the GOCE, the first space gradiometric satellite so far, operated by the European Space Agency (European Space Agency, 1999, Drinkwater *et al.*, 2003). The GOCE gradiometer is located in the centre of mass of the GOCE satellite, which is the space-borne equivalent of the torsion balance, delivering spatial variations of the gravity field, i.e. gravity gradients.

Technically, the GOCE gravity gradiometer consists of three mutually perpendicular axes, with an arm length of half a meter. At the ends of each arms of the gradiometer, there is an accelerometer; (for more details see Rummel, 2002, Rummel *et al.*, 2002, 2011]. Each accelerometer observes the local gravitation by continuously measuring the reference voltage on its capacitive sensors. By differentiating the different combinations of the observed accelerations, the whole gradient tensor can be observed.

A technical barrier of the GOCE instrumentation is that the measurements are reliable only within a certain bandwidth that is the measurement bandwidth (abbr. MBW). In the case of the GOCE gradiometer it is 5-100 mHz. Basically, there is a demand for users of the GOCE gradient data to determine optimal filtering methods for the observed gravity gradients. In the first part of this study, appropriate filters are designed to filter GOCE gravity gradient to remain strictly in the MBW.

2. Filter design

This section introduces the filters which could alternatively be used for spectral filtering of GOCE signals. The specification was established based on the literature (Schuh, 2010), but some general filter design considerations are added. The filter in Schuh (2010) has been developed at the University of Bonn, and later on will be referred as UniBonn filter. The magnitude response of the band-pass filter to be designed can be specified as it can be seen in Table I.

All the filters have been designed using the Matlab software (Matlab, 2007). The magnitude responses of the filters, including the UniBonn filter can be seen in Figure 1.

Vertical dashed lines indicate the cut-off frequencies of the band-pass filters. These figures illustrate the main features of the resulted filters, a detailed comparison will be given later.

The first filter has been designed by the windowing method. The procedure is supported in Matlab by the `fir1.m` function. The filter has 1501 coefficients and the Hamming window was used (Parks, 1987). The main advantage of this filter is that it has slightly shorter impulse response than the UniBonn filter, which has 2001 coefficients. The shape of the magnitude response can be seen in Figure 1(b).

The second filter tried to achieve equiripple behaviour in all the bands, therefore Chebyshev-approximation using the Remes algorithm was applied (Parks, 1987). The procedure is supported in Matlab by the `firpm.m` function. The algorithm guarantees that the maximum of the approximation error is minimal in all the bands. However, the design of a band-pass filter with such a narrow stop-band near to zero frequency leads to unstable iteration in Matlab. To step over this problem, the filter has been designed in such a way that a high-pass and a low-pass filter are cascaded. These filters could be designed separately, and the iteration was stable. The low-pass filter has 351 coefficients, while the high-pass has 2001. The shape of the magnitude response can be seen in Figure 1(c).

Although the second filter has better magnitude response than the UniBonn filter in a sense, because of its higher number of the coefficients (2352 altogether) was still subject of development. It is well known that IIR filters require much less coefficients than the FIR ones. However, their phase response is nonlinear, which prohibits their application for the filtering of GOCE signals. Now it can be employed that the GOCE signals are present in records of finite samples, no continuous calculation of the filter output is required. It allows the calculation of the filter response in a normal way, but the record can be filtered in the opposite direction, as well. In the latter case the magnitude response of the filter is the same, while the phase shift has opposite sign. If the record is filtered first in a normal way, then the output is filtered again backwards, the resulted output has a magnitude response which is the square of the original one accompanied by zero phase shift. Zero phase shift is a special linear phase, therefore the shape of the useful signal is preserved. Matlab supports this procedure by the `filtfilt.m` function (Matlab, 2007). Therefore an IIR filter has been designed using elliptic approximation. This type of approximation results also in equiripple magnitude response. The corresponding Matlab function is `ellip.m`. Here a cascade of a 5-order low pass and a 9-order high pass filter is designed. The shape of the magnitude response can be seen in Figure 1(d).

Figure 2 shows the magnitude response of the filters in one diagram. Here already the square of the magnitude response of the IIR filter is depicted. It can be seen that all the filters satisfy the specification, at least in the 100 mHz ... 500 mHz range. For the detailed comparison the following figures are to be investigated.

In Figure 3 the magnitude response of the filters in the 0...5 mHz range is displayed.

Each filter response can be recognized by the legend. The vertical dashed line denotes the cut-off frequency, as in Figure 1. The UniBonn filter already provides some suppression in the pass-band, while in the case of the other filters the pass-band has been remained untouched. On the other hand, the transition band of the UniBonn filter is narrower. Considering this range, only the IIR filter outperforms the UniBonn filter.

Figure 4 shows the magnitude response of the filters at the lower cut-off frequency. The notations are the same as they are in Figure 3.

Due to the Gibbs oscillation, the UniBonn filter's response has a meaningful deviation near to the cut-off frequency. The alternative filters' responses deviate within a ±0.02 dB range. A similar phenomenon can be observed in Figure 5, where the magnitude response of the filters at the higher cut-off frequency is displayed. The filters' behaviour is similar to those at the other edge of the pass-band.

3. Filtering of GOCE gravity gradients

Finally, we apply the different filters on actual GOCE gradient data. One day of observations, 02.11.2009 has been used for the tests. The observed gradients are presented on figure 6. These gradients are in the so-called Gradiometer Reference Frame (GRF), which is defined by three perpendicular axes related to the arms of the gradiometer. As it is obvious, the different gradients provide notably different characteristics: some have large bias and linear trend, others not, and also the magnitude of the signals differ.

Figure 7 shows the filtered signals. A thorough visual test of the filtered data shows that the filters provide pretty much the same result in the time domain. Statistically it is confirmed by the comparison of the signal RMS of the filtered gradients with the RMS of the residuals due to the different filtering methods (error RMS). The signal RMS (Table II) is found to be notably, at least three times larger than the error RMS (Table III).

Certainly, there must be further useful information on the gravity outside of the MBW, however this frequency band and its probable noise characteristics needs further investigations to be determined. As soon as the need of extension of the MBW is clearly defined, the above requirements on the filter can be redefined.

4. Summary

In the study, appropriate filters have been designed for GOCE gravity gradients. Generally, all proposed filters perform well. The IIR filter has a somewhat more favourable magnitude response than the UniBonn filter. Furthermore, the combined FIR filter has uniform ripple in all the bands. Further aspect of comparison is the computational efficiency, meaning that how many coefficients are required to satisfy the specification. The UniBonn, the windowed and the combined FIR filters have 2001, 1501, and 2352 coefficients, respectively. The IIR filter has 30 coefficients altogether. Considering both the magnitude response and the computational complexity of the filters, the proposed IIR filter can be a real competitor of the UniBonn filter.

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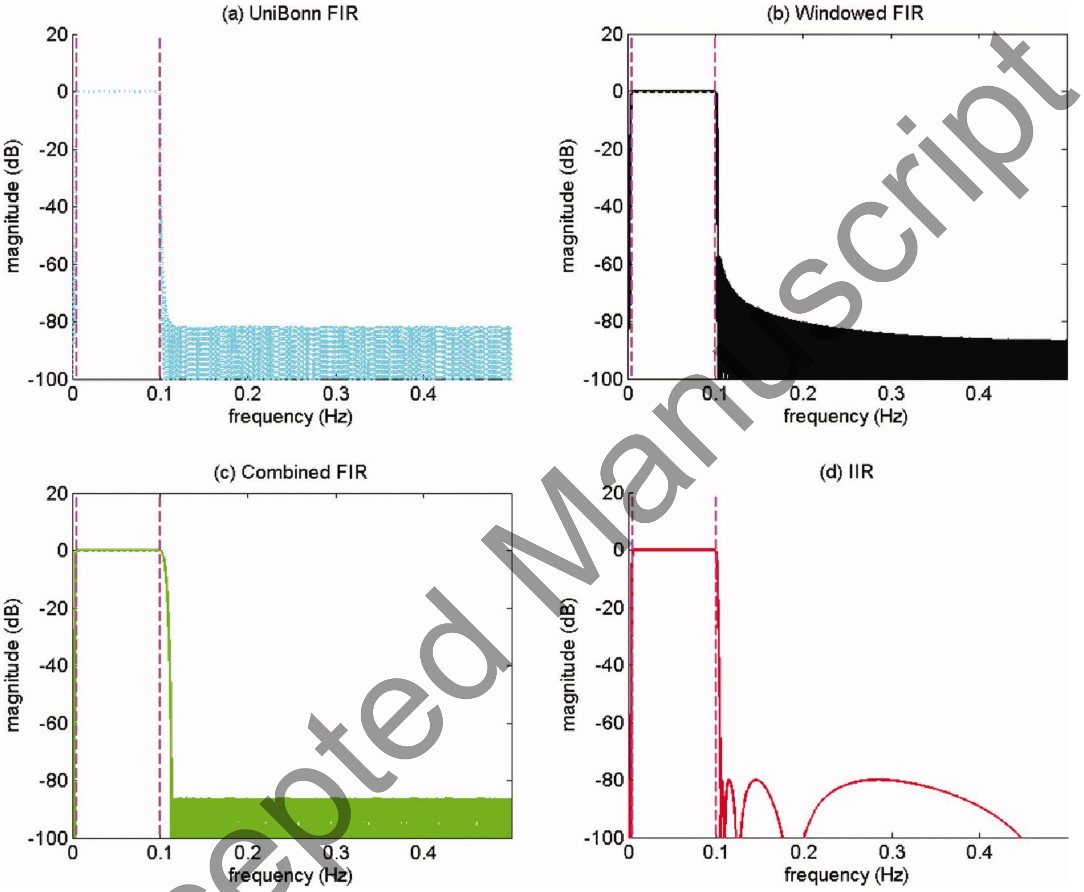

Figure 1. Magnitude response of the filters. (a) UniBonn FIR filter; (b) FIR filter designed by the windowing method; (c) combination of two FIR filters; (d) elliptic IIR filter.

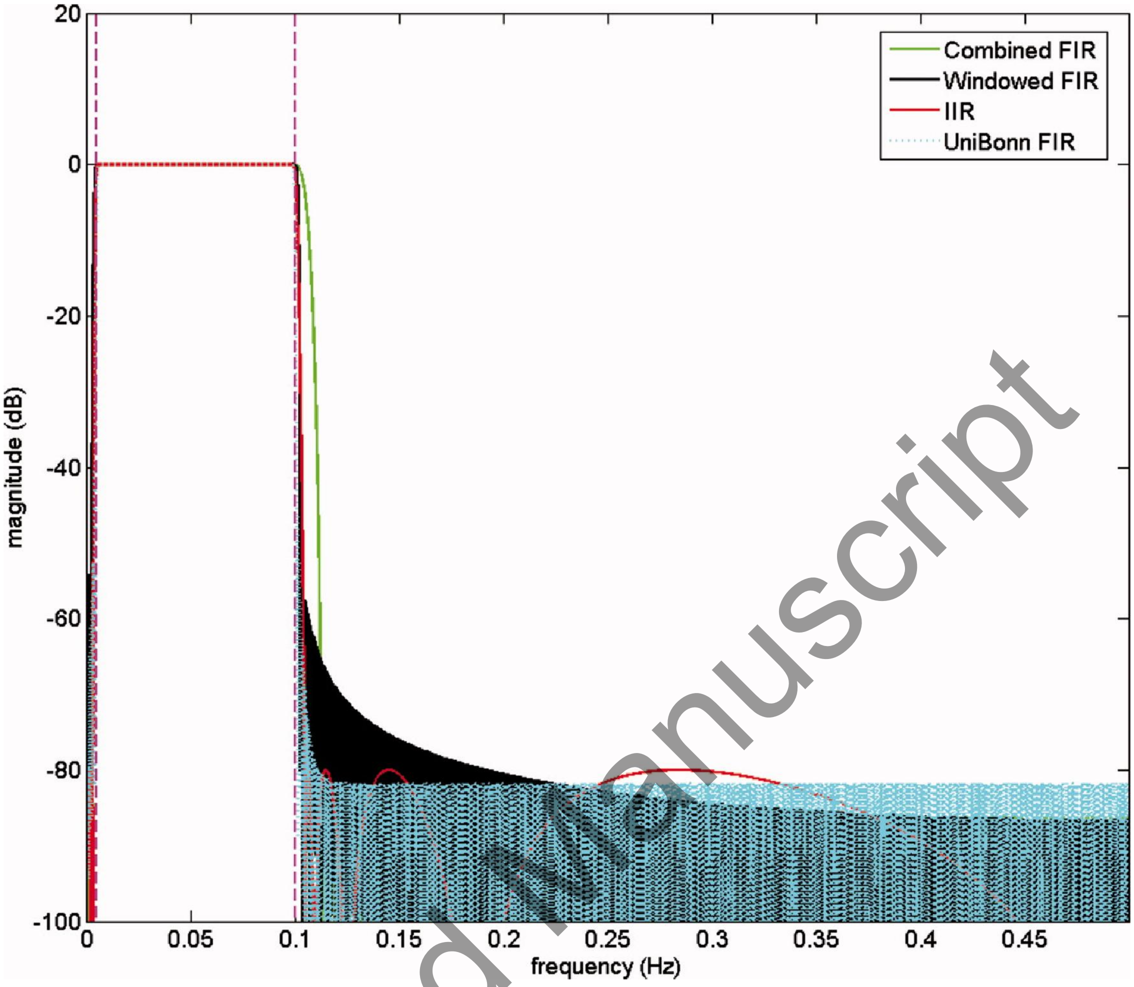

Figure 2. Magnitude response of the filters in one diagramm.

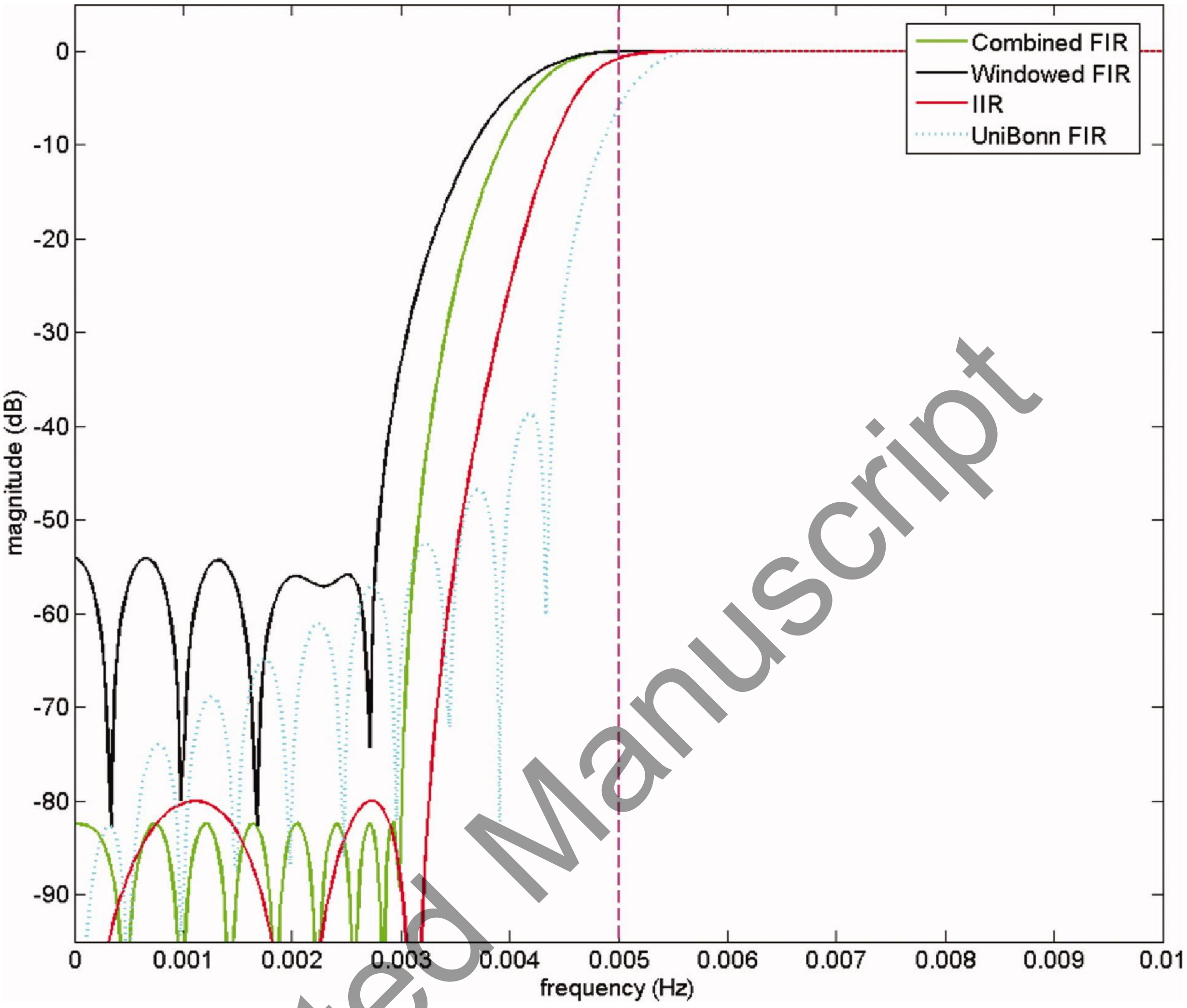

Figure 3. Magnitude response of the filters in the 0...5 mHz range.

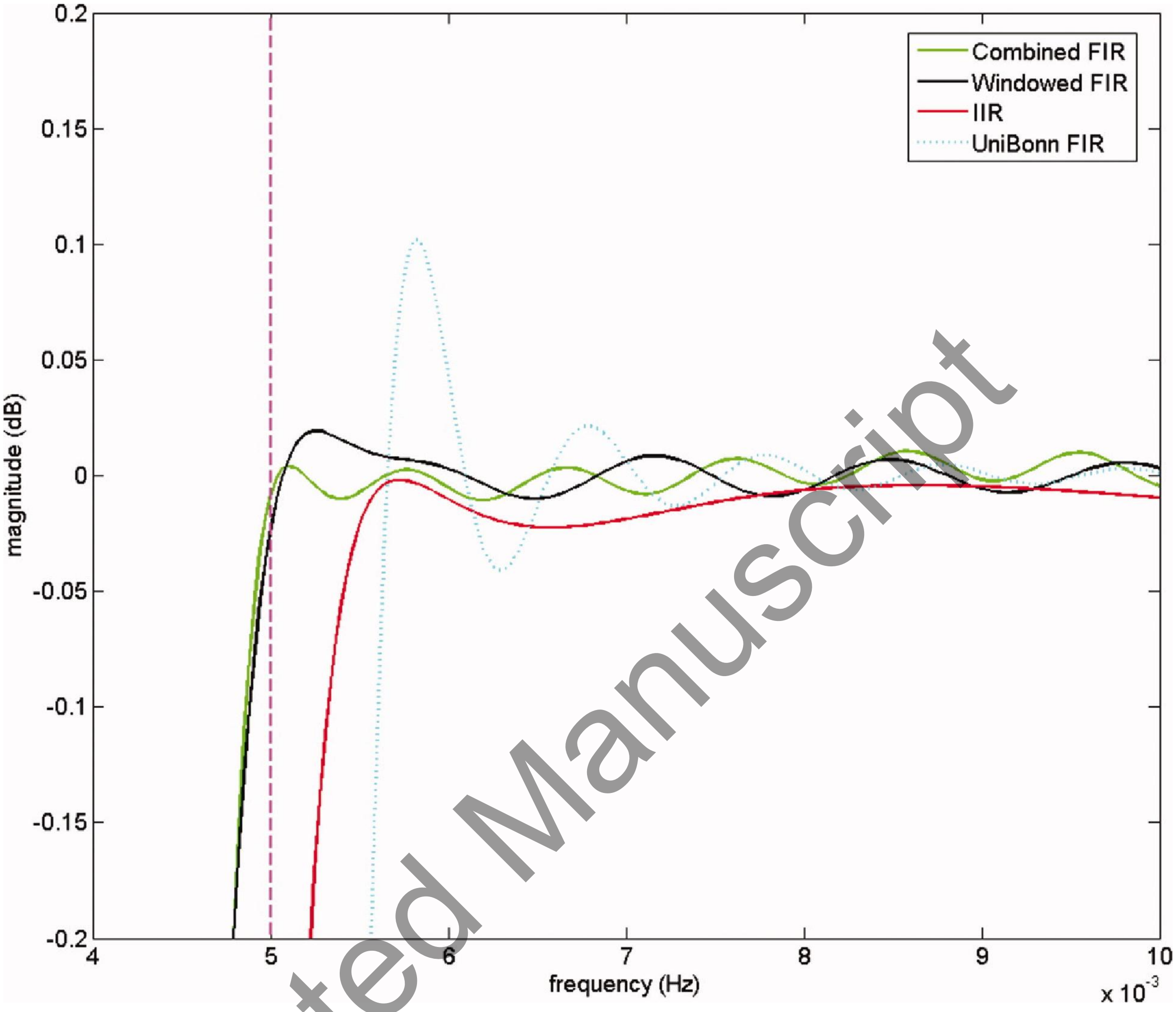

Figure 4. Magnitude response of the filters at the lower cut-off frequency.

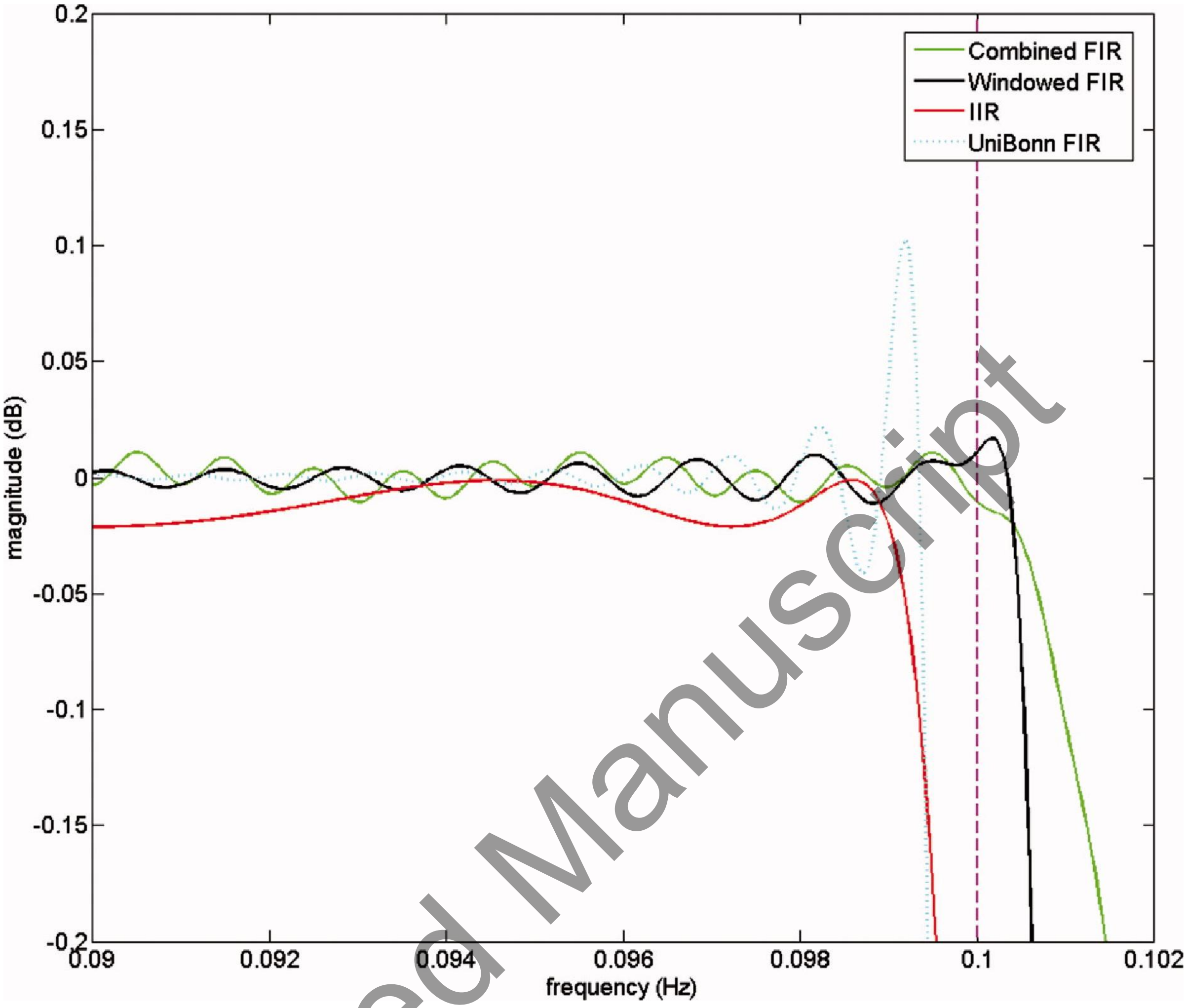

Figure 5. Magnitude response of the filters at the higher cut-off frequency.

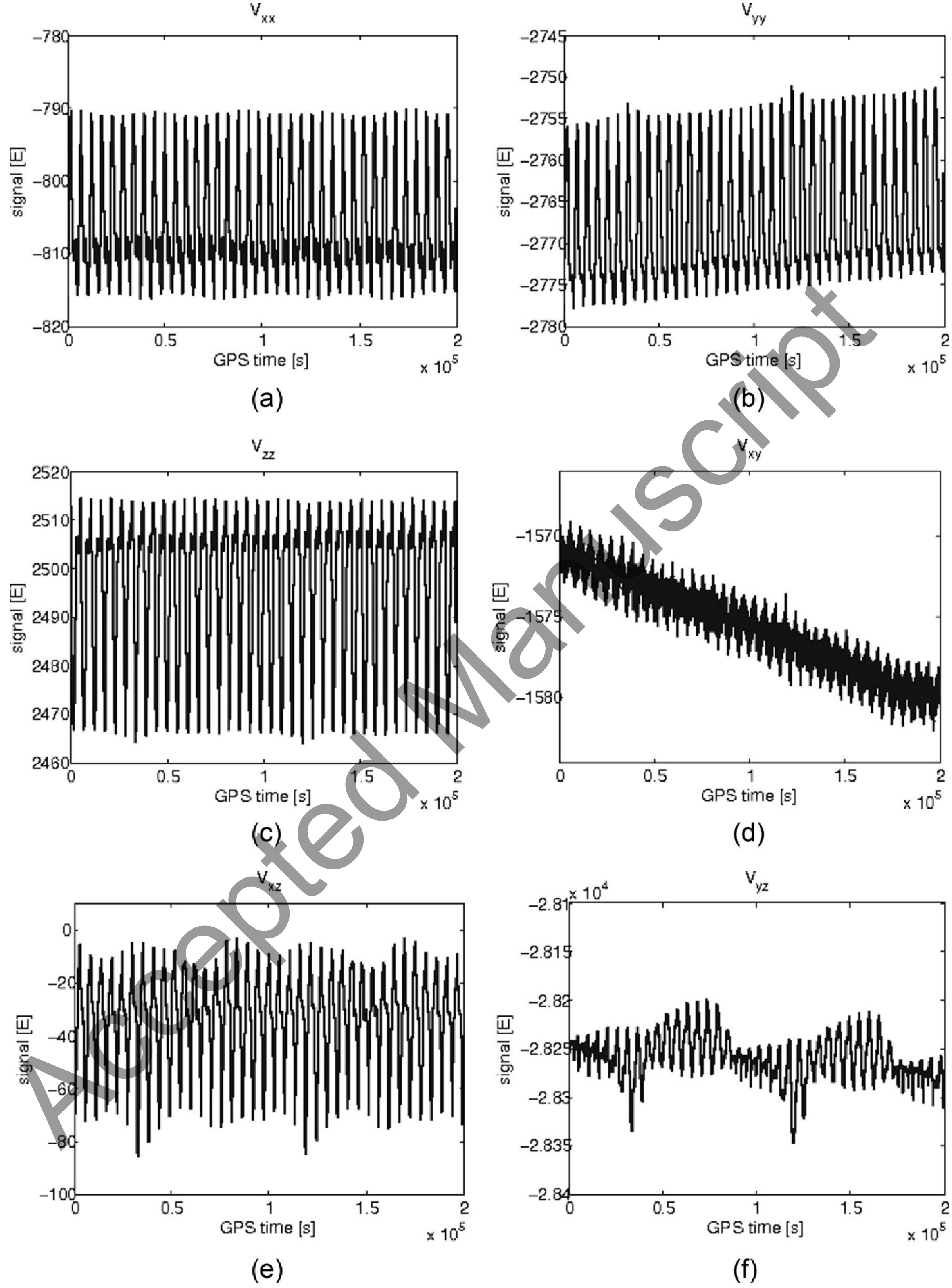

Figure 6. Observed GOCE gravity gradients on 02.11.2009; unit: 1 E = 10^{-9} s^{-2}; (a) xx, (b) yy, (c) zz, (d) xy, (e) xz, (f) yz components.

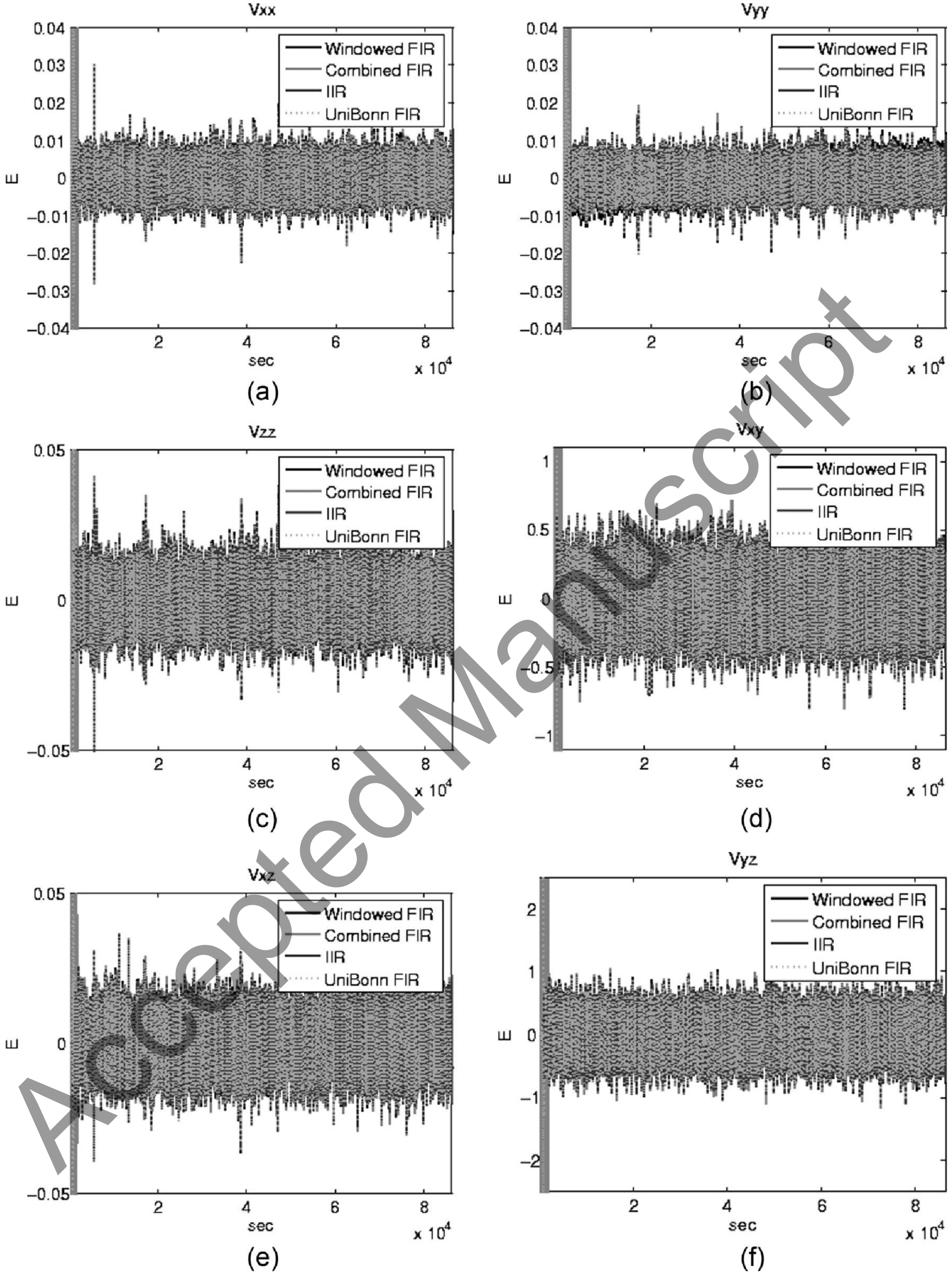

Figure 7. Filtered GOCE gravity gradients on 02.11.2009; (a) xx, (b) yy, (c) zz, (d) xy, (e) xz, (f) yz components.

Table I. Filter specification.

sampling frequency	Stop-band suppression 0...5 mHz	Stop-band suppression 100...500 mHz	Pass-band ripple 5...100 mHz
1 Hz	80 dB	80 dB	0.05 dB

Table.II. Signal RMS of the filtered gravity gradients. The unit is mE (1 mE = 10^{-12} s^{-2}).

	Windowed FIR	Combined FIR	IIR	UniBonn FIR
V_{xx}	3.7794	3.7711	3.6728	3.6504
V_{yy}	3.6761	3.5498	3.3968	3.3151
V_{zz}	6.9355	6.9529	6.7797	6.7243
V_{xy}	178.9669	181.3155	177.5242	176.8876
V_{xz}	7.0518	7.1309	6.9564	6.9221
V_{yz}	266.1894	270.489	263.7915	262.5403

Table.III. Error RMS of the filtered gravity gradients. The unit is mE (1 mE = 10^{-12} s^{-2}).

Error RMS [mE]	Windowed - Combined	IIR-Windowed	Combined-IIR	Windowed - UniBonn	Combined-UniBonn	IIR - UniBonn
V_{xx}	0.8788	0.7413	0.6882	0.9162	0.8745	0.3414
V_{yy}	1.1165	1.2016	0.7463	1.4536	1.1024	0.5184
V_{zz}	1.2228	1.0753	1.1704	1.5183	1.5976	0.6844
V_{xy}	27.6166	11.9821	29.8741	23.9000	37.5395	14.1748
V_{xz}	1.3003	0.8042	1.2780	1.2018	1.5984	0.6046
V_{yz}	45.8746	18.7293	48.4988	37.6899	60.3762	22.8707