Perceptually Motivated Audio Equalization Using Fixed-Pole Parallel Second-Order Filters

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***Abstract*—In audio, equalizer design should take into account the frequency resolution of the auditory system. In this paper, this is accomplished by the fixed-pole design of parallel second-order filters. The design process has two steps: first, the poles of the filter are set according to the desired frequency resolution. Then, the feedforward coefficients of the second-order filters are determined by a linear least squares solution. The proposed parallel filter achieves effectively the same equalization results as the Kautz filter, but requires 33% fewer multiplications and additions.**

***Index Terms*—audio signal processing, IIR digital filters, room response equalization.**

I. Introduction

AUDIO equalization using digital signal processors (DSPs) has been a subject of research for more than two decades. It generally means the correction of the magnitude (and sometimes the phase) response of an audio chain. Typical examples include loudspeaker equalization based on anechoic measurements [1], [2], [3], or the correction of loudspeaker-room response [4], [5], [6], [7]. Because the systems to be equalized are generally of significantly higher order than what is practical for an equalizer implementation, only the overall response of the system can be corrected. This overall correction should be driven by perceptual principles [5]. For example, typically the logarithmic frequency scale is used in audio engineering and a fractional-octave (e.g., third-octave) smoothed magnitude response is used to estimate perceived timbre.

Another reason for logarithmic (or logarithmic-like) frequency resolution is that an audio system often has multiple outputs, like multiple listening positions on a sofa. Transfer functions measured at different positions in space have more similarity at low frequencies than at high frequencies, due to the different wavelengths of sound. Therefore, higher resolution is required at the lower end of the spectrum compared to the upper one. An overly precise correction at high frequencies for one measurement position usually worsens the response at other points in space [5].

A straightforward choice for equalizer design is the use of standard finite impulse response (FIR) or infinite impulse response (IIR) design algorithms. Unfortunately, neither of these can meet the goal of logarithmic frequency resolution. Linear frequency resolution is inherent in FIR filters, and while IIR filters could have a theoretically higher pole density at low frequencies, the logarithmic frequency scale is so distorted compared to the linear scale that even weighted filter design cannot give satisfactory results [8].

Parametric equalizers, in which the center frequency, Q value, and gain of the sections are set manually, are commonly used for the magnitude equalization of audio systems. Automatic parameterization of parametric equalizers has been successfully demonstrated in [3] and [6], using nonlinear parameter estimation algorithms.

The most widely used method for achieving a perceptually motivated frequency resolution is the application of frequency warping (see, e.g., [2], [8], [9], [10]). In warped filters, each unit delay z^{-1} of the traditional FIR or IIR filters is replaced by a first-order allpass filter. Comparison of FIR, IIR, and warped filter equalization of loudspeakers is given in [2], showing that lower filter orders can be used compared to traditional structures, when frequency warping is applied.

Kautz filters can be seen as the generalization of warped FIR filters, where the allpass filters in the chain are not identical [11], [12]. As a result, the frequency resolution can be allocated arbitrarily by the choice of the filter poles. The Kautz structure is a linear-in-parameter model, where the basis functions are the orthonormalized versions of decaying exponentials [11], [12]. For audio applications, significantly lower order Kautz filters are sufficient compared to traditional IIR filter designs. However, the savings in filter order do not directly translate to savings in computational cost, because Kautz filters require a complicated series-parallel structure for their implementation.

Recently, a fixed-pole design method has been introduced for parallel second-order filters, for the application of instrument body modeling [13]. It has been shown that effectively the same results can be achieved by the parallel filters as with Kautz filters for the same filter order, without the disadvantage of a complicated filter structure. This letter presents the application of the method for audio equalization and proposes a technique for designing the parallel equalizer from the measured system response and desired target response directly, without inverting the system response. An illustrative example of loudspeaker–room equalization is presented.

II. The parallel filter

Implementing IIR filters in the form of parallel second-order sections has been used traditionally because it has better quantization noise performance and the possibility of code parallelization. The parameters of the second-order sections

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are usually determined from direct form IIR filters by partial fraction expansion [14].

Here the poles are set to a predetermined (e.g., logarithmic) frequency scale, leaving the zeros as free parameters for optimization. In the case of modeling a desired impulse response, the parallel filter uses the outputs of the second-order sections (exponentially decaying sinusoidal functions) as basis functions of a linear-in-parameter model. For equalizer design, the measured system response is filtered by the second-order sections, and these signals are the basis functions of the linear-in-parameter model. Thus, the parameter estimation can be done by a simple LS algorithm, similar to what was suggested for Kautz filters [12].

Note that fixed-pole IIR filters are often used in adaptive filtering (see, e.g., [15]) because of their favorable convergence properties. Instead, in this letter the motivation for fixing the poles is to control the frequency resolution of the design.

A. Problem formulation

Every transfer function of the form $H(z^{-1}) = B(z^{-1})/A(z^{-1})$ can be rewritten in the form of partial fractions:

$$H(z^{-1}) = \sum_{i=1}^{P} c_i \frac{1}{1 - p_i z^{-1}} + \sum_{m=0}^{M} b_m z^{-n} \qquad (1)$$

where p_i are the poles, either real valued or forming conjugate pairs, if the system has a real impulse response. The second sum in (1) is the FIR filter part of order M. Note that in the case of pole multiplicity, terms of higher order also appear in (1).

The resulting filter can be implemented directly as in (1), forming parallel first-order complex filters, and the estimation of the parameters can be carried out as described in [13]. However, it is more practical to combine the complex pole pairs to a common denominator. This results in second-order sections with real valued coefficients, which can be implemented more efficiently. Those fractions of (1) that have real poles can be combined with other real poles to form second-order IIR filters, yielding a canonical structure. Thus, the transfer function becomes

$$H(z^{-1}) = \sum_{k=1}^{K} \frac{d_{k,0} + d_{k,1} z^{-1}}{1 + a_{k,1} z^{-1} + a_{k,2} z^{-2}} + \sum_{m=0}^{M} b_m z^{-n} \qquad (2)$$

where K is the number of second order sections. The filter structure is depicted in Fig. 1.

The poles of the second-order sections can be determined by any method suggested for the case of Kautz filters [12]. Positioning the poles logarithmically is particularly useful for audio equalizers,

$$\vartheta_k = \frac{2\pi f_k}{f_\mathrm{s}} \qquad (3)$$

$$p_k = R^{\vartheta_k/\pi} e^{\pm j \vartheta_k} \qquad (4)$$

where ϑ_k are the pole frequencies in radians determined by the logarithmic frequency series f_k and the sampling frequency f_s. The pole radii form an exponentially damped sequence approximating constant Q resolution. The pole radius at $f_\mathrm{s}/2$ is set by the damping parameter R [12].

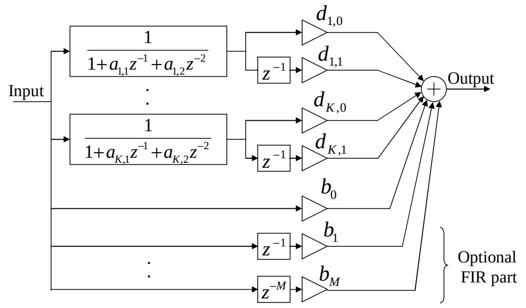

Fig. 1. Structure of the parallel second-order filter.

In general, the pole set does not have to be strictly logarithmic, but can also focus on specific frequencies by increasing the pole density in that region, which will be utilized in Section III.

B. Filter design

First, we investigate how the parameters of the parallel filter can be estimated to match a desired filter response. Because the poles of the IIR filter are predefined, (2) becomes linear in its free parameters $d_{k,0}$, $d_{k,1}$, and b_m, which can already be estimated in the frequency domain.

However, it is simpler to find the coefficients in the time domain. The impulse response of the parallel filter is given by

$$h(n) = \sum_{k=1}^{K} d_{k,0} u_k(n) + d_{k,1} u_k(n-1) + \sum_{m=0}^{M} b_m \delta(n-m) \quad (5)$$

where $u_k(n)$ is the impulse response of the transfer function $1/(1+a_{k,1}z^{-1}+a_{k,2}z^{-2})$, which is an exponentially decaying sinusoidal function, and $\delta(n)$ is the discrete unit impulse.

Naturally, (5) is linear in parameters, similar to its z-transform counterpart (2). Writing (5) in matrix form yields

$$\mathbf{h} = \mathbf{M}\mathbf{p} \qquad (6)$$

where $\mathbf{p} = [d_{1,0}, d_{1,1}, \ldots d_{K,0}, d_{K,1},\ b_0 \ldots b_M]^T$ is a column vector composed of the free parameters. The rows of the modeling signal matrix $\mathbf{M}$ contain the modeling signals, which are $u_k(n)$ and their delayed counterparts $u_k(n-1)$, and for the FIR part, the unit impulse $\delta(n)$ and its delayed versions up to $\delta(n-M)$. Finally, $\mathbf{h} = [h(0) \ldots h(N)]^T$ is a column vector composed of the resulting impulse response. The problem reduces to finding the optimal parameters $\mathbf{p}_\mathrm{opt}$ such that $\mathbf{h} = \mathbf{M}\mathbf{p}_\mathrm{opt}$ is closest to the target response $\mathbf{h}_\mathrm{t}$. If the error function is evaluated in the mean squares sense, the optimum is found by the well known LS solution

$$\mathbf{p}_\mathrm{opt} = (\mathbf{M}^H \mathbf{M})^{-1} \mathbf{M}^H \mathbf{h}_\mathrm{t} \qquad (7)$$

where $\mathbf{M}^H$ is the conjugate transpose of $\mathbf{M}$.

C. Direct equalizer design

Equalizing a system (such as a loudspeaker) by the parallel filter can be done by inverting the system response and designing the parallel filter as outlined in the previous section. This section proposes a method for designing the equalizer directly without inverting the system response. This simplifies the design significantly, and avoids many problems presented by the inversion of the measured transfer function. The basic idea of the method described here is similar to that used for Kautz filters [12].

Designing an equalizer requires that the resulting response $h(n)$, which is the convolution of the equalizer response $h_{\rm eq}(n)$ and the system response $h_{\rm s}(n)$, is close to the target response $h_{\rm t}(n)$ (which can be a unit impulse, for example). In our case, this means that the input of the parallel filter is the system response $h_{\rm s}(n)$ and its output $h(n)$ should match the target response $h_{\rm t}(n)$. The output of the parallel filter is computed as

$$\begin{aligned} h(n) = h_{\rm eq}(n) * h_{\rm s}(n) = \\ \sum_{k=1}^{K} d_{k,0} u_k(n) * h_{\rm s}(n) + d_{k,1} u_k(n-1) * h_{\rm s}(n) + \\ \sum_{m=0}^{M} b_m \delta(n-m) * h_{\rm s}(n) = \\ \sum_{k=1}^{K} d_{k,0} s_k(n) + d_{k,1} s_k(n-1) + \sum_{m=0}^{M} b_m h_{\rm s}(n-m) \end{aligned} \quad (8)$$

where $*$ denotes convolution. The signal $s_k(n) = u_k(n) * h_{\rm s}(n)$ is the system response $h_{\rm s}(n)$ filtered by $1/(1+a_{k,1}z^{-1}+a_{k,2}z^{-2})$. It can be seen that (8) has the same structure as (5). Therefore, the parameters $d_{k,0}$, $d_{k,1}$, and b_m can be estimated in the same way as presented in the previous section. Similarly, writing this in a matrix form yields

$$\mathbf{h} = \mathbf{M}_{\rm eq}\mathbf{p} \quad (9)$$

where the rows of the new signal modeling matrix $\mathbf{M}_{\rm eq}$ contain $s_k(n)$, $s_k(n-1)$, and the system response $h_{\rm s}(n)$ and its delayed versions up to $h_{\rm s}(n-M)$. Then, the optimal set of parameters is again obtained by

$$\mathbf{p}_{\rm opt} = (\mathbf{M}_{\rm eq}^H \mathbf{M}_{\rm eq})^{-1} \mathbf{M}_{\rm eq}^H \mathbf{h}_{\rm t}. \quad (10)$$

III. Design Example and Comparison

Figure 2 (a) displays the magnitude response of a two-way floor-standing loudspeaker measured at 2 m distance in a normal living room. The minimum-phase version of the measured loudspeaker–room response is used as a system response $h_{\rm s}(n)$ and the target $h_{\rm t}(s)$ is a unit impulse filtered by a fourth-order high-pass filter with a cutoff frequency of 30 Hz.

As expected, the 50th-order IIR equalization presented in Fig. 2 (b) corrects the high-frequency anomalies only due to its linear frequency resolution. Fig. 2 (c) shows a warped IIR (WIIR) filter estimated by using a warping parameter $\lambda = 0.75$. The filter has been "dewarped" to a cascade of second-order IIR filters for efficient implementation, as done in [10] for WFIR filters. With the WIIR filter, the low frequencies are still poorly equalized. By choosing higher λ values, the accuracy could be increased at low frequencies, but high-frequency accuracy would be reduced. A major problem of the WIIR equalizer is that some of the otherwise inaudible dips are compensated by sharp peaks [see Fig. 2 (e)], which leads to poor off-axis performance and audible ringing. Moreover, dewarping to second-order sections can be numerically unstable when such high Q resonances are present (i.e., poles are too close to the unit circle).

Figure 3 (a) shows the equalization of the same loudspeaker–room response by a 50th-order WFIR filter designed with $\lambda = 0.75$, then dewarped [10]. This results in a 50th-order cascade IIR filter having 50 equal poles at $p = \lambda = 0.75$. The on-axis equalization is similar to the WIIR case. However, the WFIR equalizer produces better off-axis behavior compared to its WIIR counterpart due to its smoother response displayed in Fig. 3 (d), and can be dewarped without numerical problems.

Fig. 3 (b) shows the room response equalized by a 50th order Kautz filter, which provides a flat room response when third-octave smoothed. The proposed parallel equalizer using the same pole set produces the same result as the Kautz filter, as displayed in Fig. 3 (c). This equivalence is clearly observed by comparing the magnitude responses of the Kautz and parallel equalizers in Fig. 3 (e) and (f).

The logarithmically positioned poles of the Kautz and parallel filters (displayed by vertical lines in the bottom of Fig. 3) were chosen to have higher density at low frequencies, to focus on the more problematic region of the transfer function. This demonstrates that the resolution of the equalization is controlled by the pole density, as can be observed in Fig. 3 (e) and (f). Since the two methods inherently provide a smooth equalizer response, the narrow dips of the system response are not equalized, providing better off-axis performance.

In this example, the FIR part of the parallel filter is not utilized. The FIR part has been applied to non-minumum-phase filter design in [13], and can be used for joint magnitude and phase equalization, producing results similar to Kautz filter with poles placed on the origin [12].

In summary, for the same filter order the parallel filter achieves better results than IIR, WFIR, and WIIR filters designed by the Steiglitz-McBride method. Furthermore, the parallel filter yields the same equalization as the Kautz filter. This is expected because the Kautz filter uses the orthonormalized version of the basis functions of the parallel filter; thus, the basis functions of the two methods span the same approximation space.

In contrast to the Kautz filter, the structure of the parallel filter has to be extended in the presence of pole multiplicity. Additionally, the non-orthonormality of the basis functions can make the parameter estimation more sensitive numerically. However, pole multiplicity is avoided because the poles are set by the user, and the LS parameter estimation seems to be robust even for higher filter orders (see [13] for examples). On the other hand, by giving up the orthonormality of the basis functions, the number of multiplications and additions is reduced by 33% (see Table I). Additional benefits are expected due to the potential of full code parallelization.

IV. Conclusion

This letter has presented a fixed-pole design method for parallel second-order filters as applied to perceptually mo-

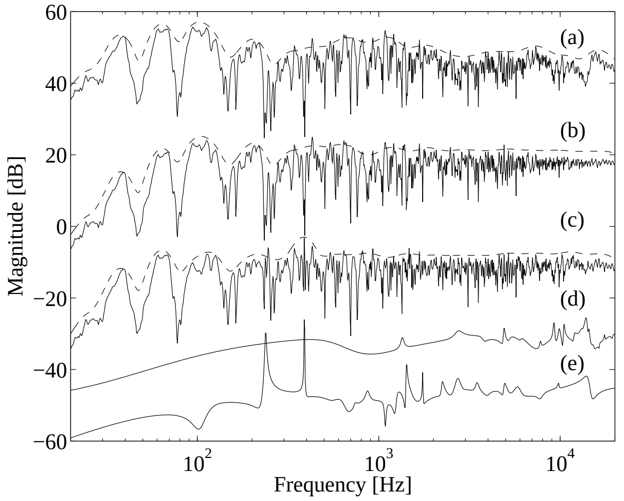

Fig. 2. Minimum-phase room response equalization: (a) unequalized loudspeaker-room response, (b) equalized by a 50th-order IIR filter estimated by MATLAB's Steiglitz–McBride method in system identification mode, and (c) by a 50th-order WIIR filter estimated by the Steiglitz–McBride method in the warped domain. The magnitude responses of the equalizers are presented by (d) for the IIR filter and (e) for the WIIR filter. In (a)–(c), the dashed lines show third-octave smoothed versions offset by 3 dB for clarity.

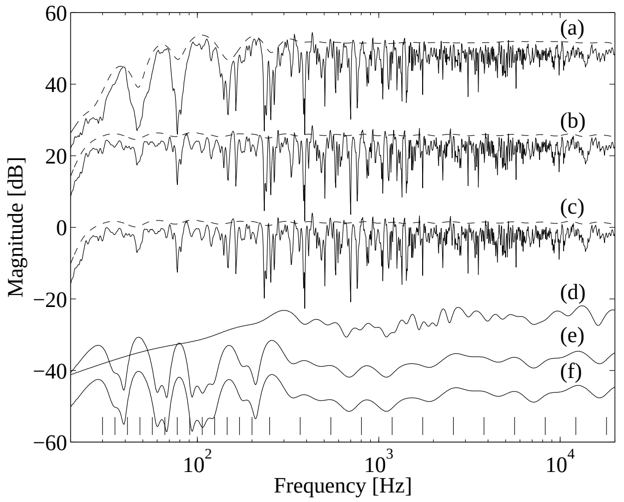

Fig. 3. Minimum-phase room response equalization for the same system as for Fig. 2. Room response (a) equalized by a 50th-order warped FIR filter estimated by the Steiglitz–McBride method, (b) equalized by a 50th-order Kautz filter, and (c) equalized by a 50th-order parallel filter. The magnitude responses of the equalizers are presented by (d) for the WFIR filter, (e) for the Kautz filter, and (f) for the parallel filter. In (a)–(c), the dashed lines show third-octave smoothed versions offset by 3 dB for clarity. The pole frequencies of the Kautz and parallel filters are displayed by vertical lines in the bottom of the figure.

tivated audio equalizer design. The design steps are similar to those of Kautz equalizers: first, the pole set is determined according to the desired frequency resolution, then the weights (zeros) of the filter are found by a closed-form LS expression from the system response and target response directly. The parallel filter produces effectively the same results as the Kautz filter, but requires one third fewer multiply-and-add operations, and has a fully parallel structure. Compared to IIR, warped FIR, and warped IIR filters estimated by the Steiglitz–McBride method, better results are achieved for the same filter order. An in-depth comparison of parallel filter with the Kautz filter and other filter-design techniques is left for future research. Matlab code for the parallel filter is available at: http://www.acoustics.hut.fi/go/spl08-parfilt.

	Multiplications	Additions
Kautz filter	$3N+2$	$3N+1$
Parallel filter	$2N+1$	$2N$

TABLE I
NUMBER OF MULTIPLICATIONS AND ADDITIONS REQUIRED FOR FILTER ORDER N.

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