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Full room equalization at low frequencies with asymmetric loudspeaker arrangements

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ABSTRACT

For rectangular rooms with symmetric loudspeaker arrangements, full room equalization can be achieved at low frequencies, as demonstrated by previous research. The method is based on generating a plane wave that propagates along the room. However, often the room is not rectangular, and/or a symmetric loudspeaker setup cannot be assured, leading to a deteriorated equalization performance. In addition, the performance of the method drops significantly above a cutoff frequency where a plane wave cannot be generated. These problems are addressed by the proposed method by prescribing only the magnitude in the control points, while the phase is determined by an iterative optimization process starting from the plane wave solution. A true "magnitude-only" variant of the method is also presented. Comparison is given to the plane-wave based methods by introducing asymmetries to the loudspeaker setup in a simulated environment, showing that the new methods result in smaller average magnitude deviations compared to the previous plane-wave based approach.

1. INTRODUCTION

The simplest form of loudspeaker or room equalization is when the sound field is generated by a single loudspeaker, and the transfer function is equalized in a single listener position, see, e.g., [1, 2, 3, 4]. Single-input single-output (SISO) equalization is effective only in a limited region of space, determined by the wavelength of sound. The effective equalization area can be extended by measuring the sound pressure at various listening positions, and finding such a compromise in the equalization filter design that improves the performance at most positions [5, 6, 7]. We may call this single-input multiple-output (SIMO) equalization.

When using multiple loudspeakers, the equalization zone can be significantly extended due to the larger degrees of freedom in the control system, see, e.g., [8, 9, 10, 11]. In this case the monophonic input signal is filtered separately before being sent to each loudspeaker in such a way that the transfer functions between the input and the control points (virtual listening positions) best match the prescribed targets. Thus, this is a multiple-input multiple-output (MIMO) problem. It has been shown that for rectangular rooms and symmetric loudspeaker arrangements, the equalization of the entire room can be achieved at low frequencies [10, 12, 13]. The idea of the method is that instead of requiring the same pressure being present in the room at all control points, a plane wave is generated that moves along the main axis of the room [10, 12]. When using the same type of loudspeakers both at the front and back wall in a symmetric arrangement, the equalization can be implemented by simply delaying the signals of the back loudspeakers by the time the sound travels along the room and inverting their sign [13]. Besides avoiding the need for digital filtering, this variant has the advantage that it does not require the measurement of room transfer functions.

A drawback of the plane-wave based methods that they rely on the assumption of a rectangular room and a symmetric loudspeaker setup, which cannot be always assured in practice. The idea of the method proposed in this paper is that instead of prescribing both magnitude and phase in the control points, only the magnitude is prescribed. It is interesting to note that in SIMO methods where the sound field is generated by a using a single loudspeaker and the goal is to obtain a sufficiently even sound field at many listener positions [5, 6, 7], the equalization is usually done by considering the magnitude response only. Interestingly, at least to the author's knowledge, the "magnitude-only" equalization has not been yet adapted to the MIMO case.

The paper is organized as follows: first, MIMO equalization by a least squares solution is summarized in Sec. 2. This is followed by the description of the new methods in Secs. 3 and 4. Finally, a comparison of the methods is given in a simulated environment in Sec. 5.

2. MIMO EQUALIZATION WITH A LEAST SQUARES SOLUTION

We formulate the equalization problem in the frequency domain for simplicity. The frequency variable ω will be omitted from the derivations for clarity, the equations are valid for a particular frequency. For equalizing the response at a wider frequency range, the problem has to be solved at all frequencies within that range.

The MIMO equalization can be formulated as follows [8]: the sound pressure $p(\mathbf{r}_n)$ in the room is measured at N control points $\mathbf{r}_n = (x_n, y_n, z_n)$. This sound pressure is generated by L sources (loudspeakers) having the source strengths q_l, and the transfer function from the lth source to position $\mathbf{r}_n$ is given by $Z_l(\mathbf{r}_n)$. Thus, the sound pressure at position $\mathbf{r}_n$ is given by

$$p(\mathbf{r}_n) = \sum_{l=1}^{L} Z_l(\mathbf{r}_n) q_l. \tag{1}$$

Equation (1) can be written in a matrix formulation

$$\mathbf{p} = \mathbf{Z}\mathbf{q}. \tag{2}$$

where $\mathbf{p}$ is a column vector composed of the pressures $\mathbf{p} = [p(\mathbf{r}_1) \dots p(\mathbf{r}_N)]^T$, $\mathbf{q}$ is a column vector composed of the source strengths $\mathbf{q} = [q_1 \dots q_L]^T$, and $\mathbf{Z}$ is an $N \times L$ matrix containing the transfer functions $Z_l(\mathbf{r}_n)$.

Now the goal is to reproduce a desired sound field $\mathbf{p}_\mathrm{d} = [p_\mathrm{d}(\mathbf{r}_1) \dots p_\mathrm{d}(\mathbf{r}_N)]$ at the control points. This is achieved by minimizing the mean squared error

$$e_\mathrm{LS} = \sum_{n=1}^{N} |p(\mathbf{r}_n) - p_\mathrm{d}(\mathbf{r}_n)|^2 = (\mathbf{p} - \mathbf{p}_\mathrm{d})^H (\mathbf{p} - \mathbf{p}_\mathrm{d}), \tag{3}$$

where $\mathbf{x}^H$ is the conjugate transpose of $\mathbf{x}$.

The minimum of Eq. (3) is found by the least squares (LS) solution

$$\mathbf{q}_\mathrm{LS} = (\mathbf{Z}^H \mathbf{Z}) \mathbf{Z}^H \mathbf{p}_\mathrm{d}. \tag{4}$$

Note that a regularization term can also be added to Eq. (4), either based on effort or power penalty [14], but it is avoided here for simplicity.

As for the desired sound field, $p_\mathrm{d}(\mathbf{r_n})$, it is a straightforward choice to prescribe a sound pressure independent of the position $p_\mathrm{d}(\mathbf{r_n}) = p_0$ [11, 15]. However, it turns out that this choice leads to a limited area of equalization, which is in a way expected since a pressure distribution with constant magnitude and phase assumes a not moving sound field, which is unphysical.

2.1. Plane-wave based equalization

Much better equalization is obtained if the desired phases are set according to a plane wave propagation:

$$p_{\mathrm{d}}(\mathbf{r_n}) = p_0 e^{-jk(L_y - y)}, \tag{5}$$

where $k = \omega/c$ is the wave number, and y is the position in the direction of the plane wave propagation. Equation (5) actually describes the phase delay of a plane wave propagating from the front wall at $y = L_y$ to the back wall at $y = 0$. In rectangular rooms and using symmetric loudspeaker arrangements which assure that only room modes along the y axis are excited, practically perfect equalization can be achieved in the entire room up to a certain cutoff frequency [10, 12, 13]. The physical reason for such a nice performance is that plane wave propagation is a “natural choice”, since the rectangular room can be considered as a part of an infinite duct, where a single plane wave propagates at low frequencies.

Note that above the cutoff frequency where not only y axis modes are excited, a plane wave cannot be anymore constructed. Above this cutoff frequency something similar happens as above the spatial aliasing frequency in wave field synthesis, and because the control system cannot reproduce the phase constellations of the target, its only choice for minimizing the error is attenuating the outputs of all loudspeakers. This phenomenon has been described in the case of sound field reconstruction in [16], which is mathematically the same problem. Therefore, since we might use our loudspeaker setup also above this cutoff frequency, the loudspeaker signals $\mathbf{q}_{\mathrm{LS}}$ obtained from Eq. (4) are scaled so that the total power generated in the control points equals to the total power of the target

$$\mathbf{q}_{\mathrm{LS,c}} = \sqrt{\frac{\sum_{n=1}^{N} |p_{\mathrm{d}}(\mathbf{r}_n)|^2}{\sum_{n=1}^{N} |p(\mathbf{r}_n)|^2}}\mathbf{q}_{\mathrm{LS}} = \sqrt{\frac{\mathbf{p}_{\mathrm{d}}^H \mathbf{p}_{\mathrm{d}}}{\mathbf{p}^H \mathbf{p}}}\mathbf{q}_{\mathrm{LS}}, \tag{6}$$

where $\mathbf{p} = \mathbf{Z}\mathbf{q}_{\mathrm{LS}}$. This correction can be considered as a SIMO equalization after the least squares MIMO control is done, since it simply scales all loudspeaker signals simultaneously so that at least the *average* sound power in the room is as desired.

Note that the large variation in the responses above cutoff frequency will still be present even with the correction term Eq. (6), since the correction scales all the responses by the same amount. In the next sections, an improved method is presented which is based on prioritizing the magnitude of the pressure at the target points during optimization, leading to more even magnitude distribution across the room.

3. MAGNITUDE-PRIORITY EQUALIZATION BASED ON ITERATIVE PHASE UPDATE

In the proposed method the optimization is started by prescribing a plane wave and designing an equalizer by the least squares method as described in Sec. 2. Then, the equalized transfer functions from the input to the control points are computed. In the next iteration, the magnitude of the specification points remains the same, while the phase is updated to match the phase of the previously equalized transfer functions. The steps are the following:

1. Prescribe the specification as a plane wave $p_{\mathrm{d}}^{(0)}(\mathbf{r}_n) = p_0 e^{-jk(L_y - y)}$.
2. Obtain the solution by the least sqaures equation
$$\mathbf{q}_{\mathrm{IT}}^{(i)} = (\mathbf{Z}^H \mathbf{Z})\mathbf{Z}^H \mathbf{p}_{\mathrm{d}}^{(i)}. \tag{7}$$
3. Update the phase of the specification to match the phase of the control points coming from the previous iteration
$$\mathbf{p}_{\mathrm{d}}^{(i+1)} = p_0 e^{j\varphi\{\mathbf{p}_{\mathrm{IT}}^{(i)}\}}, \tag{8}$$
where $\mathbf{p}_{\mathrm{IT}}^{(i)} = \mathbf{Z}\mathbf{q}_{\mathrm{IT}}^{(i)}$.
4. Go to step 2.

The convergence of the method is fast, thus, the iteration has to be repeated only around five times.

As a result of the iteration procedure, the control system will still enforce a plane wave where possible, but if this cannot be achieved, it will give up accuracy in phase in return to accuracy in magnitude, which is perceptually much more relevant. As frequency increases, the sound field continuously departs from that of a plane wave. The correction of the average power with Eq. (6) is here unnecessary, since the iteration itself assures that there is no attenuation above the cutoff frequency, as will be demonstrated in Sec. 5.

Note that this method can be considered as the extension of magnitude-priority filter design [17] (published elsewhere in these proceedings) to a MIMO problem.

4. MAGNITUDE-ONLY EQUALIZATION

As a variant of the above method, it is also possible to let the phase to be entirely free without starting from the plane wave solution. This is achieved by minimizing the cost function in terms of magnitudes

$$e_\mathrm{M} = \sum_{n=1}^{N} (|p(\mathbf{r}_n)| - p_\mathrm{d}(\mathbf{r}_n))^2 = (|\mathbf{p}| - \mathbf{p}_\mathrm{d})^T (|\mathbf{p}| - \mathbf{p}_\mathrm{d}) = (|\mathbf{Z}\mathbf{q}_\mathrm{M}| - \mathbf{p}_\mathrm{d})^T (|\mathbf{Z}\mathbf{q}_\mathrm{M}| - \mathbf{p}_\mathrm{d}) \quad (9)$$

as a function of the source strengths $\mathbf{q}_\mathrm{M}$. In MATLAB this can be done by the nonlinear least squares solver `lsqnonlin` of the Optimization Toolbox. Note that now the target sound pressures $\mathbf{p}_\mathrm{d}$ are real, since only the magnitudes are specified. In room equalization it is reasonable to prescribe the same pressure magnitude at all positions, thus, $\mathbf{p}_\mathrm{d} = p_0$.

Since the solution of Eq. (9) cannot be obtained in a closed form, the magnitude-only method is computationally heavier than the iterative solution proposed above, but it has the advantage that the positions of the control points do not have to be known since now the target does not depend on y, unlike in Eq. (5). This means that when performing measurement, the user only has to take care that he selects measurement points evenly in space, but the location of the microphones do not have to be measured.

Note that in the case of plane-wave based method of Sec. 2.1 the y positions of all measurements have to be known quite precisely, otherwise the optimization will not be able to construct a plane wave. The phase-update method of Sec. 3 is somewhere in between, since there an error in the target phase coming from a position error is corrected in the iterations. Thus, in this case a 10–20 cm error is tolerable, meaning that only a rough position measurement is required.

5. SIMULATION RESULTS AND COMPARISON

The methods are evaluated by simulations of a rectangular room using the analytical transfer functions computed from modal summation [18, 10]. The room is depicted in Fig. 1. The size of the room is $L_x = 5$ m, $L_y = 6$ m, and the height is $L_z = 2.6$ m. The positions of the loudspeakers that would correspond to a suggested symmetric setup of [13] are $x_1 = x_3 = (1/4)L_x$ and $x_2 = x_4 = (3/4)L_x$ for the x coordinates, and $y_1 = y_2 = 0.95L_y$ and $y_3 = y_4 = 0.05L_y$ for the y coordinates. The height of all speakers is $z_1 = z_2 = z_3 = z_4 = L_z/2$. The subscripts are the indexes of loudspeakers as in Fig. 1. In the first test case the only asymmetry introduced to the setup is that loudspeaker 2 is moved closer to the listener area so that $y_2 = 0.85L_y$.

The optimization is done for a grid defined by the positions $\mathbf{x}_\mathrm{opt} = L_x \times [0.2 : 0.1 : 0.8]$ and $\mathbf{y}_\mathrm{opt} = L_y \times [0.2 : 0.1 : 0.8]$, which has the grid size of 50 × 60 cm and a total size of $L'_x = 3$ m by $L'_y = 3.6$ m. The height of the grid is $z_\mathrm{opt} = L_z/2$. The total number of control points is 49, which would be small enough also for making practical measurements within acceptable time and effort. On the other hand, the results are evaluated on a two times finer grid $\mathbf{x}_\mathrm{eval} = L_x \times [0.2 : 0.05 : 0.8]$ and $\mathbf{y}_\mathrm{eval} = L_y \times [0.2 : 0.05 : 0.8]$ leading to 169 points, so that we can also see how the methods perform in between the control points.

Figure 2 (a) displays the transfer functions at the 169 evaluation points when all the four speakers are driven with the same (but frequency dependent) signal, to show what can we obtain with SIMO multipoint equalization when one filter feeds all speakers. Basically, the amplitude of the loudspeaker signals is set so that the average power in the control points is the same as that of the target. As can be seen in Fig. 2 (a), SIMO equalization cannot provide a flat response not even at low frequencies.

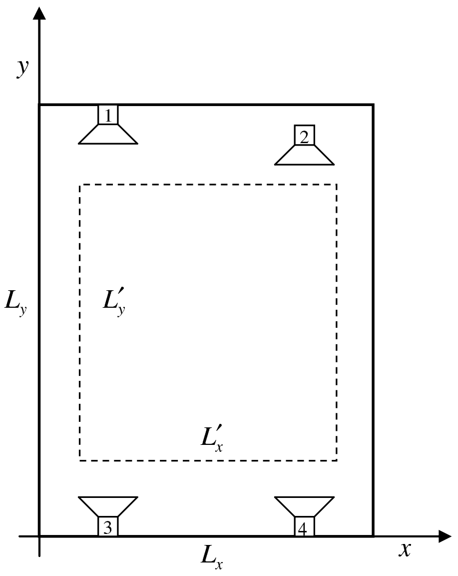

Fig. 1: The coordinate system used for room equalization. The dashed square shows the area where the equalization is performed.

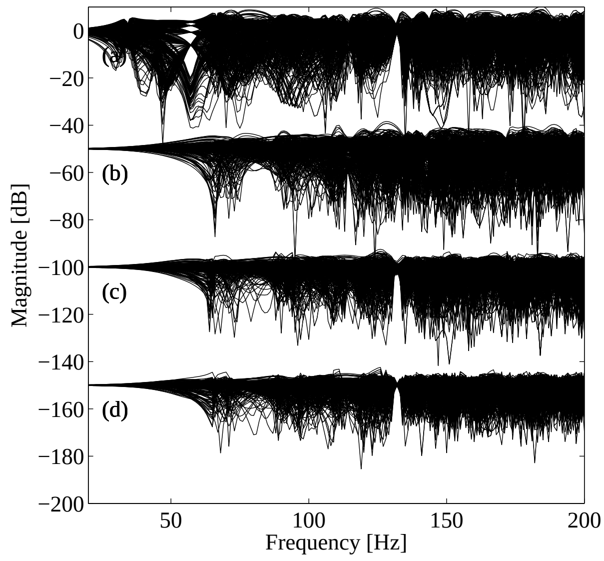

Fig. 2: The frequency responses at the 169 evaluation points with (a) SIMO multipoint equalization, (b) plane-wave based LS equalization, (c) phase-update equalization, and (d) magnitude-only equalization. The curves are offset for clarity.

The equalization is improved significantly with the plane-wave based LS equalization shown in Fig. 2 (b). Note that the correction term proposed in Eq. (6) is used in the example, otherwise the responses would be significantly attenuated above 100 Hz. It can also be seen that when using the phase-update (c) and magnitude-only (d) equalization methods, the responses are the same as with the plane-wave method (b) below the cutoff frequency of approximately 80 Hz, while above they have smaller variation.

While the frequency responses of Fig. 2 give some impression about the equalizer performance, they don't give too much indication about the perceived quality. For example, the narrow dips that populate the higher range of the figure are generally considered inaudible. Therefore, it would be beneficial to process the frequency responses with a binaural model that would give an indication of the perceived timbre. The simplest way of taking this into account is fractional-octave smoothing the measured responses. The third-octave power smoothed transfer functions are displayed in Fig. 3. Now it is clear that while the plane-wave based solution (b) is better than simple SIMO equalization (a) below 80 Hz, above it gives the same variation. The variation is significantly decreased by the phase-update method (c) and even more by the magnitude-only method (d).

To give an aggregate measure, the mean absolute deviation from the target is computed in dB using the smoothed transfer functions of Fig. 3. This is displayed in Fig. 4. The superior performance of the new methods is again clear, showing that the average absolute deviation is in the order of 1-2 dB in the entire frequency range.

It is also interesting to take a look at the 10-90 percentiles of the transfer functions computed at the 169 evaluation points. From one particular line type the upper curve shows the limit above which lie only 10% of the responses, and the lower curve shows the limit below which lie another 10% of them. In other words, 80% of the responses are in between the two lines. Especially the performance of the magnitude-only variant is notable: 80% of the responses are within +1/-2 dB, expect a smaller region where they

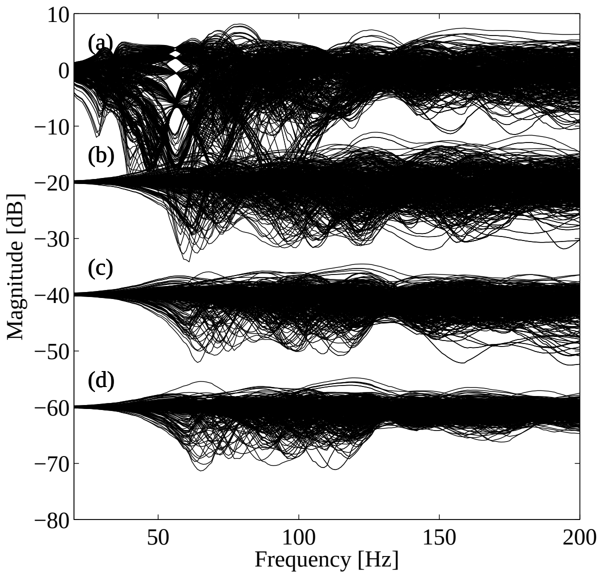

Fig. 3: Third-octave smoothed frequency responses at the 169 evaluation points with (a) SIMO multipoint equalization, (b) plane-wave based LS equalization, (c) phase-update equalization, and (d) magnitude-only equalization. The curves are offset for clarity. Note the different scale of the y axis compared to Fig. 2

are within +1/-4dB. This shows that indeed practically useful equalization can be achieved in a wide area within a normal sized room with using only four subwoofers.

The differences between the methods are similar also in the case of other loudspeaker arrangements. It is important to note that the new methods perform better than the plane-wave based method even with completely symmetric loudspeaker arrangements above the cutoff frequency (80 Hz in our examples). The symmetric setup is $x_1 = x_3 = (1/4)L_x$ and $x_2 = x_4 = (3/4)L_x$ for the x coordinates, and $y_1 = y_2 = 0.95L_y$ and $y_3 = y_4 = 0.05L_y$ for the y coordinates. The 10-90 percentiles for the symmetric case are displayed in Fig. 6.

The final example is when all the loudspeakers are moved in different directions by as much as 0.5 m, displayed in Fig. 7. The loudspeaker positions used in the simulations are $x_1 = 0.15L_x$, $x_2 = 0.75L_x$, $x_3 = 0.25L_x$, and $x_4 = 0.65L_x$ for the x coordinates, and $y_1 = 0.95L_y$, $y_2 = 0.85L_y$, $y_3 = 0.15L_y$, $y_4 = 0.05L_y$ for the y coordinates. Note that we are still able to achieve a +1/-4 dB performance in the 10-90 percentiles with the magnitude-only method, it is only that the cutoff frequency decreases so that the -4dB area gets wider.

6. CONCLUSION

In this paper a new method has been presented for global sound equalization in rooms at low frequencies. The method provides a significant improvement over the traditional plane-wave based methods, because where it is possible, it maintains the plane wave solution, while where it is not, it focuses on even magnitude distribution, which is perceptually more relevant than reproducing a correct phase. A true magnitude-only variant has also been presented, giving the advantage that the microphone positions do not have to be measured.

For the evaluation and comparison of global equalization methods, the paper proposed the analysis of fractional-octave smoothed transfer functions that eliminate the effects of inaudible narrow dips from the statistics. By performing such an analysis, it turns out that the new methods are able to equalize a normal-sized room in such a way that only 10% of the transfer functions are above the reference by

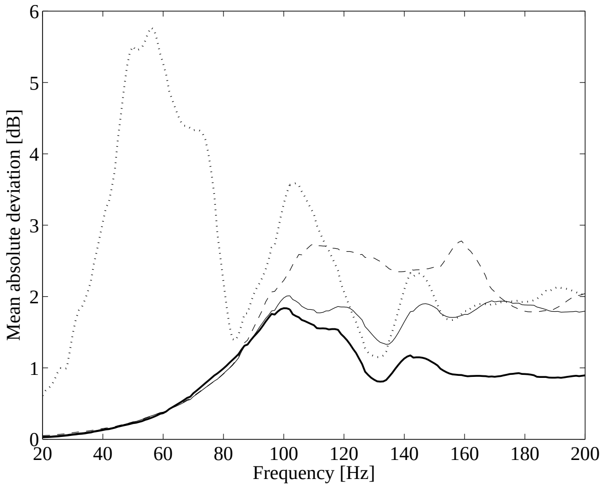

Fig. 4: Mean absolute deviation of the third-octave smoothed frequency responses from the target with (dotted line) SIMO multipoint equalization, (dashed line) plane-wave based LS equalization, (thin solid line) phase-update equalization, and (thick solid line) magnitude-only equalization.

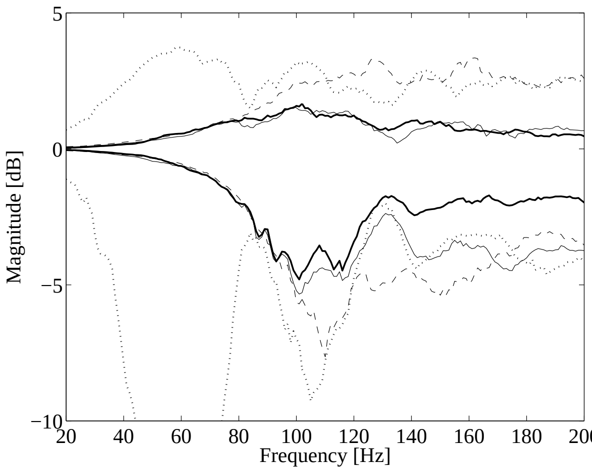

Fig. 5: 10-90 percentiles of the third-octave smoothed frequency responses with (dotted line) SIMO multipoint equalization, (dashed line) plane-wave based LS equalization, (thin solid line) phase-update equalization, and (thick solid line) magnitude-only equalization.

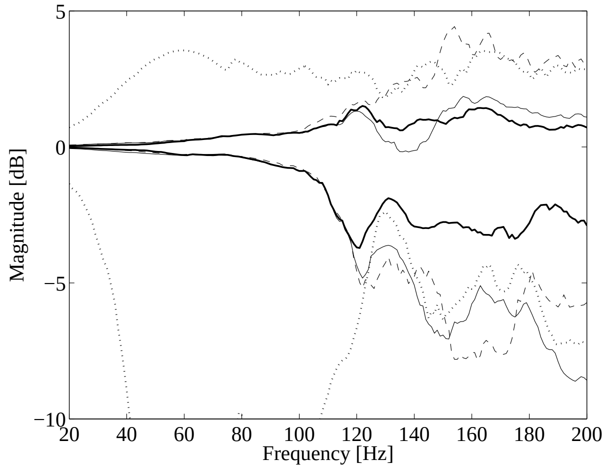

Fig. 6: Room equalization with a symmetric loudspeaker arrangement: 10-90 percentiles of the third-octave smoothed frequency responses with (dotted line) SIMO multipoint equalization, (dashed line) plane-wave based LS equalization, (thin solid line) phase-update equalization, and (thick solid line) magnitude-only equalization.

more than 1 dB, and only 10% are below by more than 4 dB up to 200 Hz, even with an asymmetry in the loudspeaker setup. This can be considered as an excellent result since just four loudspeakers are used for sound reproduction in these examples.

Future work includes the simulation of both variants of the new method in non-rectangular rooms by the use of a finite-difference room model like in [13]. It is expected that in non-rectangular rooms the benefit of the phase-update and magnitude-only methods will be even larger compared to the plane-wave based solutions. Finally, both variants should be implemented in a real room to investigate their properties from a perceptual point of view.

A related field of possible application of the phase-update method is least squares sound field reconstruction [16], where the new method would hopefully result in better behaving magnitude responses above the spatial aliasing frequency, while keeping the same accuracy below.

7. ACKNOWLEDGEMENT

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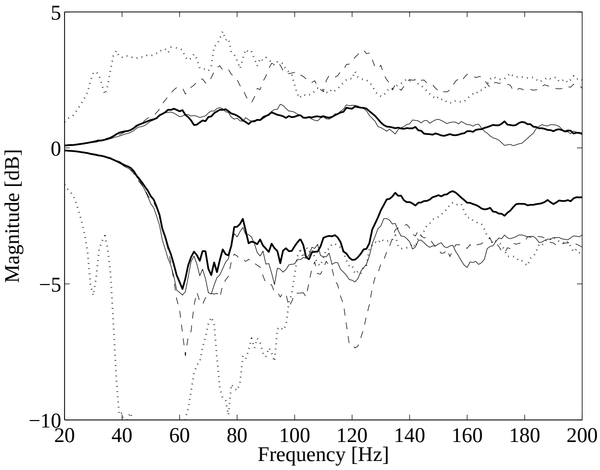

Fig. 7: Room equalization when all the speakers are moved in different directions by as much as half meter compared to the symmetric setup: 10-90 percentiles of the third-octave smoothed frequency responses with (dotted line) SIMO multipoint equalization, (dashed line) plane-wave based LS equalization, (thin solid line) phase-update equalization, and (thick solid line) magnitude-only equalization.

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