Accurate and efficient modeling of beating and two-stage decay for string instrument synthesis

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Abstract

In this paper a novel modeling method is presented for beating and two-stage decay. Here, one digital waveguide is used for each note and some resonators are run in parallel to simulate the beating and two-stage decay of those partials, where these phenomena are most prominent. The resonator bank is implemented by using the multi-rate approach, resulting in a decrease of computational cost by a factor of 10. By taking the advantage of the characteristics of the resonators, relatively simple upsampling and downsampling filters are used. Two different filtering approaches are presented and compared with respect to computational complexity. Examples are shown with the application to piano sound synthesis.

1 Introduction

Physics based synthesis of string instruments has been an active field in the recent years. The digital waveguide [1] has proven to be the most efficient tool for this purpose. The method originates from the discretization of the traveling-wave solution of the one-dimensional wave equation. The losses and the dispersion of the string are lumped to one termination of the string. Hence, the model reduces to a delay line and a filter in a feedback loop.

In struck or plucked string instruments, beating and two-stage decay appear in the sound [2]. The two-stage decay means that in the early part of the tone the decay rate is higher than in the latter. Beating refers to an amplitude modulation, which is superimposed on the exponential decay. Already the interaction of the vertical and horizontal polarizations of the string vibration produces such an effect. This is even more important in the case of the piano, where two or three strings may belong to one key. Informal listening tests show that the accurate modeling of beating and two-stage decay increases the quality of synthesized piano sounds significantly.

The present paper first describes the properties of the earlier techniques proposed for the simulation of beating and two-stage decay. Then the idea of the resonator bank is outlined. In the next sections, two alternative methods are presented for increasing the efficiency of the resonator bank by the multi-rate approach. Comparison of these methods conclude the paper.

2 Parallel waveguides

The digital waveguide is capable to generate a set of quasi-harmonic exponentially decaying sinusoids. Therefore, the basic structure of the digital waveguide has to be extended to be able to model beating and two-stage decay. For that, several methods have been proposed in the literature, based on running two coupled digital waveguides in parallel.

The simplest way is the use of constant coupling coefficients [3], or a simple coupling filter [4]. Unfortunately, these methods are unable to accurately capture the evolution of the partials. No algorithm exists for designing high-order coupling filters, which could enable precise modeling in the time domain. Maintaining the stability of such systems is not an easy issue either. When the coupled digital waveguides are implemented in the frequency domain [5], the partial envelopes are rendered precisely, but the algorithm is not suitable for real-time implementation. In general, the existing physics-based sound synthesis methods are not able to accurately model beating and two-stage decay at a low computational cost.

3 Resonator bank

In the resonator bank method [6] the beating and two-stage decay is modeled by using one digital waveguide as a basic string model and connecting some resonators in parallel, instead of using a second digital waveguide. The excitation signal is common for the digital waveguide and the resonators.

The idea comes from the fact that when two slightly mistuned strings with one polarization are coupled, their behavior can be described by a set of mode-pairs [2]. This means that every partial corresponds to two exponentially damping sinusoids with slightly different frequencies, different decay times, initial phases and amplitudes. One sinusoid of the mode-pair is now simulated by one partial of the digital waveguide and the other sinusoid by a second-order resonator. The transfer functions $R_k(z)$ of the second-order resonators are computed as follows:

$$R_k(z) = \frac{Re\{a_k\} - Re\{a_k \overline{p_k}\} z^{-1}}{1 - 2Re\{p_k\} z^{-1} + |p_k|^2 z^{-2}}$$

$$a_k = A_k e^{j\varphi_k} \quad p_k = e^{j\frac{2\pi f_k}{f_s} - \frac{1}{f_s \tau_k}} \tag{1}$$

where A_k, φ_k, f_k, and τ_k refer to the initial amplitude, initial phase, frequency and decay time parameters of the k^{th} resonator, respectively. The overline stands for complex conjugation, the Re sign for taking the real part of a complex variable, and f_s is the sampling frequency.

The efficiency of this structure comes from the fact that only those partials for which the beating and two-stage decay are prominent, are simulated precisely. For the rest of the partials, no resonators are used, they will have simple exponential decay determined by the digital waveguide. By using about five or ten resonators, perceptually adequate results can be achieved. In this case, the digital waveguide is responsible for the generation of the rich harmonic content, and the resonator bank accounts for the precise modeling of specific partial envelopes. The parameter estimation compared to the coupled waveguides gets simpler, since there is no need for coupling filter design. The stability problems of a coupled system are also avoided. Moreover, here the accuracy of the simulation can be varied by changing the number of the resonators. This way, the synthesis engine is able to use all of its resources under all conditions during playing.

4 The multi-rate approach

Although the resonator bank method presented in [6] was successfully applied for the synthesis of piano sound and provided an alternative to the methods based on a parallel waveguide, it was not superior to the earlier methods with respect to computational cost. As an example, 10 resonators corresponded to 50 multiplications and additions.

Here a multi-rate approach is presented to reduce the computational complexity of the resonator bank. It has turned out that using parallel resonators for the lowest partials gives good sonic results. In most of the cases the resonators could run at the 1/4, 1/8, 1/16, or 1/32 of the Nyquist rate. This enables modeling beating and two-stage decay at a negligible amount of extra computation compared to a basic string model with one digital waveguide. In this section, two slightly different approaches are presented for the implementation of the multi-rate system, both taking advantage of the specific characteristics of the problem.

4.1 The regular way (MR1)

Here, first the force signal coming from the hammer model is downsampled, filtered by the second-order resonators, and then upsampled to the original sampling rate. Since for one note resonators with different sampling rates are used, it is beneficial to implement the

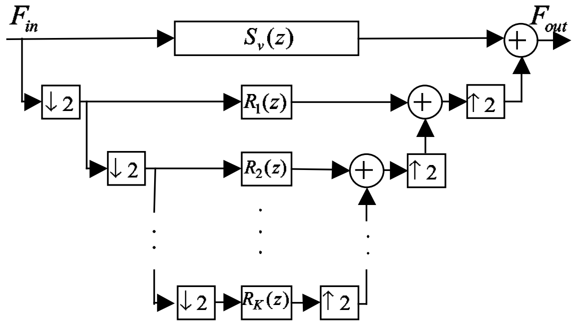

Figure 1: The multi-rate parallel resonator bank.

multi-rate system by cascading half band downsampling and upsampling filters. This also simplifies the filter design. This structure is shown in Figure 1. The sign $\downarrow 2$ refers to the downsampling operation with prior filtering, and the sign $\uparrow 2$ stands for upsampling operation with an interpolation filter. For simplicity, Fig. 1 shows only one resonator at every downsampled sampling rate, but in practice, many resonators are connected in parallel within the same branch.

In the filter design, we can take the advantage that the downsampled signal is imposed to filtering by a second order resonator, which has a very narrow amplitude response. This means that a small aliasing after downsampling is acceptable, since that leads only to a change in the initial amplitude and phase of the resonators. Having 20 dB stopband attenuation has to be found enough in practice. The upsampling filters cannot be simplified this way, there 60 dB stopband attenuation is needed to avoid audible aliasing. On the other hand, the signals of lower sampling rate of all the notes can be summed before upsampling, therefore the same interpolation filters can be used for all the notes. The filter design can be further simplified by having less tight specification in the passband. This can be done because the amplitude and phase errors of the downsampling and upsampling filters can be corrected by changing the initial amplitudes and phases of the resonators (See Section 5).

Figure 2 (a) shows an example of a resonator output for a C_4 piano note, first partial (262 Hz). The dashed line refers to the signal of a single-rate resonator as a reference (f_s = 44.1 kHz) and the solid line shows the output of the multi-rate system. The downsampling factor was 64 ($f_s' = f_s/64$), the downsampling and upsampling filters had a passband of 0 $\leq \vartheta \leq$ 0.45π, a stopband of 0.55 $\pi \leq \vartheta \leq \pi$, and an order of 9 and 26, respectively. Their passband ripple was about 5 dB. For simplicity, linear-phase FIR filters were used and they were designed by using the `remez` algorithm of MATLAB [7]. It can be seen that the signals match quite well. The latency with the multi-rate approach is about 25 ms, which can be considered inaudible. Note that by using minimum-phase downsampling and upsampling filters this delay could be shortened by a significant amount.

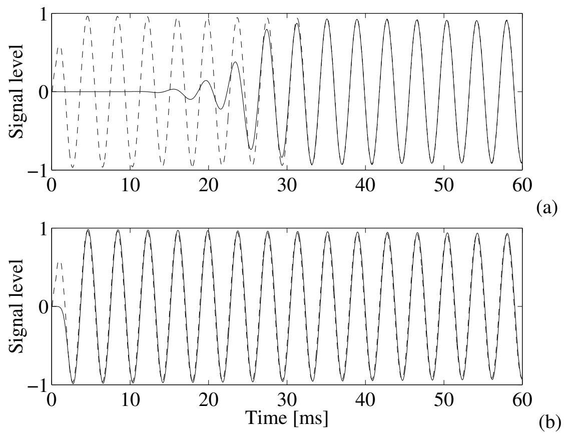

Figure 2: (a) Output of a multi-rate resonator computed by using downsampling and upsampling filters (MR1), and (b) by using upsampling filters only (MR2). The desired output is plotted with dashed line in both cases.

4.2 Skipping the dowsampling filters (MR2)

With the approach presented in the previous subsection, the complexity in terms of computations are considerably decreased compared to the single-rate approach (See Table 1). On the contrary, having separate downsampling filters for every note increases the overall complexity of the system. This can be avoided by using no downsampling filters at all. The system is the same of Fig. 1, but the $\downarrow 2$ signs refer to downsampling operation without a prior filtering.

The idea is motivated by the fact that the force signal of the hammer-string interaction is of a lowpass character [8]. This is also true for the excitation signal of other string instruments. After downsampling, the aliasing is significant only in the higher frequency region of the downsampled signal. If the resonators are present only in the lower half of the downsampled frequency band, aliasing can be avoided. The upsampling filters will have a passband of $0 \leq \vartheta \leq 0.25\,\pi$ and a stopband of $0.75\,\pi \leq \vartheta \leq \pi$ with 60 dB stopband rejection. This specification can be met by already a 6^{th} order FIR filter. However, for extremely low sampling rates, using the half of the band could still lead to aliasing. As a rule of thumb, in the case of $f_s = 44.1$ kHz, not going under a downsampling factor of 16 ($f'_s = f_s/16$) always avoided this problem.

Figure 2 (b) shows the output of the multi-rate resonators by using the second method (MR2), with the same resonator parameters as for Fig. 2 (a). The downsampling factor was 16 ($f'_s = f_s/16$) in this case. It can be seen that the amplitude of the sinusoid is almost as desired, and the long delay of Fig. 2 (a) is also avoided.

The first five partial envelopes of a synthesized C_4 (262 Hz) piano note are depicted in Fig. 3. The example was generated by using the multi-rate resonator bank without downsampling filters (MR2).

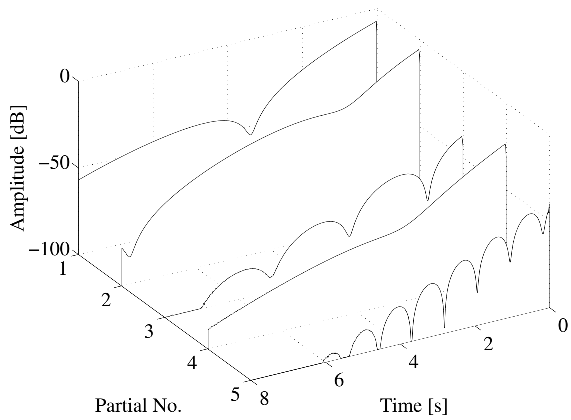

Figure 3: Partial envelopes of synthesized C_4 piano note, generated by using the method MR2.

5 Parameter estimation

The parameters of the resonators are determined by first removing the general exponential decay of the partial envelopes and then fitting an exponentially decaying or growing sinusoid on this deviation. See [9] for details. As an alternative, a technique based on standard filter design tools could be also used [5].

After having the resonator parameters computed, their initial phase and amplitude parameters have to be changed in order to correct the errors of the downsampling and upsampling filters. The total transfer function error E_k of the filters for the $\mathrm{k^{th}}$ resonator is computed as follows:

$$E_k = \prod_{i=1}^{N} H_{dn}\left(\frac{2\pi f_r 2^{i-1}}{f_s}\right) H_{up}\left(\frac{2\pi f_r 2^{i-1}}{f_s}\right) \quad (2)$$

where $H_{dn}(\vartheta)$ and $H_{up}(\vartheta)$ are the transfer functions of the downsampling and upsampling filters, and they are computed by substituting $z = e^{j\vartheta}$. The downsampling factor is 2^N, i.e., $f'_s = f_s/2^N$. In the case of the method of Subsection 4.2, where no downsampling filters are used, the formula is computed by setting $H_{dn}(\vartheta) = 1$. The amplitude A_k and phase φ_k parameters of the resonators are then modified to correct the error of E_k, and their transfer function is calculated by Eq. (1).

Note that when using linear-phase downsampling and upsampling filters, their phase delay differences could be compensated by inserting delay lines into the different branches of Fig. 1. However, this solution would not lead to better sound quality, meanwhile it would increase the complexity of the system.

5.1 Comparison

The two approaches for implementing the multi-rate resonator bank of Section 4 are compared together with the single resonator bank with respect to computational complexity. As a rough measure, Table 1 shows the number of multiplications needed for each method for computing one sample for three different notes. The number of additions are of the same order. The resonators correspond to the lowest 10, 5, or 3 partials of every note. The upsampling filters are not taken into account, since they are common for all the notes. The numbers are computed assuming normal filtering operations. Polyphase implementation of the downsampling filters would lead to slightly different results.

Table 1: Number of multiplications for the single-rate model (SR), and for the multi-rate approach with (MR1) and without (MR2) downsampling filters.

	SR	MR1	MR2
Note C_2 (65 Hz), 10 reson.	50.0	3.6	3.4
Note C_4 (262 Hz), 5 reson.	25.0	6.1	3.1
Note C_6 (1050 Hz) 3 reson.	15.0	8.7	4.3

As it can be seen in Table 1, the approach MR2 of Subsection 4.2 without downsampling needs somewhat less multiplication than the method MR1 of Subsection 4.1. The resonators of MR2 use only the half of the downsampled frequency band, thus they run at a double sampling rate compared to the corresponding resonators of MR1. This increases the number of computations, but the saving by removing the downsampling filters is larger. Moreover, removing the downsampling filters help to keep the implementation simple. Mostly because of the latter reason, the author suggests the use of the method MR2 of Subsection 4.2. By doing so, the only difference from the single-rate model is that the code of the resonators run only in every 2^{nd}, 4^{th}, 8^{th}, or 16^{th} cycle.

6 Conclusion

Previously, methods based on two parallel waveguides were used for beating and two-stage decay simulation. Their advantage is that they convey a physical meaning. On the other hand, they suffer from parameter estimation problems, since the design of the coupling filter is not trivial. Moreover, special care has to be taken to assure the stability of such systems.

In the resonator bank approach the physical properties of the coupling remain hidden, since the amplitude envelopes of the different partials are controlled separately. Its advantage is that the accuracy of the simulation can be controlled in a flexible way. The parameter estimation of such a system is robust and simple.

Two alternative methods were presented for increasing the efficiency of the resonator bank by the multi-rate approach. For the downsampling and upsampling, low-order decimation and interpolation filters are used and their passband errors are corrected by changing the parameters of the resonators. By comparing the two methods, it has been shown that using only the half of the frequency band and removing the downsampling filters can lead to smaller number of computations. By taking this approach, the general complexity of the system (code) compared to the single-rate resonator bank is increased by a negligible amount, while the number of computations are reduced by about a factor of 10. This enables the precise modeling of beating and two-stage decay of string tones at low computational cost.

As for future work, perceptual studies would be of great help in determining how many resonators should be used and whether the sound quality could be improved by implementing more complex models, e.g., by using more then one resonator per partial.

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