Improved Pole Positioning for Parallel Filters Based on Spectral Smoothing and Multi-Band Warping

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***Abstract*—The use of second-order parallel filters with predefined pole locations has been recently proposed for equalization and transfer function modeling. This letter presents an improved method for obtaining the pole positions of the parallel filter. The steps of the new method are the following: first, the target frequency response is smoothed to the required resolution. Then, multiple warped IIR filters with different warping parameters are designed to the smoothed target divided into frequency bands. Finally, the united pole set of the warped IIR designs is used for parallel filter design. The method leads to increased accuracy compared to earlier pole positioning techniques, and can also be used for Kautz filter designs. Examples of loudspeaker–room response modeling and equalization are presented.**

***Index Terms*—IIR digital filters, logarithmic frequency resolution, frequency warping, audio signal processing, loudspeaker–room response equalization.**

I. Introduction

ONE of the common tasks in audio signal processing is to design a digital filter for modeling or equalizing an audio system. While the use of traditional FIR and IIR design algorithms results in a linear frequency resolution, audio applications usually require a logarithmic (or logarithmic-like) frequency resolution that better fits the properties of human hearing. One of the most common audio filter design applications is the equalization of the audio reproduction chain. This usually involves the compensation of the anechoic loudspeaker frequency response or the joint transfer function of the loudspeaker–room system. Loudspeaker- or loudspeaker–room response equalization provides an additional reason for using logarithmic frequency resolution in filter design: acoustical transfer functions vary significantly as a function of space at high frequencies, while less at low frequencies due to the different wavelengths of sound. Since the equalization should improve the sound in a finite region of space (e.g., multiple seating positions on a sofa), it should be more precise at low frequencies where the transfer function is less dependent on the listening position, while at high frequencies it should only correct the overall trend of the response [1].

For obtaining filters with a logarithmic-like frequency resolution, various design approaches have been developed, including frequency warped filter design [2], [3], [4], [5], [6], Kautz filters [7], [8], and iteratively optimized cascade second-order filters [9], [10]. Recently, the fixed-pole design of parallel second-order filters has been presented [11]. It was demonstrated in [12] that for a given pole set the parallel filter produces the same results as the Kautz filter. However, it requires 33% percent fewer arithmetic operations and it is better suited for code parallelization. In addition, the resulting parallel structure is also beneficial from the point of view of quantization noise performance [13].

The frequency resolution of the parallel filter is controlled by the suitable choice of the filter poles. Two different ways have been proposed for choosing the pole positions of the parallel filter, depending on the system to be modeled or equalized. In general, if the filter order is in the same range as the system order, the filter poles should correspond to system poles for best accuracy. This is achieved by designing a warped IIR filter to the target response and using the poles of this warped IIR filter as the poles of the parallel filter [14]. On the other hand, if the filter order is significantly smaller than the order of the system, then only the average trend of the system response can be modeled. In this case better results are achieved if the pole set is predetermined, based on how much resolution we wish to achieve in the different frequency regions. For example, placing the poles according to a logarithmic frequency-scale results in a parallel filter response that is basically the logarithmically-smoothed version of the target response [12]. It has been demonstrated in [11] that already using this predetermined pole set results in better performance compared to IIR, warped FIR, and warped IIR filters estimated by the Steiglitz–McBride method [15]. The present letter proposes a new pole positioning method that provides a significant improvement over using the predetermined pole set.

The remainder of this letter is organized as follows: Section II outlines the structure of the parallel filter and the least-squares estimation of filter weights. Section III proposes the smoothed multi-band pole positioning method, and Sec. IV provides a comparison to previous pole positioning techniques. Section V presents a loudspeaker–room equalization example and gives comparison to various equalizer design methods. Finally, Sec. VI concludes the letter.

II. Filter structure and weight estimation

The general form of the parallel filter consists of a parallel set of second-order sections and an optional FIR filter path [11]:

$$H(z^{-1}) = \sum_{k=1}^{K} \frac{d_{k,0} + d_{k,1} z^{-1}}{1 + a_{k,1} z^{-1} + a_{k,2} z^{-2}} + \sum_{m=0}^{M} b_m z^{-m} \quad (1)$$

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where K is the number of second order sections, and M is the order of the FIR part.

Let us now assume that the poles of the filter are given. Once the denominator coefficients are determined by the poles p_k ($a_{k,1} = p_k + \overline{p}_k$ and $a_{k,2} = |p_k|^2$, where $\overline{p}_k$ is the complex conjugate of p_k), the problem becomes linear in its free parameters $d_{k,0}$, $d_{k,1}$ and b_m. Their values can be estimated either in the time-domain [11], or in the frequency-domain [16]. Here we review the frequency-domain algorithm.

Writing (1) in matrix form for a finite set of ϑ_n angular frequencies yields

$$\mathbf{h} = \mathbf{M}\mathbf{p} \tag{2}$$

where $\mathbf{p} = [d_{1,0}, d_{1,1}, \ldots d_{K,0}, d_{K,1}, b_0 \ldots b_M]^T$ is a column vector composed of the free parameters. The rows of the modeling matrix $\mathbf{M}$ contain the transfer functions of the second-order sections $1/(1 + a_{k,1}e^{-j\vartheta_n} + a_{k,2}e^{-j2\vartheta_n})$ and their delayed versions $e^{-j\vartheta_n}/(1 + a_{k,1}e^{-j\vartheta_n} + a_{k,2}e^{-j2\vartheta_n})$ for the ϑ_n angular frequencies. The last $M+1$ rows of $\mathbf{M}$ are the transfer functions of the optional FIR part $e^{-jm\vartheta_n}$ for $m = [0 \ldots M]$. Finally, $\mathbf{h} = [H(\vartheta_1) \ldots H(\vartheta_N)]^T$ is a column vector composed of the resulting frequency response.

Now the task is to find the optimal parameters $\mathbf{p}_{\mathrm{opt}}$ such that $\mathbf{h} = \mathbf{M}\mathbf{p}_{\mathrm{opt}}$ is closest to the target frequency response $\mathbf{h}_{\mathrm{t}} = [H_{\mathrm{t}}(\vartheta_1) \ldots H_{\mathrm{t}}(\vartheta_N)]^T$. If the approximation error is evaluated in the mean squares sense, the minimum is found by the well-known least-squares solution

$$\mathbf{p}_{\mathrm{opt}} = (\mathbf{M}^H\mathbf{M})^{-1}\mathbf{M}^H\mathbf{h}_{\mathrm{t}} \tag{3}$$

where $\mathbf{M}^H$ is the conjugate transpose of $\mathbf{M}$. Note that (3) assumes a filter specification $H_{\mathrm{t}}(\vartheta_n)$ given for the full frequency range $\vartheta_n \in [-\pi, \pi]$.

Note that using the FIR part is advantageous for non-minimumphase filter design, where it results in a lower overall computational complexity compared to using second-order sections alone [14]. Since in this letter only minimum-phase modeling and equalization examples are provided, the optional FIR part is not utilized.

III. Improved pole positioning based on smoothed responses and multi-band warped filter design

The first step of the new method is that the target response is smoothed to the required resolution (sixth- to twelfth-octave smoothing is typically used in loudspeaker equalization, while other applications, such as instrument body modeling [14] may require higher resolution). Next, this smoothed response is used to determine the pole set of the parallel filter by a warped IIR design. Because the effective resolution of warped IIR filters is limited to a certain frequency range depending on the warping parameter λ [4], [5], multiple (in practice, two) warped IIR filters are designed to the separate parts of the transfer function using different warping parameters, and their pole sets are united. The steps of the procedure are outlined below using a loudspeaker–room modeling example (the sampling frequency used in this work is $f_{\mathrm{s}} = 44100$ Hz):

1 Target response smoothing: The target response is made minimum-phase and smoothed to the required resolution, e.g.

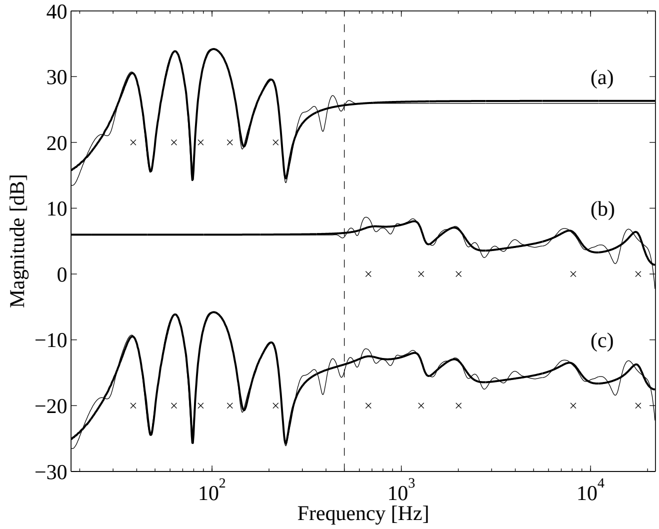

Fig. 1. Parallel filter design with dual-band pole positioning: (a) the specification of the low-frequency warped IIR filter (thin line) and the filter response (thick line), (b) the specification of the high-frequency warped IIR filter (thin line) and the filter response (thick line), and (c) the smoothed target response (thin line) and the final parallel filter response (thick line) using the united pole set of the two warped IIR filters. The pole frequencies of the filters are displayed by crosses. The vertical dashed line indicates the crossover frequency $f_{\mathbf{c}} = 500$ Hz of the two warped IIR designs.

by applying the fractional-octave smoothing technique proposed in [17]. This basically involves convolving the transfer function by a variable-width smoothing function. Naturally, not only fixed fractional-octave (i.e. logarithmic), but any other resolution profiles can be used. Then the smoothed response is cut to separate frequency bands.

The sixth-octave smoothed version of the target response is displayed by a thin line in Fig. 1 (c). In this example we use a dual-band design with a crossover frequency of 500 Hz, which is approximately in the middle of the audio band on a logarithmic scale, indicated by a vertical dashed line. The low-frequency part of the smoothed specification is displayed by a thin line in Fig. 1 (a), while the high-frequency part by a thin line in Fig. 1 (b). As can be seen in Fig. 1, the out-of-band parts of the transfer functions are crossfaded to a constant gain. This assures that the warped IIR designs will only produce poles in their respective frequency bands.

2 Warped IIR filter design: The smoothed-flattened target responses are used as the specification for warped IIR filter design [3]. We use the warping parameters $\lambda = 0.986$ and $\lambda = 0.65$ for the low- and high-frequency parts, respectively. These were chosen so that the warped filters have the maximal logarithmic resolution (minimal $\Delta f/f$) [6] in the middle of their respective bands by finding such λ values where the minimum of

$$\frac{\Delta f}{f} = \frac{1 + \lambda^2 - 2\lambda\cos(2\pi f/f_{\mathrm{s}})}{(1 - \lambda^2)f} \tag{4}$$

is at $f = 100$ Hz for the low-frequency band and $f = 3160$ Hz for the high-frequency band. The warped filters are designed by the Steiglitz–McBride algorithm [15] from the time-domain versions of the warped transfer functions (i.e., frequency-warped impulse responses).

The frequency response of the low-frequency warped IIR filter (filter order is 10) is displayed by thick solid line in Fig. 1 (a), while the frequency response of the high-frequency warped IIR filter (filter order is 10) is shown by thick solid line in Fig. 1 (b).

3 Pole finding and dewarping: Finally, the poles $\tilde{p}_k$ of the warped IIR filters are found as the roots of the filter denominators. The warped poles $\tilde{p}_k$ are converted back to linear frequency scale by

$$p_k = \frac{\tilde{p}_k + \lambda}{1 + \lambda \tilde{p}_k}. \quad (5)$$

The pole frequencies of the warped IIR filters are indicated by crosses in Fig. 1 (a) and (b), while their united pole set is shown by crosses in Fig. 1 (c). The frequency response of the final 20th-order parallel filter designed using this united pole set is displayed by thick line, overlaid on the smoothed target.

We have found that it is sufficient to divide the full audio frequency range into two parts and use a dual-band design, since the warped IIR designs handle the half of the audio band properly, as can also be seen in Fig. 1. However, it is also possible to use more bands, and thus smaller filter orders within each band.

Note that even though the pole set has been obtained from a minimum-phase smoothed response, the method can also be used for non-minumumphase filter design. In this case once the pole set is obtained from the minimum-phase target, the parallel filter is designed by (3) using the original (non-minimumphase) response.

By looking at Fig. 1 it is clear that by connecting the two warped IIR filters in series one would obtain practically the same response as that of the parallel filter. This means that the first two steps of the above procedure can also be used for improved warped IIR filter design, similarly to [5]. In addition, coming from the theoretical equivalence of parallel and Kautz filters [12], the poles obtained in Step 3 can also be used for Kautz filter design, resulting in exactly the same response.

IV. Comparison to Previous Pole Positioning Techniques

Figure 2 provides the comparison of the new method to previous pole positioning techniques for a minimum-phase loudspeaker–room response modeling example. Figure 2 (a) shows a 20th-order parallel filter response, where the poles are obtained from a warped IIR design ($\lambda = 0.95$) according to [14]. Since the filter order is significantly smaller than the order of the system, the warped IIR design does not model the overall response but picks a few resonances instead. It can also be seen that the frequency resolution of the filter is concentrated to a limited region of the full frequency scale due to the restricted frequency range of the warping effect. The objective measure used in this study is the mean absolute dB error [9] computed between the target and filter response, and evaluated on a logarithmic frequency scale. The error values are displayed in Fig. 2 above the corresponding responses.

Figure 2 (b) shows a parallel filter design with a predetermined pole set [11], where 10 pole pairs are placed between 30 Hz and 20 kHz on a logarithmic scale. However, for this low filter order the pole density is too low for modeling the problematic low frequency behavior of the loudspeaker–room response. The fit of Fig. 2 (b) can be improved by increasing the pole density in the problematic frequency region. Figure 2 (c) shows an example with the same filter order but stepwise logarithmic pole positioning, where six pole pairs are placed between 30 and 200 Hz, and four pole pairs between 400 Hz and 20 kHz, providing a much better result.

Finally, Fig. 2 (d) shows a 20th-order parallel filter design using the new smoothed multi-band pole positioning method. It can be seen that a significantly better fit is achieved compared to the previous techniques for the same filter order.

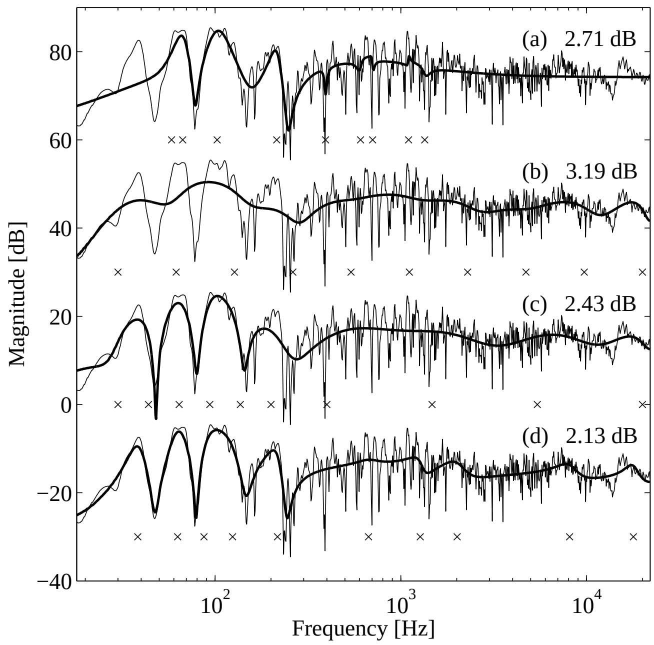

Fig. 2. Parallel filter design with various pole positioning techniques. The thin lines show the minimum-phase target (loudspeaker–room response), and the filter order is 20 in all cases. The thick lines show the parallel filter responses obtained by using (a) pole positioning based on a warped IIR filter design with $\lambda = 0.95$, (b) logarithmic pole positioning, (c) stepwise logarithmic pole positioning with higher resolution at low frequencies, and (d) by the proposed smoothed multi-band pole positioning method. The crosses indicate the pole frequencies of the parallel filters. The dB values indicate the mean absolute errors [9].

V. Equalizer Design Example

Figure 3 presents an equalizer design example using the same loudspeaker–room response that was used in Fig. 2. All the filters designed by the various methods have a filter order of 32. Figure 3 (a) thin line displays the unequalized response, while the thick line shows its sixth-octave smoothed version. A dashed line displays the desired loudspeaker–room response. The loudspeaker–room response is first equalized by a (b) warped FIR and a (c) warped IIR filter [3], both designed with $\lambda = 0.95$ and using the Steiglitz–McBride method [15]. The equalization is excellent in the middle frequency range, but the responses have strong anomalies at low- and high-frequencies, due to the limited frequency range of the effective resolution of the warped filters. The resulting mean

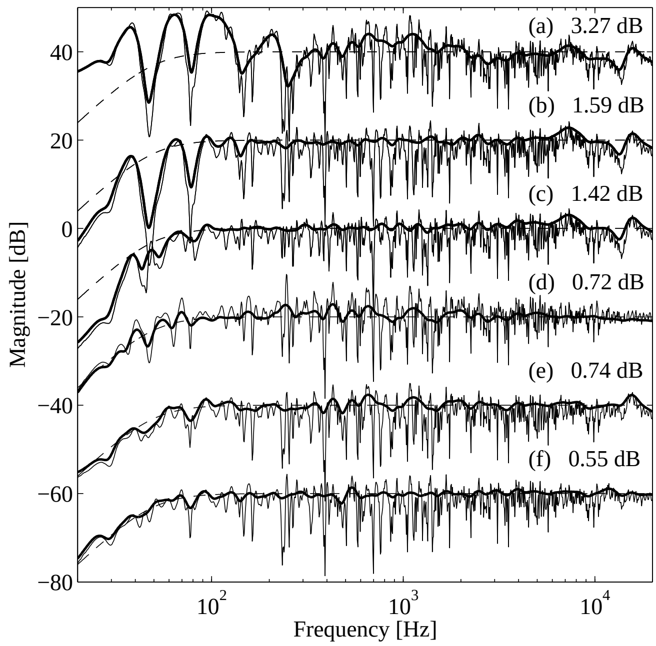

Fig. 3. Minimum-phase loudspeaker–room equalization comparison: (a) unequalized response. Equalized responses using (b) a warped FIR filter, (c) a warped IIR filter, (d) combined IIR and warped IIR filters, (e) automatically optimized parametric filters, and (f) the proposed method. The filter order is 32 in all cases. The thin lines show the original transfer functions, while the thick lines display the sixth-octave smoothed versions. The desired response is displayed by dashed lines. The dB values indicate the mean absolute errors computed between the desired and sixth-octave smoothed curves.

absolute dB errors computed between the target and sixth-octave smoothed responses are displayed in Figure 3.

In Fig. 3 (d) the response is equalized by combined warped and linear filters [6] with warped IIR filter order of 18 ($\lambda = 0.985$) and IIR filter order of 14, leading to an improved fit. A similar result is obtained by using iteratively optimized parametric equalizers [9], displayed in Fig. 3 (e). However, a drawback of the latter method is its complexity, since it combines a direct search procedure with a random optimization.

In Fig. 3 (f) the system is equalized by a parallel filter where the pole positioning is based on the new pole positioning technique with 9 pole pairs below 500 Hz and 7 pole pairs above. The filter weights were estimated by the direct equalizer design approach of [16]. The performance of the new method (f) is superior compared to the earlier methods. In addition, the resulting parallel filter structure is more advantageous from the point of view of quantization noise performance [13], and it is well suited for full code parallelization.

VI. Conclusion

This letter has presented an improved pole positioning method for parallel filter design. The method is based on designing separate warped IIR filters to the different parts of the fractional-octave smoothed version of the target response. The warped IIR designs have different warping parameters so that they maximize the warping effect in their respective bands. Then, the pole sets of the warped IIR filters are united, and this united pole set is used for parallel filter design. The method outperforms previous pole positioning techniques and can also be used for Kautz filter design. It provides superior equalization compared to warped FIR, warped IIR, and combined linear-warped equalizers, and it also outperforms the method of iteratively optimized parametric filters, while requires a much simpler design algorithm. Matlab code for parallel filter design is available at http://www.mit.bme.hu/∼bank/parfilt.

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