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Spectral Observer with Reduced Information Demand

György Orosz, László Sujbert, Gábor Péceli
Department of Measurement and Information Systems, Budapest University of Technology and Economics
Magyar tudósok krt. 2., H-1521 Budapest, Hungary
Phone: +36 1 463-2057, fax: +36 1 463-4112, E-mail: orosz@mit.bme.hu

Abstract *– Spectral observer is appropriate for investigating the frequency domain properties of signals. Since the spectral analysis is a very common method, this observer can be used widely in many applications. The paper introduces a novel resonator based spectral observer and control algorithm that require reduced amount of data for the operation, so they are appropriate for the utilization in signal processing systems with limited resources (e.g. wireless sensor networks). The data reduction is achieved by the deployment of the sign error principle. For the improvement of the transient properties of the sign error structures a new method is proposed that tunes the speed of the adaptation according to the norm of the error. This algorithm makes possible to find a trade-off between the settling time and the amount of data required for the operation. The algorithm can be used for recursive Fourier analysis of signals and control of linear systems. As a practical application a wireless active noise control system is presented.*

Keywords *– adaptive signal processing, spectral observer, wireless sensor network, networked signal processing*

I. INTRODUCTION

The spectral observer [2] is an adequate structure for the processing of periodic signals. Due to its simple conformation it is easy to deploy on many processors [12]. Since frequency domain analysis is a very common task in the engineering, the spectral observer can be widely used in many applications e.g. in precision measurement technology. The basic observer can be extended to realize more complex algorithms, e.g. digital filters [13], adaptive Fourier-analysis [11]. It can also be used in control applications [7]. Due to the internal feedback it provides better properties (e.g. measurement noise suppression) than general methods. In this paper a modified resonator based spectral observer structure is introduced, which shows advantageous features in connection with the implementation of the algorithm in systems with limited resources.

The modified resonator based spectral observer structure is a kind of sign error adaptive algorithm. It is based on the relationship between the least mean square (LMS) [1] and the original resonator based observer algorithms [2][3], but utilizes the sign error LMS algorithm [4] for estimating the state variables of the observed system. Since the algorithm uses the signum of the error of the estimation, significant reduction in the computational demand and in the amount of required data can be achieved, so the utilization of this algorithm reduces design constraints in systems with limited resources. The proposed algorithm can be used e.g. in wireless feedback signal processing systems, where the bandwidth of communication channel can be a restriction factor in the case of the deployment of plenty of sensors.

The drawback of the sign error adaptive algorithms is their poor transient properties compared to the original algorithms [5], since in the adaptation the magnitude of the error is neglected. In the paper a new method is also introduced in order to resolve this problem. This algorithm offers the possibility for the tuning of the transient properties with the adjustment of the amount of data used for the adaptation in the algorithm, so makes possible to find a trade-off in the system design.

An application example for the proposed algorithms is an active noise control (ANC) system [8] that uses wireless sensor network (WSN) for noise sensing [6]. This is a straightforward field for the deployment of this algorithm, since ANC systems require lots of sensors and relatively high sampling frequency taking into account the typical bandwidth of the WSN's radio standards (e.g. ZigBee), so data reduction plays important role.

The paper is structured as follows. Section II summarizes the operation of the original spectral observer that serves as basis for the new algorithm. In Section III the sign error resonator based spectral observer is introduced that uses reduced amount of information for the operation compared to the original algorithm. A new method is also described that improves the properties of the sign error structure. For these algorithms some transient and steady state properties are also derived. In this section a sign error observer based controller algorithm is also described. In Section IV the practical application of the controller algorithm is introduced that is a wireless active noise control system.

II. REVIEW OF THE TRADITIONAL RESONATOR BASED OBSERVER

The resonator based observer was designed to follow the state variables of the so-called conceptual signal model [2]. The signal model is described as follows:

$$\mathbf{x}_{n+1} = \mathbf{x}_n\,; \qquad \mathbf{x}_n = [x_{i,n}]^{\mathrm{T}}, \tag{1}$$

$$y_n = \mathbf{c}_n \mathbf{x}_n = \sum_{i=-L}^{L} c_{i,n} \cdot x_{i,n}\,, \tag{2}$$

$$\mathbf{c}_n = [c_{i,n}];\; c_{i,n} = e^{j\cdot\omega_i\cdot n} = e^{j\cdot i\omega_1\cdot n}\,,\;\; i=-L \ldots L, \tag{3}$$

where $\mathbf{x}_n$ is the state vector of the signal model at time step n, y_n is its output (the input of the observer), $\mathbf{c}_n$ represents the basis functions.

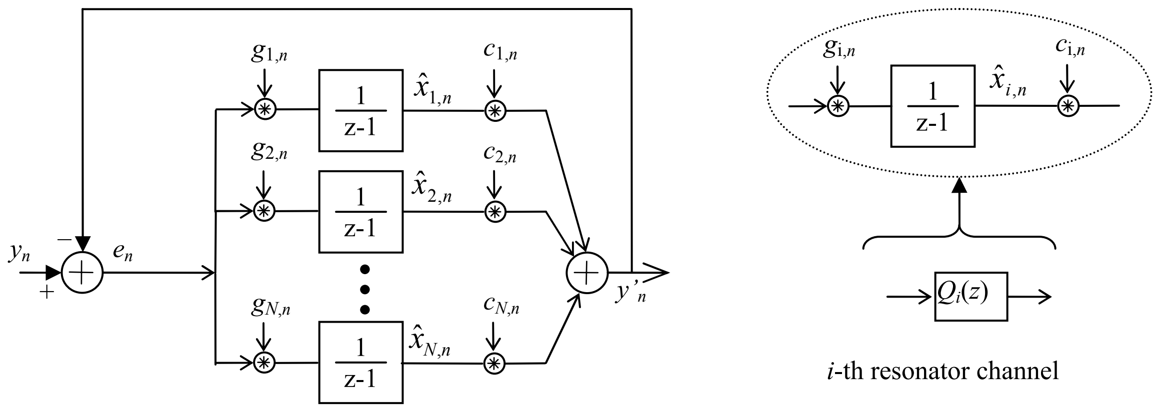

Figure 1. Basic configuration of the resonator based observer

To generate a real signal $\omega_{-i}=-\omega_i$ shall be satisfied. This restriction is not necessary, but advantageous in most cases. Obviously, in these cases the corresponding state variables shall form complex conjugate pairs. The conceptual signal model can be considered as a summed output of resonators which can generate any multisine with components up to the half of the sampling frequency. The corresponding observer can be seen in Fig. 1, and the system equation is the following:

$$\hat{\mathbf{x}}_{n+1}=\hat{\mathbf{x}}_n+\mathbf{g}_n\,(y_n-\mathbf{c}_n\cdot\hat{\mathbf{x}}_n)=\hat{\mathbf{x}}_n+\mathbf{g}_n\,e_n\,, \tag{4}$$

where $\mathbf{g}_n=[g_{i,n}]^{\mathrm{T}}=[r_i\,c^*_{i,n}]^{\mathrm{T}}$, $\{\hat{\mathbf{x}}_n=[\hat{x}_{i,n}]^{\mathrm{T}};\ i=1\ldots N;\ N=2L+1\}$ is the estimated state vector, $\{r_k;\ k=1\ldots N\}$ are free parameters to set the poles of the system, and * denotes the complex conjugate operator. N is the number of harmonic components. Due to the complex exponentials, the channels of the observer can be considered as time-invariant systems with a pole on the unit circle. This is why they are called resonators. If the resonator poles are arranged uniformly on the unit circle, and $\mathbf{g}_n=1/N\,\mathbf{c}_n^{\mathrm{H}}$ ($^{\mathrm{H}}$ denotes the conjugate transpose), the observer has finite impulse response, and the observer corresponds to the recursive discrete Fourier transform (RDFT) [2]. If the alignment of the resonators is not uniform, the settling is no longer deadbeat, but the system is still stable [11].

Due to the formal correspondence, (4) can be interpreted as the state variables were updated by the complex LMS algorithm, where the reference signal is $\mathbf{c}_n$. Using this relationship between the observer and LMS [3] in the proposed new observer structure the sign error LMS (SE-LMS) algorithm is used for updating the state variable $\hat{\mathbf{x}}_n$.

III. INTRODUCTION TO THE MODIFIED OBSERVER STRUCTURES

A. The simple sign error observer

The proposed modification of the original algorithm is, that in the updating process of the state variables only the signum of the observation error is utilized, and its magnitude is neglected. This can be called sign error structure, and it is depicted in Fig. 2, where $v=1$. The update procedure is the following:

$$\hat{\mathbf{x}}_{n+1}=\hat{\mathbf{x}}_n+\mathbf{g}_n\,\mathrm{sgn}(e_{\mathrm{n}})\,, \tag{5}$$

and

$$\mathbf{g}_n=[g_{k,n}]^{\mathrm{T}}=[\alpha\,c^*_{k,n}]^{\mathrm{T}}=\alpha\,\mathbf{c}_n^{\mathrm{H}}, \tag{6}$$

where $e_n=(y_n-y'_n)$ is the error of the estimation, and sgn(x) is the signum function:

$$\mathrm{sgn}(x)=\begin{cases}+1 \text{ if } x>0\\ \ \ 0 \text{ if } x=0\\ -1 \text{ if } x<0\end{cases} \tag{7}$$

This updating requires only the knowledge of the sign of the error, so it needs less computation than the original algorithm defined by (4), and the amount of data required for the operation is reduced. This is advantageous if it is implemented in systems with constrained resources. α is used for setting the transient and steady state behavior of the observer.

The steady state error of the observer can be determined by adapting the results in [4] for this structure:

$$E_a(n)=\frac{1}{n}\sum_{k=0}^{n-1}|e_k| \ \le\ \frac{\|\mathbf{x}\|^2}{2\alpha n}+\frac{N}{2}\alpha, \tag{8}$$

where E_a is the absolute mean error. (8) implies that if $n\to\infty$ (system is in steady state), the average absolute error is bounded by $N\alpha/2$ that is proportional to the convergence parameter α.

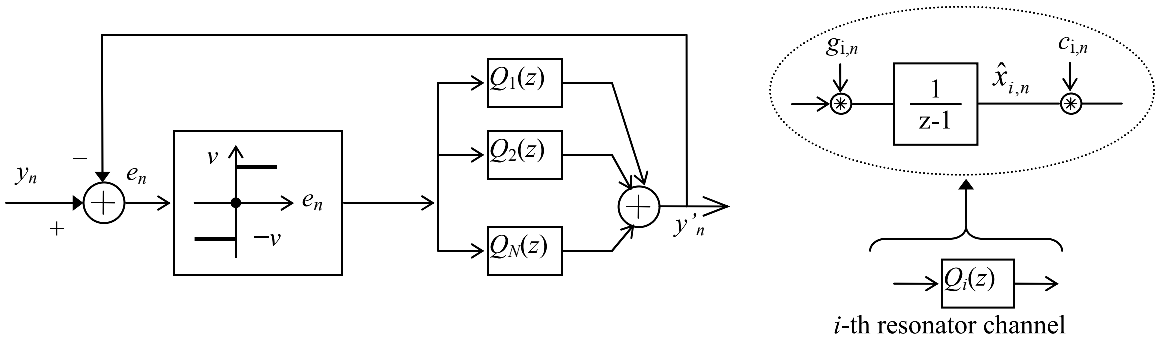

Figure 2. Basic configuration of the resonator based sign-error observer

The settling time M of the observer can be estimated by the recursive expansion of (5):

$$\hat{\mathbf{x}}_M = \sum_{j=0}^{M-1} \alpha \cdot \mathbf{c}_j^{\mathrm{H}} \cdot \mathrm{sgn}(e_j) + \hat{\mathbf{x}}_0. \tag{9}$$

Taking the absolute value, and assuming that the initial state $\hat{\mathbf{x}}_0 = 0$ we get:

$$\|\hat{\mathbf{x}}_M\| = \left\| \sum_{j=0}^{M-1} \alpha \cdot \mathbf{c}_n^{\mathrm{H}} \cdot \mathrm{sgn}(e_j) \right\| \le \sum_{j=0}^{M-1} \left\| \alpha \cdot \mathbf{c}_n^{\mathrm{H}} \cdot \mathrm{sgn}(e_j) \right\|, \tag{10}$$

and

$$\sum_{j=0}^{M-1} \left\| \alpha \cdot \mathbf{c}_n^{\mathrm{H}} \cdot \mathrm{sgn}(e_j) \right\| = \sum_{j=0}^{M-1} \alpha \cdot \left\| \mathbf{c}_n^{\mathrm{H}} \right\| = M\alpha\sqrt{N}. \tag{11}$$

From (10) and (11) with the assumption that $\hat{\mathbf{x}}_M \approx \mathbf{x}$ (the observer is in steady state at time instant M) the estimation of the settling time is:

$$M \ge \frac{\|\mathbf{x}\|}{\alpha\sqrt{N}}, \tag{12}$$

(8) and (12) pose contradictory conditions for the observer, namely for appropriate steady state error a low value for the parameter α is required, but this results in longer settling time. The following subsection introduces the improved version of the observer which ensures faster convergence with small steady state error.

B. The improved sign error observer structure

The proposed new method resolves the contradictory conditions for the settling time and steady state error with the adaptive tuning of the convergence parameter:

$$\beta = \alpha\, v = \alpha\, \|\mathbf{e}_m\|_1\ ; \quad \mathbf{e}_m = [e_m\ e_{m-1}\ \ldots\ e_{m-V+1}]^{\mathrm{T}}, \tag{13}$$

where β is the new convergence parameter. $v = \|\mathbf{e}_m\|_1$, $\mathbf{e}_m$ is a vector consisting of the last V values of the error signal at the time instant m when β is modified. $\|\cdot\|_1$ denotes the absolute norm. The updating algorithm is the following:

$$\hat{\mathbf{x}}_{n+1} = \hat{\mathbf{x}}_n + \alpha \cdot \mathbf{c}_n^{\mathrm{H}} \cdot \|\mathbf{e}_m\|_1 \cdot \mathrm{sgn}(e_n)\ ; \quad \mathbf{g}_n = \alpha\, \|\mathbf{e}_m\|_1 \mathbf{c}_n^{\mathrm{H}}, \tag{14}$$

It can be called normalized sign error spectral observer. If the value of the error signal is high, then $v = \|\mathbf{e}_m\|_1$ is also high, so the state variables are updated more radically (with larger steps), thus the convergence is faster. If the estimation error is low—the estimated and real value of $\mathbf{x}$ are near to each other—$\hat{\mathbf{x}}_n$ is updated with lower modifications so decreasing the error of the observation. These facts mean that the utilization of the norm of the error improves the behavior of the sign error observer. The frequently the parameter v is calculated the faster the convergence is. If $V = 1$, the original observer is obtained.

The optimal value of α in (13) and (14) can be calculated for the basic case when resonators are aligned uniformly and β is updated in each period of y_n. Practical applications apply approximately uniform resonator set [7][11]. An observer with N resonators corresponds to a periodic signal with period N. It means that V=N, and m=kN in (13), so $\mathbf{e}_m$=$\mathbf{e}_{kN}$=$[e_{kN} \ldots e_{kN-N+1}]$, since for uniformly aligned resonators the length of one period of the signal is N. Using these conditions an optimal value for α can be chosen in such a way, that it minimizes the ratio of the squared norm (power) of the error signal between two consecutive periods. The ratio of the error vectors' norm is: $\lambda = \left\|\mathbf{e}_{(k+1)N}\right\|^2 \big/ \left\|\mathbf{e}_{kN}\right\|^2$. α is optimal if the decay of the error is the fastest, i.e.:

$$\alpha_{\mathrm{opt}} = \frac{1}{N \cdot N_{\mathrm{NZ}}}, \tag{15}$$

where N_{NZ} is the number of nonzero elements of $\mathbf{e}_m$. In practice α_{opt} can be used as an initial value when the refreshing of the convergence factor is taken place with other period or resonators are placed unevenly.

The convergence speed of the algorithm depends on the properties of the observed signal. With the utilization of α_{opt} N step convergence can be achieved if all elements of the error signal $\mathbf{e}_{kN}$ in (13) have the same absolute value:

$$|e_i| = |e_j| : \forall i, j \in [kN..kN-N+1]. \tag{16}$$

In worst case the error signal is a periodic impulse: except for one dominant element of the period the other elements are nearly zero: $e_i \to 0$, but $|e_i|>0$ that is important in the calculation of $\mathrm{sgn}(e_i)$. In this case the ratio of the mean square values of consecutive error periods is:

$$\lambda = \left\|\mathbf{e}_{(k+1)N}\right\|^2 \big/ \left\|\mathbf{e}_{kN}\right\|^2 = \left(1 - \frac{1}{N}\right). \tag{17}$$

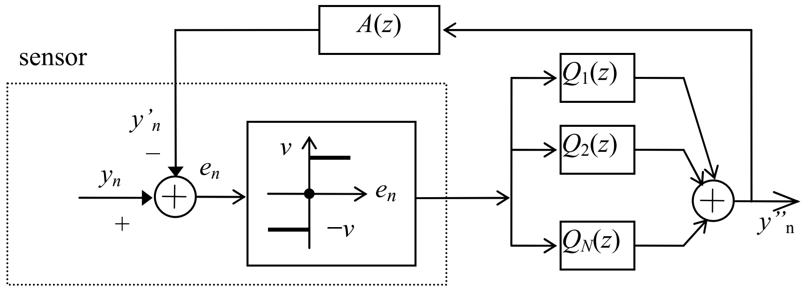

Figure 3. Adaptive control structure

Using this worst case value of the decay ratio a higher bound for the settling time can be given. Let M denote the number of periods during which the power of the error decreases to its ρ-th part. Using the above described conditions:

$$M \le -\frac{\lg(\rho)}{\lg(\lambda)}. \tag{18}$$

These facts mean, that the normalized sign error algorithm has generally worst transient property for signals with high crest factor.

C. The resonator based sign error controller

Sign error algorithms can be effectively used in the resonator based control algorithm [7]. This structure provides excellent behavior for periodic reference signal. The structure in Fig. 3 realizes an adaptive controller using the resonator based observer. A practical application will be shown in Section IV.

The plant to be controlled is $A(z)$. Since the loop gain at the resonator frequencies tends toward infinity, the steady state error is zero for periodic signals. This ensures, that the output of $A(z)$ tracks the periodic reference signal y_n. Note that the algorithm can also be used for constant y_n, since this is a special case of periodic signals with ω=0. The updating procedure is the following:

$$\hat{x}_{i,n+1} = \hat{x}_{i,n} + A^{-1}(z_i)\cdot\alpha\cdot c^*_{i,n}\cdot\|\mathbf{e}_m\|_1\cdot\mathrm{sgn}(e_n), \tag{19}$$

thus $\mathbf{g}_n = [g_{i,n}]^T = \alpha\cdot[A^{-1}(z_i)\cdot c^*_{i,n}]^T$. In (19) $z_i = \exp(j\cdot\omega_i)$ and $A^{-1}(z_i)$ denotes the inverse of the transfer function on the i-th resonator frequency. $A^{-1}(z_i)$ compensates the effect of $A(z)$ in the feedback path, so it ensures negative feedback with unity gain required for the stability [7]. $A(z)$ should be identified in advance. The stability and the appropriate operation of the system require the accurate identification of the plant. If the norm $\|\mathbf{e}_m\|_1$ is not used in (19), a simple sign error controller is obtained. This results in lower amount of data, but slower transients. The steady state error and the settling times can be influenced by the convergence parameter α.

The utilization of the sign error structure is advantageous in such control applications where the sensor and the controller are physically separated, and the sensor is connected to the controller via a low bandwidth data transmission channel. This is the situation when the control system is based on a wireless sensor network where the sensors are realized by the nodes of the networks, while the controller works in a control unit. If the error signal is known on the sensor, it can perform the calculation of the sign and the norm of the error. The error signal e_n can be measured directly by the sensor, or it can be calculated on the sensor if the reference signal y_n is known and the output of the plant (y'_n) is measured. Since the signum of e_n has three possible values $\{-1,0,1\}$, and the norm of the error is required for the algorithm only once in each V sampling interval long periods, the amount of the data to be transmitted from the sensor to the controller decreases significantly.

IV. TEST RESULTS OF THE RESONATOR BASED SIGN ERROR CONTROLLER

The practical test of the sign error observer was carried out in a test application [6] that is a wireless active noise control (ANC) system [8]. The main goal in the ANC systems is to suppress low frequency acoustic disturbances by means of the phenomenon of destructive interference. The noise to be suppressed is sensed by microphones, and the so-called anti noise is radiated by loudspeakers. Since the noise suppression is restricted to a limited surrounding of the microphones, in the ANC systems generally more sensors are used, which results in large amount of data to be transmitted from the sensors to the central data processing unit.

In our test system the microphone is situated on a ZigBee compatible wireless sensor node—a Crossbow micaz mote [9]—, the communication bandwidth of which is 250 kilobit per second.

Taking into account the minimal required sampling frequency that is in the range of 1-2 kHz, the deployment of a data compression method is inevitable, if more than 3 or 4 sensor have to be used in the system. The proposed controller offers the possibility for the data reduction.

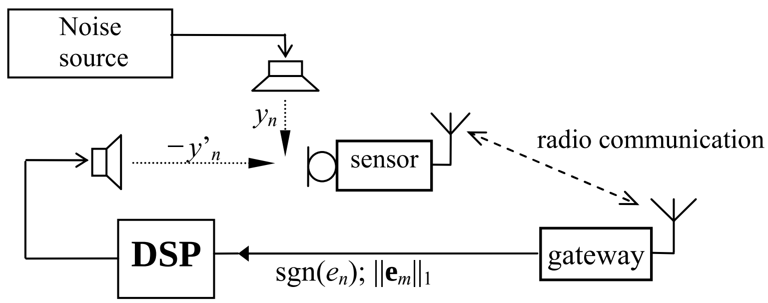

Figure 4. Configuration of the wireless ANC system

The structure of the test system can be seen in Fig 4. ANC systems can be regarded as control systems in which the plant to be controlled—$A(z)$ in Fig. 3—is an acoustic plant, the input of which is a loudspeaker, and the reference signal y_n is the noise to be suppressed. If the signal y_n is predicted correctly, the remaining noise e_n decreases, so reduction in the power of noise is achieved. The error signal e_n evolves as the superposition of the noise (y_n) and the inverted estimated noise ($-y'_n$), and it is sensed by the microphone. Since superposition of the sound waves is a sum of the signals, so the subtraction can be achieved with the multiplication of y'_n by −1 in the DSP. The microphone is situated on the wireless sensor node that samples the error signal (i.e. remaining noise). It performs the calculation of the signum function and the norm of the error signal and sends the data through a gateway to a DSP that implements the observer structure. In our system the DSP is an ADSP-21364 32 bit floating point processor from the Analog Devices [10]. Since in this case the error signal can be sensed directly, the signum and the norm can be calculated on the sensors. The sampling frequency of the error signal is 1.8 kHz.

In the test system the simple sign error, the normalized sign error and the original resonator based noise control algorithms were implemented and tested under similar conditions. Compared to the normal algorithm the amount of data to be transmitted from the sensor to the DSP was one eighth and one sixth in case of simple sign error and normalized sign error algorithms, respectively. The reason is that instead of the current value of the signal (that is converted by an 8 bit AD) only the sign of the error and the absolute norm of the error in V=32 sample long intervals were transmitted. Data reduction decreases the load of the radio network, so it makes possible either the expansion of the number of sensors with the same sampling frequency, or increasing the sampling frequency.

During the test an external noise y_n with the fundamental frequency of 95 Hz was radiated by an extra loudspeaker. y_n consisted of 5 harmonic components. The remaining noise e_n was measured directly at the noise sensing microphone. Measurement results of the realized systems can be seen in Fig. 5 and Fig. 6.

In steady state each algorithm produced for each of the five harmonics at least 20 dB suppression. The noise suppression is influenced by the precision of the AD conversion, and the ubiquitous external acoustic noise.

The algorithms differ from each other mainly in the transient performances. In the following measurements settling time is interpreted as a time interval during which the transient decreases under the level of the measurement noise. Fig. 6.a. shows the transient of the original algorithm.

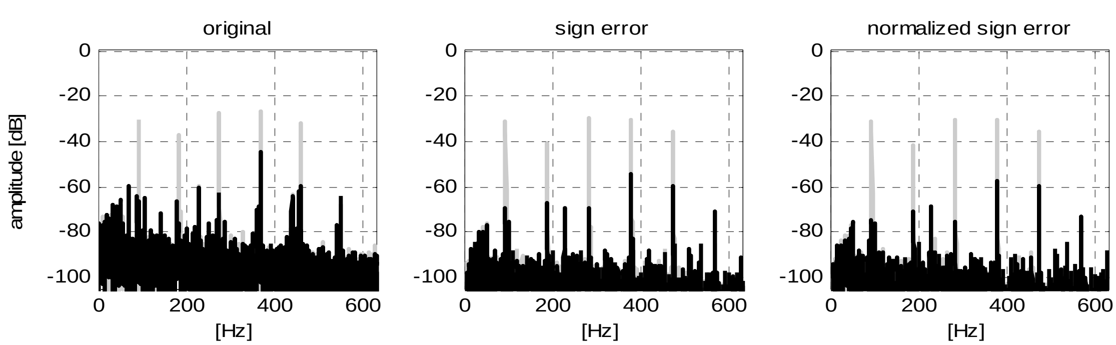

Figure 5.a. Figure 5.b. Figure 5.c.

Figure 5. Spectra of the steady state error signal of the resonator based control systems. Black lines: controller is on; grey lines: controller is off
a: original controller, b: simple sign error controller, c: normalized sign error controller

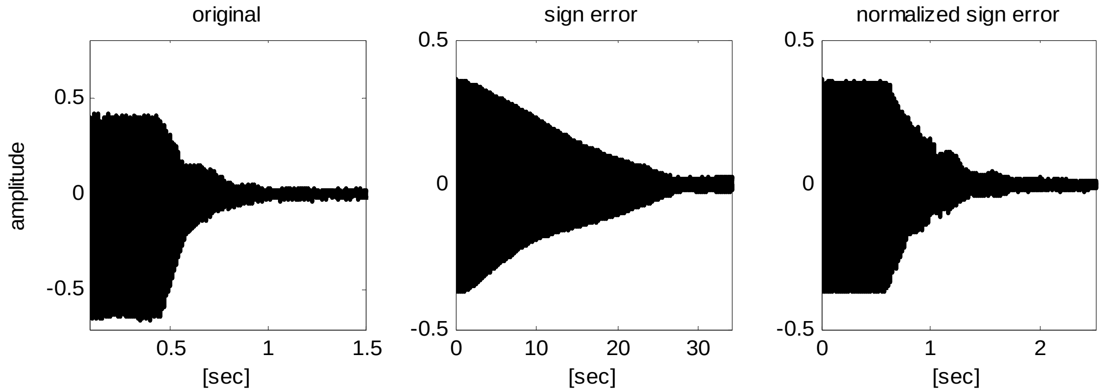

Figure 6.a. Figure 6.b. Figure 6.c.

Figure 6. Transient properties of the resonator based control systems
a: original controller, b: simple sign error controller, c: normalized sign error controller

The settling time was 0.3 sec, and it was the fastest transient as it was expected, since in this system the magnitude of the error signal was used in each sampling time instant. Since this is the fastest transient it will be used as reference for the comparison of the other algorithms.

In Fig. 6.b. the transient of the simple sign error controller is shown. The settling time was 25 sec. The convergence parameter was set in such a way that the system provided appropriate steady state error. Since the updating of the state variables were performed with constant steps using only the signum of the error, the error converges fairly slowly to the steady state with nearly linear envelop.

According to Fig. 6.c. the settling time of the normalized sign error controller was 1 sec. However, this settling time is slower than that of the original algorithm, this is much better than the transient of the simple sign error observer, which was reached by the utilization of the norm of the error. This extra information increased the amount of data by 30% compared to the simple sign error algorithm, but resulted in significantly shorter settling time. This can be explained with the time interval between the transmissions of the norm of the error which was performed once in each 32 sampling interval. Since the sampling frequency was 1.8 kHz, it means, that the error norm was calculated for each T_n = 18 ms long interval. Since T_n is relatively small compared to the reference time that can be achieved at all with the original structure, so it characterises the signal appropriately during the transient.

V. CONCLUSIONS AND FUTURE PLANS

In this paper a sign error spectral observer and controller structure was introduced which can be effectively used in sensor networks where the bandwidth of the communication channel is limited. A method was also proposed for the adaptive tuning of the convergence factor in order to improve the transient and steady state properties of the algorithms. This so-called normalization was introduced in a resonator based observer, but it can also be used in other sign error adaptive algorithms (e.g. sign error LMS).

A control system utilizing the sign error and normalized sign error observers was also presented. The test application was a wireless active noise control system, and the results show that the algorithm can be applied successfully in these control systems. Although the settling time of the system increases to some extent, it is reasonable in certain applications.

In the future the expansion of the theory and their implementation to MIMO systems can be expected.

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