Comparison of LMS-based Adaptive Audio Filters

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***Abstract*—In the field of audio signal processing, logarithmic frequency resolution IIR filters, such as fixed-pole parallel filters and Kautz filters, are commonly used. These proven structures can efficiently approximate the frequency resolution of hearing, which is a highly desired property in audio applications. In recursive adaptive filtering however, the FIR structure with LMS algorithm is the most commonly used. Since the linear frequency resolution of FIR filters is less than ideal for audio applications, in this paper we explore the possibility of combining the logarithmic frequency resolution IIR filters with the LMS algorithm. To this end the LMS algorithm is applied to fixed-pole parallel and Kautz filters, and the resulting structures are compared against each other and to the FIR-LMS filters in terms of convergence time and remaining error.**

***Index Terms*—audio signal processing, LMS, fixed-pole parallel filters, Kautz filters**

I. Introduction

Infinite impulse response (IIR) filters are commonly used in audio signal processing [1], where logarithmic frequency resolution is highly desired when modeling a transfer function. To achieve this, specialized filter design methodologies have been developed, including warped filters [2], second-order fixed-pole parallel filters [3], and Kautz filters [4].

In adaptive filtering, finite impulse response (FIR) filter structures with least mean squares (LMS) method are popular choices. The reason for their popularity is their global convergence, however, they require more parameters to model a given response, as opposed to IIR filters. Another drawback is that their residual error (misadjustment) is related to the step-size coefficient (μ), and thus, a trade-off must be made between convergence time and residual error [5].

Common applications for adaptive audio filters, such as compensation, or noise reduction, contain an adaptive filter that identifies a given signal path. Thus, as a first step for comparing logarithmic frequency resolution IIR filters in adaptive context, this paper explores the identification capabilities of the different IIR structures using LMS algorithm.

In this paper, the LMS algorithm is applied to the parallel and Kautz filters, and the resulting adaptive IIR filters are compared to each other and to the common FIR-LMS filters.

II. The LMS Algorithm

The Least Mean Squares (LMS) algorithm is a stochastic grade descent method where the coefficients are adapted based on the current error in time [5]. It uses the estimate of the mean square error (MSE) gradient vector from the available data, to make successive corrections to the filter coefficients in the direction of the negative of the gradient vector. This iterative procedure eventually leads to minimum mean square error.

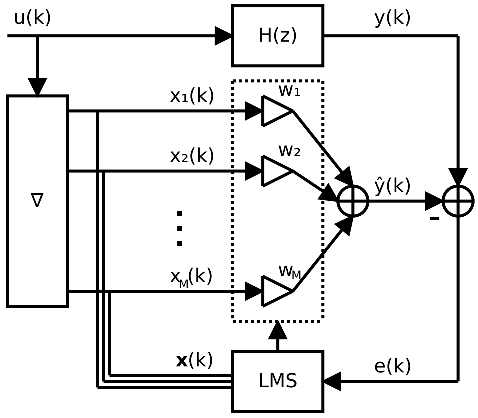

Fig. 1. LMS-based adaptive filter used for identification.

The block scheme of the LMS filter can be found in Fig. 1. The common input of the system to be identified and the adaptive filter is denoted by $u(k)$, and the outputs are marked by $y(k)$ and $\hat{y}(k)$ respectively.

The output of the adaptive filter is computed as

$$\hat{y}(k) = \mathbf{w}^\top(k)\mathbf{x}(k). \tag{1}$$

The recursive function for coefficient adaptation is the following:

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu e(k)\mathbf{x}(k), \tag{2}$$

where $\mathbf{w}$ denotes the filter coefficients, k is the discrete time, μ is the step-size parameter, $\mathbf{x}$ is the estimated gradient and e is the output error, where $e(k) = y(k) - \hat{y}(k)$.

The input vector $\mathbf{x}(k)$, which acts as the estimated gradient vector, is unique for every filter structure. For FIR filters, it is a delay line; for other structures it can be deduced using Equation 1.

Note that each element of $\mathbf{x}(k)$ is a function of time, and they span the space of the output function. Because they act as base functions, their correlation has an impact on the convergence time: the lower the eigenvalue spread of the correlation matrix $\mathbf{R}$, the faster the convergence [5].

The estimated autocorrelation matrix of $\mathbf{x}(k)$ is calculated as:

$$\hat{\mathbf{R}} = \frac{1}{L}\sum_{k=0}^{L}\mathbf{x}(k)\cdot\mathbf{x}^{\top}(k), \quad (3)$$

where L denotes the number of samples.

The main drawback of the LMS algorithm is that the gradient vector scales with the input, which can cause instability in the adaption. As a remedy, the Normalized-LMS (NLMS) method is used, which normalizes the power of the input [5]:

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu\frac{e(k)\mathbf{x}(k)}{\alpha + \mathbf{x}^{\top}(k)\mathbf{x}(k)}, \quad (4)$$

where α is a small positive number used to avoid the denominator to become zero.

In this paper we used the NLMS algorithm for realizing adaptive filters.

III. Adaptive IIR Filters

Adaptive IIR filters require fewer parameters compared to FIR filters, however, early research showed that adaptively varying both the poles and zeros can lead to suboptimal performance caused by multimodal error surfaces [6] or because they require satisfaction of a strict positive real condition [7].

Alternatively, the poles of the IIR filter can be fixed at predetermined values, which preserves the linearity in parameters and leads to well-behaved adaptation properties [8].

In audio signal processing, fixed-pole filters are commonly used. The Kautz (Fig. 3) and the fixed-pole parallel filters (Fig. 2) are proven to have equivalent transfer functions when designed off-line [3]. The main difference between them lies in the computational demand (see Table I): the fixed-pole parallel filter need approximately 47% less operations compared to the Kautz filter. The tap outputs of the two filters span the same space, but the base functions of the Kautz filter are orthonormal [9]. This results in convergence properties similar to that of FIR filters [8].

The general structure of the parallel second-order structure can be found in Fig. 2. The second-order sections can be implemented as either direct-form, or other structures [10]. Note that the structure of the second-order sections have direct impact on the parameters, and thus, affects the convergence properties if the second-order section is used in an adaptive filter realization.

Adapting the aforementioned fixed-pole audio filters using the LMS algorithm can be done by substituting the IIR filter to the ∇ block in Fig. 1, with the output multiplications and summation replaced by the adaptive linear combination of the LMS algorithm. For example, in case of the Kautz filter in Fig. 3 it means that the c_i coefficients are the tuned parameters.

IV. Orthogonal Second-order Section

In order to improve convergence, we present a new second-order structure (Fig. 4), which, to our knowledge, has not been presented before. The new structure is equivalent to a second-order Kautz filter, therefore its two tap outputs are orthogonal. As this structure is more complex than the direct-form implementation, its usage in parallel filters result in computational demand between the direct-form parallel filter and the Kautz filter.

TABLE I
Number of arithmetic operations requied for the tested adaptive IIR filters having N conjugate-complex pole pairs implemented using direct-form 2 (DF2) or orthogonal second-order sections.

	Multiplication	Addition
Fixed-pole parallel filter (DF2)	$6N$	$3N-1$
Fixed-pole parallel filter (orth.)	$6N$	$5N-1$
Kautz filter (DF2)	$9N+2$	$8N+1$

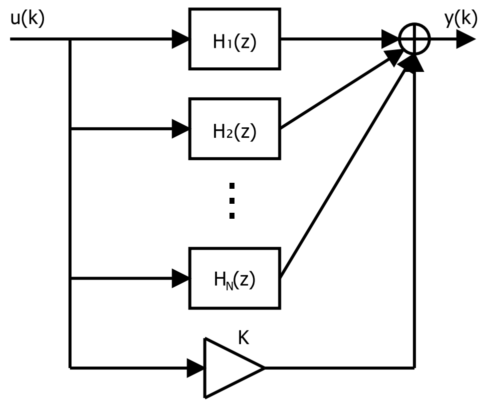

Fig. 2. Parallel second-order filter. Note that in our investigations we omitted the constant K section.

The parameters a_1 and a_2 are the same as in the direct form. The p and q coefficients can be computed from the direct-form parameters b_0 and b_1 with the following formulas:

$$p = \frac{b_0 - b_1}{2}, \quad (5)$$

$$q = \frac{b_0 + b_1}{2}. \quad (6)$$

The estimates of the autocorrelation matrices can be found in Fig. 5. It can be seen that the orthonormal property of the Kautz filter results in a unity autocorrelation matrix. In fixed-pole parallel filters however, the neighboring tap outputs have high levels of cross-correlation. This effect is lower when the orthogonal second-order sections are used: only the tap outputs of the different sections are correlated, resulting in a periodic pattern.

V. Normalizing the Tap Outputs

The convergence rate of the LMS algorithm is related to the eigenvalues of the $\mathbf{R}$ matrix [5]. It is shown that if the eigenvalue spread of the $\mathbf{R}$ matrix is the minimum over all possible matrices, the maximum convergence rate can be achieved. As a consequence, the tap outputs of the filter (denoted by $\mathbf{X}(k)$) having the same output power is a necessary condition. This criterion is inherently satisfied for orthonormal filters [8], but not for fixed-pole second-order

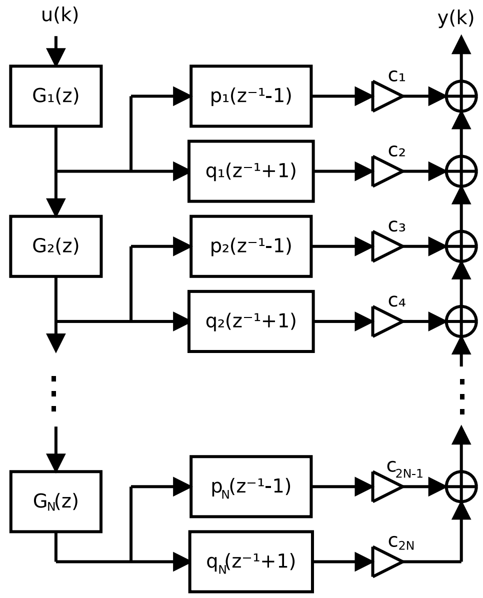

Fig. 3. Kautz filter structure.

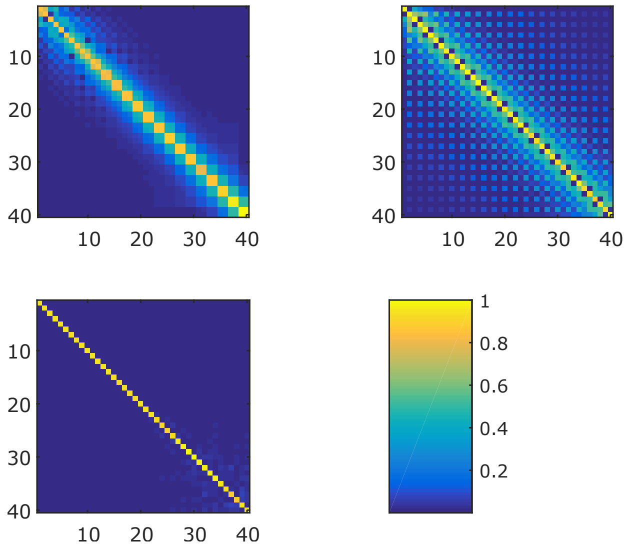

Fig. 5. Visualization of $\hat{\mathbf{R}}$ matrices. Top left: parallel filter with direct-form sections; top right: parallel filter with orthogonal second-order sections; bottom left: Kautz filter.

filters. Therefore, the tap outputs of the second-order sections need to be scaled.

To determine the normalizing coefficients, we compute the impulse responses between the input and the tap outputs. The scaling factors are then determined by the sum of squares of the impulse responses:

$$s_i = \frac{1}{\sum_{k=0}^{\infty} \left(h_i(k)\right)^2}, \tag{7}$$

where h_i denotes the impulse response between the filter input and the i-th filter tap output. Using this scaling, the tap outputs will have the same power when the input is white noise.

VI. Comparisons

In our investigation we used the NLMS algorithm as a method for system identification (Fig. 1). The input was a white noise uniformly distributed in range $[-1; +1]$. The system to be identified was implemented using a 10000-tap long FIR filter, whose coefficients were based on actual impulse response measurements.

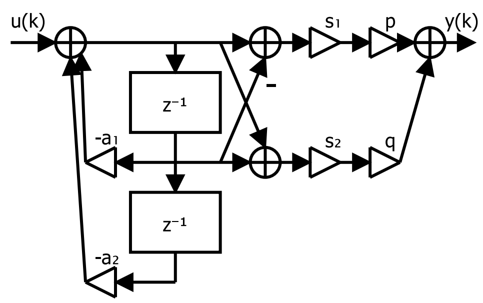

Fig. 4. Orthogonal second-order structure, with normalizing terms s_1 and s_2.

The filters to be compared are fixed-pole second-order parallel filters (without FIR section), implemented using both direct-form and improved second-order sections, a Kautz filter and as a reference, a FIR filter. The IIR filters have 20 conjugate complex pole pairs, placed along a logarithmic scale between 20 Hz and 20 kHz, assuming 44.1 kHz sampling frequency. The quality factors of the poles were chosen that the neighboring sections had their magnitude response cross at their -3 dB point [11]. The FIR filter has 40 taps, thus the filters have the same amount of free parameters.

The mean square error (MSE) of adapted filter parameters are computed on a logarithmic scale: the error, denoted by $e(k)$ in Fig. 1, has its DFT spectrum sampled at certain frequencies having logarithmic distribution. The samples are then squared and summed from 20 Hz to 20 kHz, assuming $f_s = 44.1$ kHz sampling rate:

$$E(j\omega) = \mathrm{DFT}\{e(k)\}, \tag{8}$$

$$\mathrm{MSE} = \sum_{f=20\mathrm{Hz}}^{f=20\mathrm{kHz}} \left|E(j2\pi f/f_s)\right|^2. \tag{9}$$

For comparison, the MSE was calculated for all structures at every 256 samples and then plotted.

In our investigation, we used two example transfer functions for testing the algorithms: a minimumphase one-way loudspeaker (Fig. 6 top) and a larger, two-way loudspeaker with non-minimumphase response (Fig. 6 bottom). In the figures, we marked the result of the off-line LS design as well as the magnitude response of the adaptive fixed-pole parallel filter that is implemented using orthogonal second-order sections. Note that the transfer function of the adaptive Kautz is omitted because it fits the LS solution after the simulation time (65536 samples).

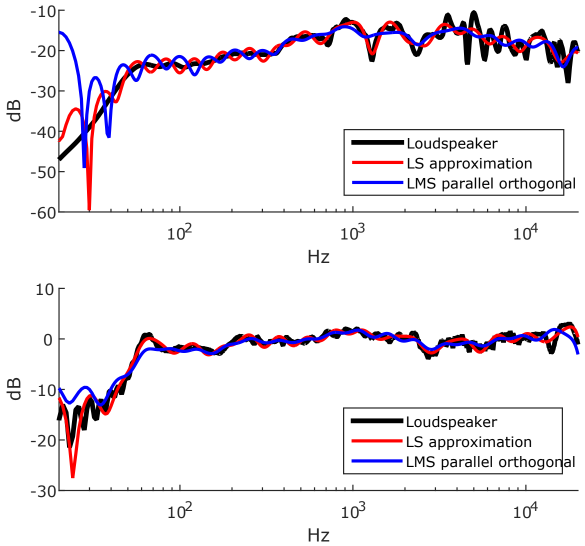

Fig. 6. Magnitude plots of the example transfer functions (black lines). Top: minimumphase one-way loudspeaker; bottom: non-minimumphase two-way loudspeaker. The LS approximations are plotted using red lines. The magnitude responses of the fixed-pole parallel filters using orthogonal second-order sections, after 65536 samples, are also plotted (blue lines).

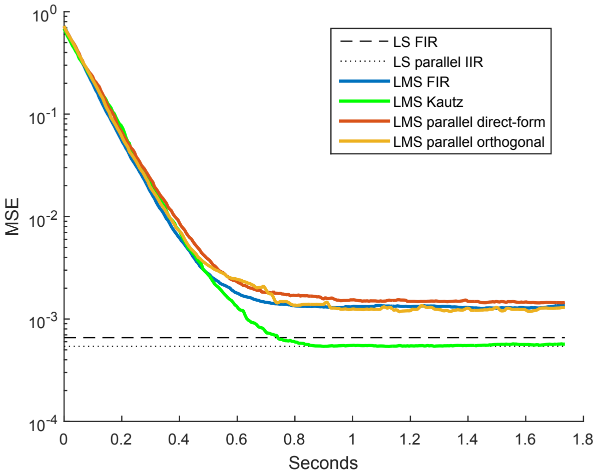

Fig. 7. MSE by time, for a minimumphase one-way loudspeaker response.

The MSE plots of the systems identifying the example transfer functions can be found in Fig. 7 and 8. For each of the filters, the μ step-size parameter is tuned in a way that the curves would have the best fit with each other on the first 12800 samples. As reference, the MSE of offline designed filters, based on the LS approximation, are shown on the figures using dashed and dotted horizontal lines.

According to figures 7 and 8, the Kautz filter has the best convergence: for the minimumphase system its MSE is on par with the LS approximation, and for the non-minimumphase system it has the fastest convergence among the tested structures.

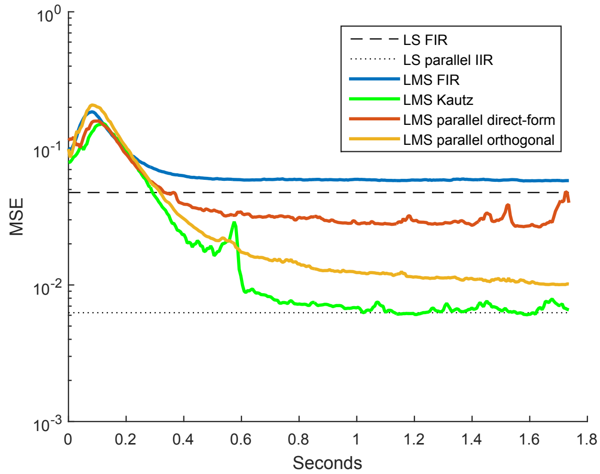

Fig. 8. MSE by time, for a non-minimumphase two-way loudspeaker response.

VII. Conclusion

This paper compared the LMS-based adaptive implementations of the most common fixed-pole IIR filters used in audio. As a result, we recommend to use the Kautz structure in LMS-based adaptive audio filters, if its computational demand can be satisfied.

Future research includes the usage of other filter structures: the delayed fixed-pole parallel filter, a modified Kautz structure with FIR component, and the resonator-based filter.

References

[1] V. Välimäki and J. D. Reiss, “All about audio equalization: Solutions and frontiers,” *Applied Sciences*, vol. 6, no. 5, 2016, art. no. 129, doi: https://doi.org/10.3390/app6050129.

[2] A. Härmä, M. Karjalainen, L. Savioja, V. Välimäki, U. K. Laine, and J. Huopaniemi, “Frequency-warped signal processing for audio applications,” *J. Audio Eng. Soc.*, vol. 48, no. 11, pp. 1011–1031, Nov. 2000.

[3] B. Bank, “Audio equalization with fixed-pole parallel filters: An efficient alternative to complex smoothing,” *J. Audio Eng. Soc.*, vol. 61, no. 1/2, pp. 39–49, Jan. 2013.

[4] T. Paatero and M. Karjalainen, “Kautz filters and generalized frequency resolution: Theory and audio applications,” *J. Audio Eng. Soc.*, vol. 51, no. 1–2, pp. 27–44, Jan./Feb. 2003.

[5] S. S. Haykin, B. Widrow, and B. Widrow, *Least-mean-square adaptive filters*. Wiley Online Library, 2003, vol. 31.

[6] S. Stearns, “Error surfaces of recursive adaptive filters,” *IEEE Transactions on Circuits and Systems*, vol. 28, no. 6, pp. 603–606, 1981.

[7] C. Johnson, M. Larimore, J. Treichler, and B. Anderson, “Sharf convergence properties,” *IEEE Transactions on Circuits and Systems*, vol. 28, no. 6, pp. 499–510, 1981.

[8] G. A. Williamson and S. Zimmermann, “Globally convergent adaptive iir filters based on fixed pole locations,” *IEEE transactions on signal processing*, vol. 44, no. 6, pp. 1418–1427, 1996.

[9] J. Cousseau, G. Sentoni, P. Diniz, and O. Agamennoni, “On orthogonal parallel realization for adaptive iir filters,” in *Proceedings of Third International Conference on Electronics, Circuits, and Systems*, vol. 2. IEEE, 1996, pp. 856–859.

[10] K. Horváth and B. Bank, “Optimizing the numerical noise of parallel second-order filters in fixed-point arithmetic,” *Journal of the Audio Engineering Society*, vol. 67, no. 10, pp. 763–771, 2019.

[11] B. Bank, “Audio equalization with fixed-pole parallel filters: An efficient alternative to complex smoothing,” in *Proc. 128$^{\text{th}}$ AES Conv., Preprint No. 7965*, London, UK, May 2010.