NONLINEAR INTERACTION IN THE DIGITAL WAVEGUIDE WITH THE APPLICATION TO PIANO SOUND SYNTHESIS

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ABSTRACT

The paper discusses a crucial part of the piano model, the hammer model – digital waveguide interaction. The discontinuity problem arising when feeding the interaction force into the digital waveguide is investigated, and a solution for its avoidance is proposed. The stability problem of the hammer model is overcome by a novel multi-rate implementation.

1 INTRODUCTION

The digital waveguide (Smith 1992) has been of high interest in the recent years, mainly because of its computational efficiency. It is based on the spatial and time domain discretization of the wave equation. When the model consists of linear elements, the so called "commuted synthesis" (Karjalainen and Välimäki 1993; Smith 1993) technique can be used, which eliminates the need for a complicated body or soundboard filter. On the contrary, the introduction of nonlinear interaction can lead to more natural sounds, since it reacts to the intervention of the musician in a physical manner. The influence of the model parameters becomes more meaningful as well.

By inserting interaction force to the digital waveguide, a discontinuity arises on the string, which cannot be interpreted physically, it is due to the sampling of the wave equation. The digital waveguide describes the physical phenomena in a correct way everywhere but at the excitation point. When there is a feedback from the string to the excitation, this will lead to incorrect results.

Another problem in the hammer model comes from the discretization of the differential equation of the hammer. When the assumption that the variables of the model change only a little with respect to time is not met, the stability of the model is at risk.

The paper first overviews the different hammer models found in the literature. Then, the discontinuity problem, which arises when feeding the interaction force into the digital waveguide is discussed. After that, the hammer model is presented, which overcomes the discontinuity problem and ensures the stability of the model.

2 PRIOR WORK

The piano hammer is generally considered as a mass connected to a nonlinear spring (see, e.g., Boutillon 1988). As it has an initial velocity, it hits the string. The spring compresses, and the interaction force pushes the hammer away from the string. The equations for the hammer behavior are the following:

$$F(t) = f(\Delta y) = \begin{cases} K(\Delta y)^p & \text{if } \Delta y > 0 \\ 0 & \text{if } \Delta y \leq 0 \end{cases} \quad (1)$$

$$F(t) = -m_h \frac{d^2 y_h(t)}{dt^2} \quad (2)$$

where $F(t)$ is the interaction force, $\Delta y = y_h(t) - y_s(t)$ is the felt compression, i.e., the difference of hammer and string displacement, K is the stiffness coefficient, and p is the stiffness exponent. In Eq. (2) m_h refers to hammer mass and $y_h(t)$ denotes the hammer displacement.

The most straightforward approach for modeling the hammer is discretizing the differential equation of the mass. This approach was taken in (Chaigne and Askenfelt 1994), where the hammer was connected to a finite difference model of the string. Borin et al. (1992) applied the same kind of power law model to the digital waveguide. Note that $f(\Delta y)$ can be arbitrary, it does not have to follow the power law of Eq. (1).

For avoiding the stability problems of the simple model, Borin and De Poli (1996); Borin et al. (1997) proposed a method which is based on the separation of known and unknown terms. The drawback of the model is that the computation of the interaction force is quite complicated, and that the model can only be used with special $f(\Delta y)$ felt characteristics.

An elegant solution for the hammer model based on the traveling wave decomposition of the mass-spring system was proposed in (Van Duyne et al. 1994). There, a distributed hammer model was attached to the string by a scattering junction. The disadvantage of this technique lies in its complexity.

In (Smith and Van Duyne 1995; Van Duyne and Smith 1995), a linear hammer model was used. The hammer was modeled by a linear filter, whose parameters were determined by nonlinear simulation. The advantage of the linear approach is that the soundboard filter can be commuted through the string and the hammer, hence it can be implemented as a wavetable. A drawback is that the nice feature of the nonlinear model, that it responds to the initial velocity in a physically meaningful way, is lost. Moreover, the restrike of the string cannot be simulated correctly.

3 THE DISCONTINUITY PROBLEM

There are two different tasks when connecting the hammer to the string. One is introducing the interaction force to the digital waveguide and the other is determining the displacement of the string, since that is the feedback signal to the hammer model. These can be done as shown in Fig. 1, where $M_{out} = M_{in}$ for this case. The displacement of the string can be determined by integrating the velocity signal v_{out}. However, there is a problem with this simple approach. The present paper deals with the problem thoroughly, since it has not been discussed in the literature.

Let us assume that the string is infinite, or terminated by a Z_0 impedance, i.e., there is no reflection from the terminations. There are two possibilities for connecting the hammer model to the digital waveguide, according to Fig. 1. One is first to read the string velocity from the delay lines and then to add the excitation signal

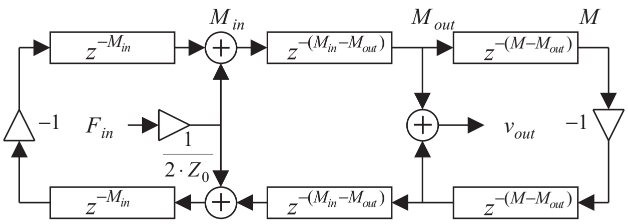

Figure 1: Digital waveguide model (Smith 1992) of the ideal string.

to the cells. After this, the delay lines are shifted (*method No. 1*). In this case, the velocity which is read from the cells at the hammer position will always be zero. Consequently, all parts of the string will be moving, but not the excitation point. Since the feedback to the hammer model is coming from the displacement of the excitation point, the hammer will behave as if it was bouncing to a rigid wall. Obviously, this method is not appropriate.

The other approach is the opposite: first we add the excitation signal to the delay cells at the position of the hammer, then read the string velocity, and shift the delay lines (*method No. 2*). Now the problem is that the string velocity and displacement at the excitation point will be twice the value of any other cells. This is illustrated in Fig. 2 (a), where $T_s = 1/f_s$ is the sampling period. Assume that a discrete unit impulse is added to the cells corresponding to the hammer position, and the delay lines of the digital waveguide are consecutively shifted. The velocity of the string, which is displayed in the figure, is calculated by summing the content of the delay cell pairs corresponding to the same spatial position. Note that the velocity of the excitation point will be twice that of the impulse. However, as this pulse travels further in the delay lines, the velocity values will be the same as the pulse amplitude. Integrating the string velocity in the time domain shows a discontinuity in the string displacement in Fig. 2 (a).

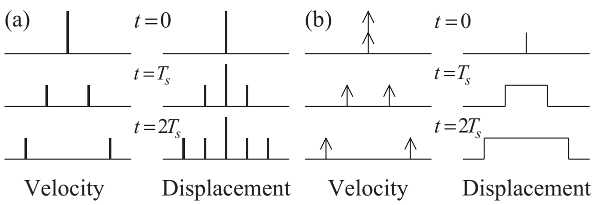

Figure 2: String velocities and displacements: (a) discrete and (b) continuos case.

The continuos string does not show this behavior: Fig. 2 (b) reveals the velocity and the displacement of a string excited with a continuos time Dirac velocity impulse. The velocity and displacement values of the string are displayed for the same time instants as the discrete model, and the arrows refer to Dirac impulses. The difference comes from the fact that although there are two continuos Dirac pulses in the region corresponding to the spatial sample of the excitation position, these stay within this region only a half time step. They travel from the midpoint of this section to its borders within $T_s/2$. Thereafter, in every spatial section of the string there will be only one impulse, staying for T_s. Since in the discrete model an impulse cannot be in a cell for less than the sampling period, we have to diminish the amplitude of the impulse by a factor of two at the interaction position. Nevertheless, the amplitude of the impulse in the other cells should not be altered.

This can be done by first adding only half of the excitation to the delay lines at the hammer position M_{in}, and after one temporal sampling interval, when the impulses moved further, we can add to them the remaining part (Bank 2000). Another solution is keeping track of the string velocity at the excitation point separately. This approach is taken in the proposed hammer model.

4 THE HAMMER MODEL

The proposed model is illustrated in Fig. 3. Now the displacement of the string is not calculated directly by integrating the output of the digital waveguide, but with the help of a separate variable inside the hammer model. Consequently, it does not matter that the digital waveguide cannot compute the string velocity at the excitation point properly.

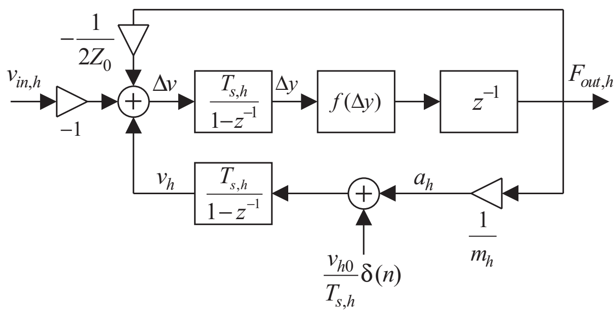

Figure 3: The core of the proposed hammer model.

The hammer model of Fig. 3 first computes the velocity difference of the the hammer and the string $\Delta v = v_h - v_s$, where v_h is the velocity of the hammer. The string velocity v_s is calculated inside the hammer model: $v_s = v_{in,h} + F_{out,h}/(2Z_0)$, where $v_{in,h}$ is the incoming string velocity, Z_0 is the string impedance, and $F_{out,h}$ is the force signal computed by the power law in the previous time instant. Note that with the traditional methods discussed in Section 3 the string velocities at the excitation point would be $v_s = v_{in,h}$, and $v_s = v_{in,h} + F_{out,h}/Z_0$, respectively. The felt compression Δy is calculated by integrating Δv with respect to time. The interaction force is computed by the felt characteristic $f(\Delta y)$, here the power law of Eq. (1) was used. A unit delay z^{-1} has been inserted to avoid a delay-free loop. The velocity of the hammer v_h is calculated by integrating the hammer acceleration $a_h = F_{out,h}/m_h$. The initial velocity v_{h0} of the hammer is controlled by sending an appropriate acceleration pulse to the integrator, or by setting the initial value of integrator to v_{h0}.

The model of Fig. 3 can be directly connected to the digital waveguide model of Fig. 1, by setting $v_{in,h} = v_{out}$, $F_{in} = F_{out,h}$, and $M_{out} = M_{in}$. The sampling rate of the hammer model now equals to that of the entire system, $T_{s,h} = T_s$. It works as follows: first the cells at position M_{in} are read and used as the input $v_{in,h}$ of the hammer model. Then, the hammer model computes the force input F_{in} to the string, and this value is added to the delay lines. After that, the delay lines are shifted. Since the string velocity $v_s = v_{in,h} + F_{out,h}/(2Z_0)$ is computed inside the hammer model, the discontinuity problem mentioned in Section 3 is avoided.

Fig. 4 shows the output $F_{out,h}$ of the hammer model. The parameters of the hammer and the string were taken from (Chaigne and Askenfelt 1994), C_4 note, and the impact velocity was set to 4 m/s. The solid line shows the interaction force of the finite difference string and hammer model of (Chaigne and Askenfelt 1994), but the stiffness of the string was set to zero. The dash-dotted line shows the hammer force when the same hammer model is connected to lossless, nondispersive digital waveguide in the traditional way (Section 3, *method No. 2*). This differs from the force of the finite difference method largely. The force signal of *method No. 1* of Section 3 is not shown because the model becomes unstable, i.e., the force signal approaches infinity, when such a method is applied. The dashed line displays the interaction force when the structure of Fig. 3 is used. The force curve is now close to that of the finite difference method.

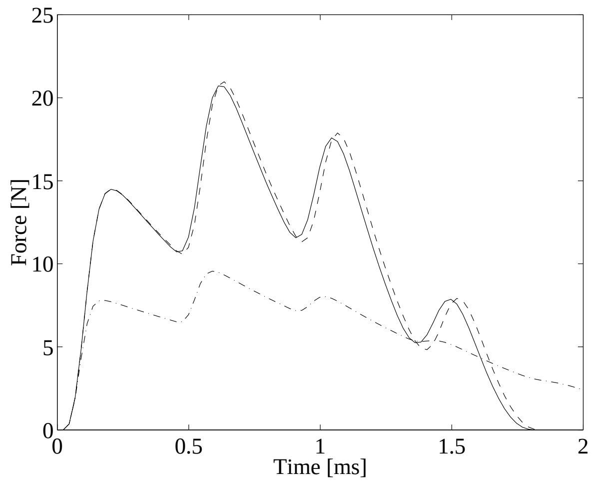

Figure 4: Interaction force for note C_4 calculated by the finite difference method of Chaigne and Askenfelt (1994) (solid line), by a digital waveguide using the traditional technique (Section 3 *method No. 2*) (dash-dotted line), and by the hammer model of Fig. 3 (dashed line).

The method based on discretizing the differential equation of the hammer works well for the low and middle range of the piano, but for the high notes with large impact velocities the model becomes unstable. This is because the assumption, that the interaction force changes only a little in one temporal sampling interval, is no longer valid for those cases. This was also noted by Borin and De Poli (1996), but their solution to the problem seems to be too complicated.

5 THE MULTI-RATE HAMMER

Here a novel multi-rate hammer model is proposed, which overcomes the stability problems. The idea comes from the fact that by increasing the sampling rate of the whole string model, the instability can be avoided. The hammer model is based on the discretization of a differential equation. It is stable when the variables change only a little in every temporal sampling interval. The stability of such a system can be always maintained by choosing a sufficiently large sampling rate, assuming that the corresponding continuos time system was stable. When the sampling period converges to zero, the discrete system will behave as the original differential equation.

Unfortunately, increasing the sampling rate by a factor of two of the whole string model would double the computation time as well. Nevertheless, if only the hammer model operates at a double rate, the computational complexity is raised by a negligible amount. Therefore, in the solution proposed here the string model operates at normal, but the hammer model runs at double sampling frequency. This is illustrated in Fig. 5.

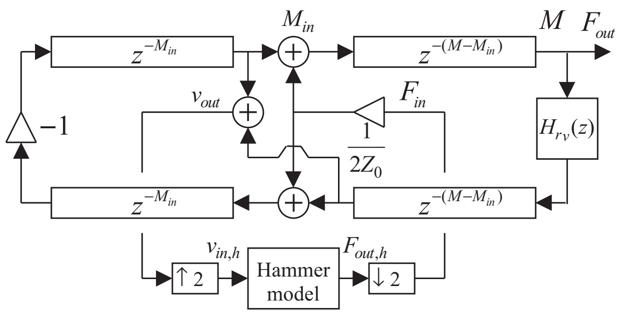

Figure 5: The multi-rate hammer and the digital waveguide.

In the proposed implementation, the core of the hammer model (Fig. 3) runs at a double sampling rate, that is, $T_{s,h} = T_s/2$. Upsampling (↑ 2 in Fig. 5) is done by linear interpolation (Schafer and Rabiner 1973). In this manner, the unknown samples will be the average of two consecutive known values. To be able to do this without introducing a delay, one should know the next incoming sample. This is easy in the case of the digital waveguide, since the upcoming values at the excitation point are already in the delay lines, exactly one spatial sampling interval away. Hence, the input for the hammer model can be calculated using linear interpolation for upsampling by the following equations:

$$\begin{aligned} v_{in,h}(nT_s) &= y^+(n, M_{in}) + y^-(n, M_{in}) \\ v_{in,h}(nT_s + T_s/2) &= \frac{y^+(n, M_{in}) + y^+(n, M_{in}-1)}{2} + \\ &+ \frac{y^-(n, M_{in}) + y^-(n, M_{in}+1)}{2} \end{aligned} \quad (3)$$

where $y^+(n, m)$ and $y^-(n, m)$ refer to the content of the upper and lower delay lines of the digital waveguide, at the time instant n and position m, respectively.

The force input for the string is computed by simply averaging the two output samples of the force model, i.e., $F_{in}(nT_s) = (F_{out,h}(nT_s) + F_{out,h}(nT_s + T_s/2))/2$.

In Fig. 6 the interaction force is shown for note C_5 (522 Hz). For the simulation, an ideal digital waveguide model was used, without any dispersion or losses. The parameters of the hammer were taken from (Chaigne and Askenfelt 1994), C_4 hammer. The impact velocity was $v_{h0} = 6$ m/s. The dash-dotted line refers to the single-rate hammer model with $f_s = 44.1$ kHz. The solid line shows the force of the single-rate model, but both the digital waveguide and hammer models operate at a double sample rate, that is, $f_s = 88.2$ kHz. This is our reference structure. The dashed line in Fig. 6 is the force of the multi-rate implementation, by using $f_s = 44.1$ kHz for the waveguide model. It can be seen that the traditional technique operating at normal sampling rate becomes unstable, while the output of the proposed multi-rate hammer model is close to the output of the single-rate model operating at double sampling frequency.

For note C_7 (2090 Hz) of the piano model (with losses and dispersion), the multi-rate hammer starts to be unstable for an impact velocity as much as 20 m/s, while the single-rate model is already unstable at around 5 m/s.

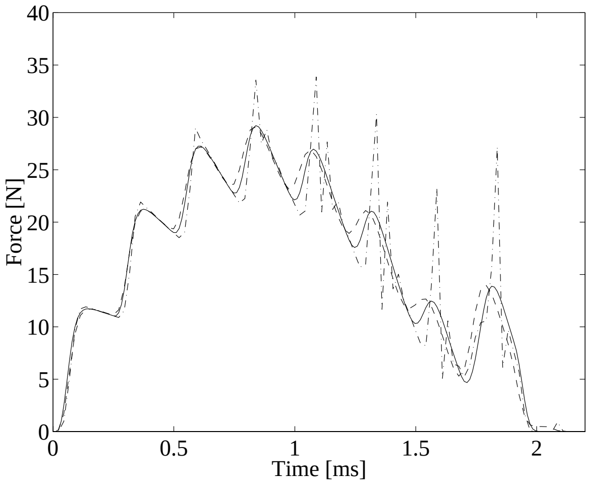

Figure 6: Simulated interaction forces for note C_5 (522 Hz), $v_{h0} = 6$ m/s, computed by the single-rate model (dash-dotted line), the single-rate model operating at double sampling frequency (solid line), and the multi-rate model (dashed line).

Surprisingly, the multi-rate method is more stable than the single-rate operating at double sampling rate. For the same string and hammer parameters as in the previous example, the single-rate model at double sampling frequency becomes unstable for $v_{h0} > 18$ m/s, the multi-rate model is still stable until $v_{h,0} = 42$ m/s. This comes from the up- and downsampling: the hammer model operates at $2f_s$, but its input $v_{in,h}$ is upsampled from a signal v_{out} whose sampling rate is f_s. Consequently, the input cannot contain strong frequency components higher than $f_s/2$. This way, the incoming high frequency components, which are mainly responsible for stability problems, are suppressed.

6 CONCLUSION

The paper dealt with connecting a nonlinear excitation to the digital waveguide with the application to piano hammer. The discontinuity problem was pointed out, which arises when feeding the interaction force into the digital waveguide. This leads to incorrect sonic results when there is a feedback from the excitation point of the string to the model of the excitation. A solution to the problem is computing the string velocity of the excitation point separately, e.g., within the excitation model. Detailed analysis was presented with the application to the piano hammer, although this treatment is valid for other instruments in which a nonlinear interaction between the digital waveguide and the excitation is present (e.g., in the violin).

With the application to the piano, a new multi-rate hammer model was proposed. The model overcomes the discontinuity problem discussed in Section 3. This is done by computing the string displacement at the excitation point within the hammer model. The stability of the hammer model is ensured by the multi-rate approach. This makes the model more stable since higher sampling rate yields a better discrete time approximation of the differential equation. It is also because the high frequency components of the input signal, which are mainly responsible for instability problems, are suppressed by the linear interpolation. Another benefit of the model is that it is simple, easy to implement, and any kind of nonlinear function can be used as a felt characteristic $f(\Delta y)$.

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