Audio Engineering Society

Convention Paper

Presented at the 130th Convention
2011 May 13–16 London, UK

Warped IIR filter design with custom warping profiles and its application to room response modeling and equalization

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ABSTRACT

In traditional warped FIR and IIR filters, the frequency-warping profile is adjusted by a single free parameter, leading to a less flexible allocation of frequency resolution. As an example, it is not possible to achieve a truly logarithmic frequency resolution, which would be often desired in audio applications. In this paper a new approach is presented for warped IIR filter design where the filter specification is transformed by any desired (e.g., logarithmic) frequency transformation, and a standard IIR filter is designed to this transformed specification. Then, the poles and zeros of this transformed filter are found and mapped back to the original frequency scale. Due to the approximations in mapping back the poles and zeros, the resulting transfer function shows some discrepancies from its optimal version. This is resolved by an additional optimization of the zeros of the final filter. Examples of loudspeaker–room response modeling and equalization are presented.

1. INTRODUCTION

In audio, filter and equalizer design should take into account the frequency resolution of hearing for achieving the best possible sound quality at a given computational cost. Since traditional FIR and IIR filter design methods result in a linear frequency resolution, specialized filter design methodologies have been developed. One of the most often used methodology is warped filter design [1, 2], where the unit delay of traditional FIR or IIR filters is replaced by a first-order allpass filter, resulting in the transformation of the frequency axis. The allocation of the frequency resolution is controlled by the pole λ of the allpass filter. A drawback of warped filter design is that the frequency resolution is determined by a single parameter λ, resulting in a limited range of warping profiles. For example, there is no such λ value that would result in a truly logarithmic frequency resolution. This can be partly resolved by designing multi-band warped filters, where the warping parameter is different in the various subbands [3, 4], at the expense of design complexity.

To overcome the limitation of a single parameter controlling the frequency resolution, various other design approaches have been developed. For example, in [5] logarithmic frequency-scale warping is realized by using a parallel set of all-pass filters of increasing order. However, a disadvantage of the method is its heavy computational load making it impractical for real-time applications. Kautz filters [6, 7] can be seen as the generalizations of warped FIR filters, where the allpass poles can be different for all the sections. The frequency resolution of Kautz filters is controlled by the pole positions. For example, setting the pole frequencies to a logarithmic scale results in a logarithmic frequency resolution [7]. Recently, the fixed-pole design of second-order parallel filters [8, 9] have been proposed, resulting in the same transfer function as that of Kautz filters [10], while requiring 33% less arithmetic operations for the same filter order.

This paper presents a new method for warped IIR filter design that can use arbitrary warping profiles. The method starts with defining a custom frequency mapping function that will determine the allocation of frequency resolution. Then, the filter specification is transformed by this mapping function, and an IIR filter is designed to the transformed specification by standard IIR filter design tools, e.g., by the `invfreqz` command in Matlab. Next, the poles and zeros of this transformed filter are found and mapped back to the original frequency scale. Finally, the filter is implemented as a series or parallel set of second-order filters.

The organization of the paper is as follows: Sec. 2 overviews traditional warped filter design, Sec. 3 presents the custom warping method, and Sec. 4 provides loudspeaker–room modeling and equalization examples, and comparison to other filter design techniques. Finally, Sec. 5 concludes the paper and gives directions for future research.

2. WARPED FILTER DESIGN

The most commonly used perceptually motivated design technique is based on frequency warping [1, 2]. The basic idea of warped filters is that the unit delay z^{-1} of traditional FIR or IIR filters is replaced by an allpass filter

$$z^{-1} \leftarrow D(z) = \frac{z^{-1} - \lambda}{1 - \lambda z^{-1}}. \tag{1}$$

Such a substitution results in a transformation of the frequency axis

$$\tilde{\vartheta} = \nu(\vartheta) = \arctan \frac{(1-\lambda^2)\sin(\vartheta)}{(1+\lambda^2)\cos(\vartheta) - 2\lambda}, \tag{2}$$

where ϑ is the original and $\tilde{\vartheta}$ is the warped angular frequency in radians [2]. Accordingly, a filter originally having the transfer function of $H(\vartheta)$ will have the transfer function of $H(\nu(\vartheta))$ after substituting its delay elements by the first order allpass of Eq. (1).

In the time domain, the design of warped filters starts with warping the target impulse response $h_{\mathrm{t}}(n)$ by the use of an allpass chain, which can be considered as the frequency-dependent resampling of the impulse response [2]. Then, warped FIR (WFIR) filters can be obtained by truncating or windowing the warped target response $\tilde{h}_{\mathrm{t}}(n)$. Warped IIR filters are designed by traditional filter design algorithms (e.g., Prony, Steiglitz-McBride) using this warped $\tilde{h}_{\mathrm{t}}(n)$. In the frequency-domain, the target specification $H(\vartheta)$ is first transformed by inverse of Eq. (2), then an FIR or IIR filter is designed based on this mapped specification.

The WFIR filters have a similar structure as FIR filters, but the unit delays are replaced by the allpass filter $D(z)$. That is, the WFIR filter is an allpass chain, where the signals between the first-order allpass blocks are tapped and weighted by b_k. Because of the allpass elements, WFIR filters are actually IIR filters, and only their structure and design resemble to that of FIR filters. For WIIR filters the replacement of unit delays by $D(z)$ leads to delay-free loops, and the filter structure has to be modified for practical implementation [2].

Because of the specialized filter structures, WFIR and WIIR filters require 2–4 times higher computational complexity compared to normal FIR and IIR filters of the same order [2]. For WIIR filters, this additional complexity can be avoided if the filters are "dewarped" to a direct form IIR filter, but this can be done only up to filter orders of 20 due to numerical problems coming from pole clustering at low frequencies [2]. Another option is to find the poles $\tilde{p}_k$ and zeros $\tilde{m}_k$ of the warped IIR filter, dewarp and them by the expression

$$p_k = \frac{\tilde{p}_k + \lambda}{1 + \lambda \tilde{p}_k}, \quad m_k = \frac{\tilde{m}_k + \lambda}{1 + \lambda \tilde{m}_k}. \tag{3}$$

Finally, the filter is implemented as a series of second-order sections, computed from the dewarped (linear frequency-scale) poles p_k and zeros m_k [11].

3. CUSTOM WARPING

This paper presents a new method for warped IIR filter design that can use arbitrary warping profiles. The filter design steps are explained by using a loudspeaker–room response modeling example.

3.1. Frequency mapping

The method starts with defining a custom frequency mapping function that will determine the allocation of frequency resolution. In the examples of the paper, a logarithmic frequency transformation is used, leading to logarithmic frequency resolution, but it is emphasized that any other profiles can be used. We should map the original angular frequencies $\vartheta \in [0, \pi]$ to the warped frequencies $\tilde{\vartheta} = \nu(\vartheta) \in [0, \pi]$ by a smooth function. That is, $\nu(\vartheta)$ and its first derivative $\nu'(\vartheta) = d\nu(\vartheta)/d\vartheta$ should be continuous, because we will use these functions for pole dewarping. Here we use the warping function proposed in [5] which is linear below a frequency limit ϑ_c and logarithmic above. The linear function is chosen so that the derivative does not jump at ϑ_c:

$$\tilde{\vartheta} = \nu(\vartheta) = \begin{cases} a\vartheta & \text{if } 0 \leq \vartheta < \vartheta_\mathrm{c} \\ \pi \frac{\ln(b\vartheta)}{\ln(b\pi)} & \text{if } \vartheta_\mathrm{c} \leq \vartheta < \pi \end{cases}, \quad (4a)$$

$$a = \frac{\pi}{\vartheta_\mathrm{c}(1 + \ln(\pi/\vartheta_c))}, \quad (4b)$$

$$b = \frac{e}{\vartheta_\mathrm{c}}, \quad (4c)$$

where $e = e^1 = \exp(1)$. Let us also define the inverse mapping $\nu^{-1}(\tilde{\vartheta})$ so that $\vartheta = \nu^{-1}(\nu(\vartheta))$. The mapping function $\nu(\vartheta)$ is depicted in Fig. 1 for $\vartheta_c = 0.0071$, which corresponds to 50 Hz with $f_\mathrm{s} = 44.1$ kHz.

Then, the filter specification is transformed by this mapping function so that the original specification points $H_\mathrm{t}(\vartheta_n)$ are moved to the frequencies $\tilde{\vartheta}_k = \nu(\vartheta_n)$. Mathematically, this mapping is described by $\tilde{H}_\mathrm{t}(\tilde{\vartheta_n}) = H_\mathrm{t}(\nu^{-1}(\tilde{\vartheta_n}))$.

It may also be interesting to compute the equivalent $\lambda(\vartheta)$ values that would correspond to the mapping of Fig. 1. This can be done by solving Eq. (2) for λ for each angular frequency pair ϑ, $\tilde{\vartheta}$ given by the logarithmic mapping

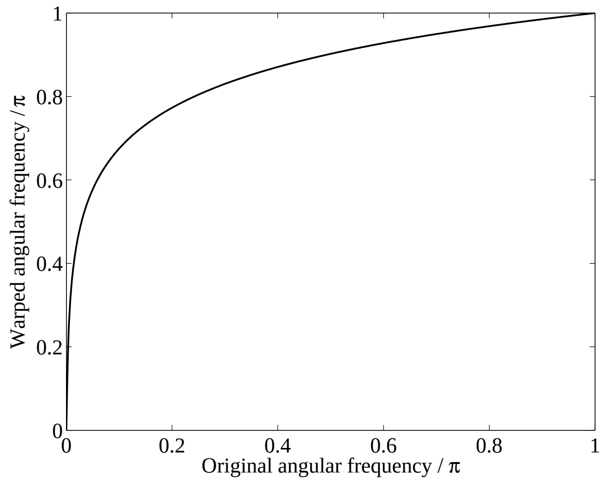

Fig. 1: The frequency mapping function $\nu(\vartheta)$ of Eq. (4).

of Eq. (4) and Fig. 1. This is displayed in Fig. 2, which also explains why straightforward warped designs with a single λ cannot provide a truly logarithmic resolution. Note that in multi-band warped methods the "varying" λ is approximated by using different λ parameters in different frequency bands [3, 4, 12].

The target specification used for illustrating the design steps is a loudspeaker–room response, which is smoothed to a 12th octave resolution by the complex smoothing method of [13]. The logarithmically warped filter specification is displayed in Fig. 3 thin line. Note that the warped angular frequency is plotted in a linear scale.

3.2. Filter design and pole dewarping

At this step an IIR filter is designed to the warped specification $\tilde{H}_\mathrm{t}(\tilde{\vartheta})$ by any of the traditional filter design methods. Here the `invfreqz` command of MATLAB is used to design a 32th order IIR filter. The resulting response is shown in Fig. 3 thick line. To avoid unstable filters, the warped specification is made minimum-phase prior to filter design.

Then, the poles and zeros of this warped filter $\tilde{H}(\tilde{\vartheta})$ are found and mapped back to the original frequency scale. For complex poles, we first compute the pole frequencies $\tilde{\vartheta}_{p,k} = \varphi\{\tilde{p}_k\}$ and radii $\tilde{r}_{p,k} = |\tilde{p}_k|$. Then, the dewarped

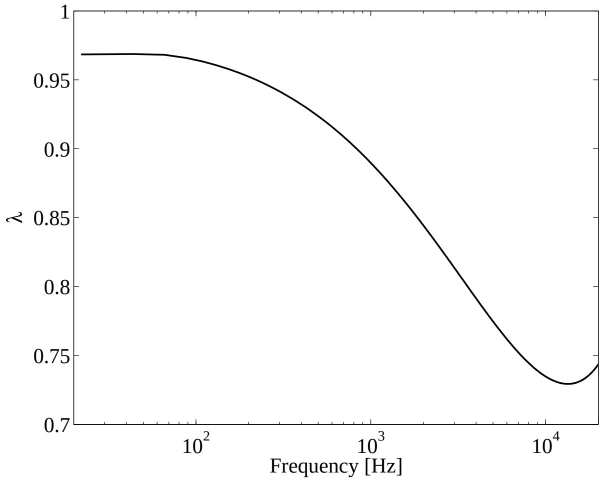

Fig. 2: Equivalent frequency warping parameter values $\lambda(\vartheta)$ values as a function of frequency that would give the logarithmic mapping of Fig. 1. Note that the x axis is in Hz ($f = f_\mathrm{s}\vartheta/(2\pi)$ with $f_\mathrm{s} = 44.1$ kHz), and in a logarithmic scale.

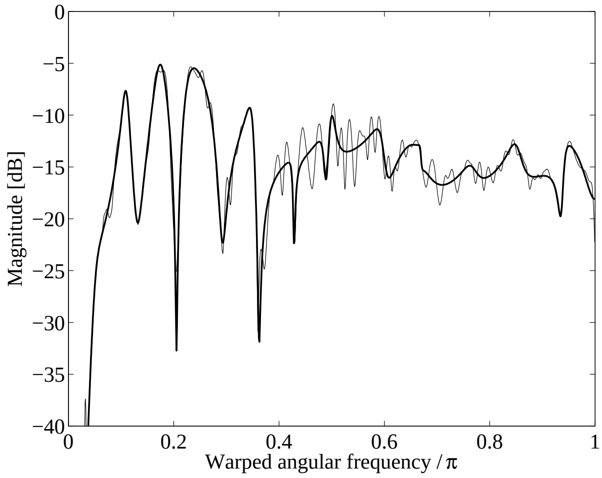

Fig. 3: Logarithmically warped target specification (12th octave smoothed minimum-phase loudspeaker–room response, displayed by thin line) and a 50th order IIR filter designed by `invfreqz` in MATLAB (thick line). Note that the warped angular frequency is displayed in a linear scale.

poles p_k arise as

$$\vartheta_{p,k} = \nu^{-1}(\tilde{\vartheta}_{p,k}), \tag{5a}$$

$$r_{p,k} = \tilde{r}_{p,k}^{\nu^{-1'}(\tilde{\vartheta}_{p,k})}, \tag{5b}$$

$$p_k = r_{p,k} e^{j\vartheta_{p,k}}, \tag{5c}$$

that is, the pole frequencies are mapped according to the inverse mapping function $\nu^{-1}(\tilde{\vartheta})$ and the radii are raised to the power according to the derivative of the inverse mapping function $\nu^{-1'}(\tilde{\vartheta})$. The complex zeros are mapped in exactly the same way. For real poles and zeros we compute their frequencies (the -3dB point of their transfer functions) and remap them by $\nu^{-1}(\tilde{\vartheta})$.

Then, the poles and zeros are paired to form a filter with a series of second-order sections. The resulting response is displayed in Fig. 4 thick dashed line, together with the specification (thin solid line) in the original frequency scale. (Note the logarithmic frequency axis in Fig. 4 as opposed to the linear one in Fig. 3.)

It can be seen that the resulting filter response (thick dashed line) is tilted compared to the specification (thin line). This comes from the inaccuracies of pole-zero remapping. More specifically, when a pole is dewarped in normal warped filters, a zero arises at λ, while when a zero is dewarped, a pole arises at λ. Since λ is constant in traditional warped filters, these additional poles and zeros cancel out. However, in our case every dewarping corresponds to a different equivalent λ value, and the effects of these not-implemented poles and zeros accumulate.

3.3. Response correction

In principle, the frequency response could be corrected by running a post-optimization in pole-zero form, where the dewarped poles and zeros are used as starting values. A simpler solution is to design a low-order correcting filter and put this in series with the original filter. In practice, filter orders of 4–8 are needed to correct the filter behavior. A trivial drawback of using a correction filter is the increase of computational complexity.

Here a different solution is proposed that is both computationally simple and does not require the increase of filter order. The idea is to keep the poles as they are, and optimize only the zeros instead. This is most easily done when the filter is converted to a parallel form, since in that case the problem becomes linear in its free parameters, so they can be computed by the well-known LS

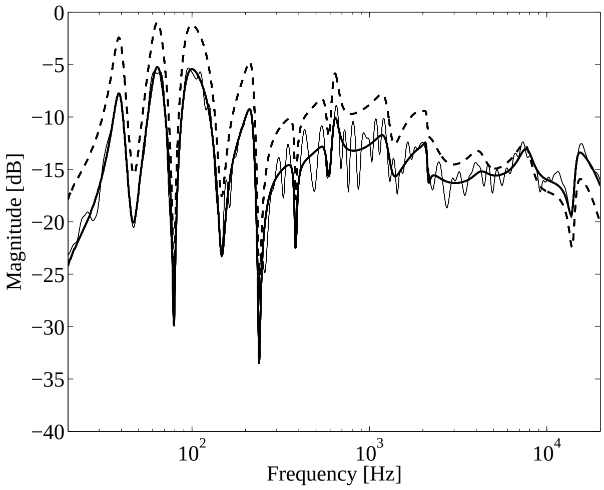

Fig. 4: Minimum-phase loudspeaker–room response modeling: original specification (thin line), 32nd order series second-order filter after pole-zero dewarping (thick dashed line), and a 32nd order parallel second-order filter after the optimization of the zeros (thick solid line).

solution in a closed form. This variant can also be considered as a new pole positioning strategy for the fixed-pole design of parallel second-order filters [8, 9], since now only the poles are used from the custom warped filter design, and the zeros are estimated by the linear least squares equations.

The filter consists in a parallel set of second-order sections:

$$H(z^{-1}) = \sum_{k=1}^{K} \frac{d_{k,0} + d_{k,1}z^{-1}}{1 + a_{k,1}z^{-1} + a_{k,2}z^{-2}} \qquad (6)$$

where K is the number of sections. Once the denominator coefficients are determined by the dewarped poles ($a_{k,1} = p_k + \overline{p}_k$ and $a_{k,2} = |p_k|^2$), the problem becomes linear in its free parameters $d_{k,0}$, $d_{k,1}$.

Writing (6) in matrix form for a finite set of ϑ_n angular frequencies yields

$$\mathbf{h} = \mathbf{M}\mathbf{p} \qquad (7)$$

where $\mathbf{p} = [d_{1,0}, d_{1,1}, \dots d_{K,0}, d_{K,1},\ b_0 \dots b_M]^T$ is a column vector composed of the free parameters. The rows of the modeling matrix $\mathbf{M}$ contain the transfer functions of the second-order sections $1/(1 + a_{k,1}e^{-j\vartheta_n} + a_{k,2}e^{-j2\vartheta_n})$ and their delayed versions $e^{-j\vartheta_n}/(1 + a_{k,1}e^{-j\vartheta_n} + a_{k,2}e^{-j2\vartheta_n})$ for the ϑ_n angular frequencies. Finally, $\mathbf{h} = [H(\vartheta_1) \dots H(\vartheta_N)]^T$ is a column vector composed of the resulting frequency response.

Now the task is to find the optimal parameters $\mathbf{p}_{\mathrm{opt}}$ such that $\mathbf{h} = \mathbf{M}\mathbf{p}_{\mathrm{opt}}$ is closest to the target frequency response $\mathbf{h}_{\mathrm{t}} = [H(\vartheta_1)_{\mathrm{t}} \dots H(\vartheta_N)_{\mathrm{t}}]^T$. If the error is evaluated in the mean squares sense

$$e_{\mathrm{LS}} = \sum_{n=1}^{N} |H(\vartheta_n) - H(\vartheta_n)_{\mathrm{t}}|^2 = (\mathbf{h} - \mathbf{h}_{\mathrm{t}})^H(\mathbf{h} - \mathbf{h}_{\mathrm{t}}), \qquad (8)$$

the minimum of (8) is found by the well-known least-squares (LS) solution

$$\mathbf{p}_{\mathrm{opt}} = (\mathbf{M}^H\mathbf{M})^{-1}\mathbf{M}^H\mathbf{h}_{\mathrm{t}} \qquad (9)$$

where $\mathbf{M}^H$ is the conjugate transpose of $\mathbf{M}$.

Note that (9) assumes a filter specification $H_{\mathrm{t}}(\vartheta_n)$ given for the full frequency range $\vartheta_n \in [-\pi, \pi]$. Thus, for obtaining filters with a real impulse response, we have to ensure that the frequency domain specification is conjugate-symmetric, that is, $H_{\mathrm{t}}(-\vartheta_n) = \overline{H}_{\mathrm{t}}(\vartheta_n)$, where $\overline{H}_{\mathrm{t}}$ is the complex conjugate of H_{t}.

Finally, the parallel filter might be converted to a series of second-order sections, or, implemented directly in the parallel form. An advantage of the parallel form is that it possesses favorable numerical properties [14] and it has the potential for full code parallelization. Also, this way we avoid the numerical problems that may arise during conversion.

The frequency response of a 32th order parallel second-order filter designed using the dewarped poles p_k is displayed in Fig. 4. It can be seen that now the filter response matches the target specification quite precisely.

4. DESIGN EXAMPLES AND COMPARISON

4.1. Loudspeaker–room response modeling

Figure 5 provides the comparison of the new method to previous filter design techniques for a 12th-octave smoothed minimum-phase loudspeaker–room response modeling example (the specification is the same as in the example of Sec. 3). Figure 5 (a) shows a 32nd-order warped IIR design ($\lambda = 0.9$) estimated by the Steiglitz-McBride method [15]. It can be seen that the frequency resolution of the filter is concentrated to a limited region of the full frequency scale. The objective measure used here is the mean absolute dB error [4] computed between the target and filter response, and evaluated on a logarithmic frequency scale between 20 Hz and 20 kHz. The error values are displayed in Fig. 5 above the corresponding responses.

Figure 5 (b) shows a parallel filter design with a predetermined pole set [9], where 16 pole pairs are placed between 30 Hz and 20 kHz on a logarithmic scale. Now the frequency resolution is spread evenly on a logarithmic scale, but modeling at low frequencies is still not very accurate. Figure 5 (c) shows a 32nd-order parallel filter design using the multi-band pole positioning method of [12], where the poles of the parallel filter are obtained by designing separate warped filters for the low- and high-frequency range with different warping parameters ($\lambda_{\mathrm{LF}} = 0.986$ and $\lambda_{\mathrm{HF}} = 0.65$). It can be seen that a very good fit is achieved.

Finally, Figure 5 (d) shows the response of the 32nd-order filter designed by the custom warping method of Sec. 3, which slightly outperforms the already excellent fit of the multi-band parallel filter method (c). Besides the small increase in accuracy, the benefit of the new method compared to the multi-band parallel filter method [12] is its simplicity. Moreover, it can be used with arbitrary warping profiles, while adapting the multi-band method for non-logarithmic profiles would be quite complicated.

To show the robustness of the design, a 1000th order filter is designed to the original (unsmoothed) loudspeaker–room response. The filter specification is displayed in Fig. 6 (a), while the modeled response is shown in in Fig. 6 (b). Since the filter is directly designed in a parallel second-order form, it can be implemented without numerical problems, despite its high order.

4.2. Loudspeaker–room response equalization

Next, the custom warping method is applied to loudspeaker–room response equalization. The equalizer is designed using a 12th octave complex-smoothed version of the measured loudspeaker–room response. The desired response is a second-order highpass filter with a cutoff frequency of 50 Hz. The equalizer is designed so that both the loudspeaker–room response $H_{\mathrm{r}}(\vartheta)$ and the desired response $H_{\mathrm{d}}(\vartheta)$ are mapped by the frequency mapping function $\nu(\vartheta)$. Next, an IIR filter $\tilde{H}(\tilde{\vartheta})$ is identified between the warped responses $\tilde{H}_{\mathrm{r}}(\tilde{\vartheta})$ and $\tilde{H}_{\mathrm{d}}(\tilde{\vartheta})$

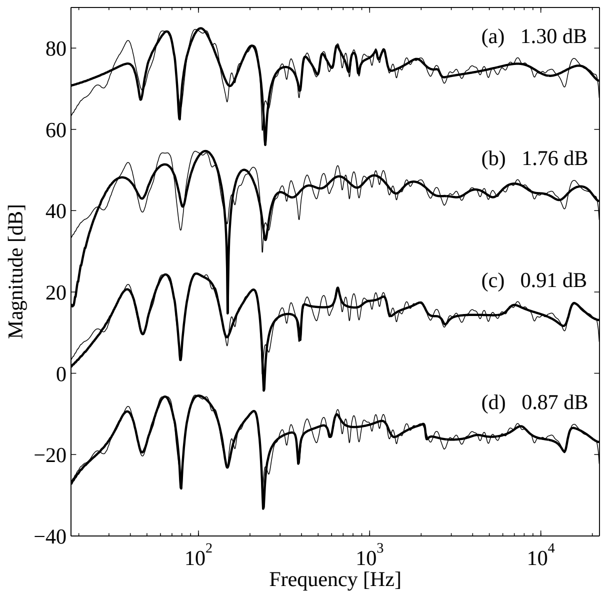

Fig. 5: Minimum-phase loudspeaker–room response modeling with various techniques. The thin lines show the minimum-phase target, and the filter order is 32 in all cases. The thick lines show the filter responses for (a) the warped IIR filter designed with $\lambda = 0.9$, (b) the parallel filter with logarithmic pole positioning, (c) the parallel filter with multi-band pole positioning, and (d) the filter designed by the proposed custom warping method. The dB values indicate the mean absolute errors [4].

by the frequency-domain Steiglitz-McBride method [16] so that the equalized response $\tilde{H}(\tilde{\vartheta})\tilde{H}_{\mathrm{r}}(\tilde{\vartheta})$ best matches the desired response $\tilde{H}_{\mathrm{d}}(\tilde{\vartheta})$. Then the poles and zeros are dewarped, and the zeros are corrected, as described in Sec. 3.

Figure 7 shows the smoothed loudspeaker–room response (a), and its equalized versions (b)–(e) using filters of increasing order (32, 64, 96 and 128, respectively). Note that for practical room equalization applications, the lower-order responses (b) and (c) would be sufficient, and (e) and (f) are displayed only to show the capabilities of the method.

5. CONCLUSION AND FUTURE RESEARCH

This paper has presented a new warped IIR filter design method where the warping profile can be arbitrary, as

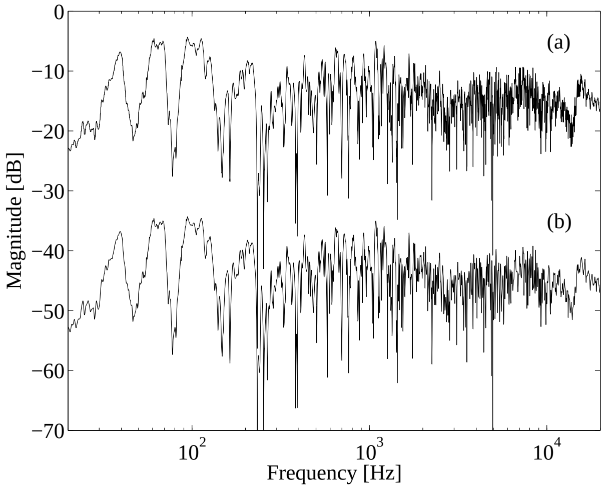

Fig. 6: High-order loudspeaker–room response modeling example: (a) the measured frequency response and (b) the frequency response of the 1000th order IIR filter designed by the proposed custom warping method.

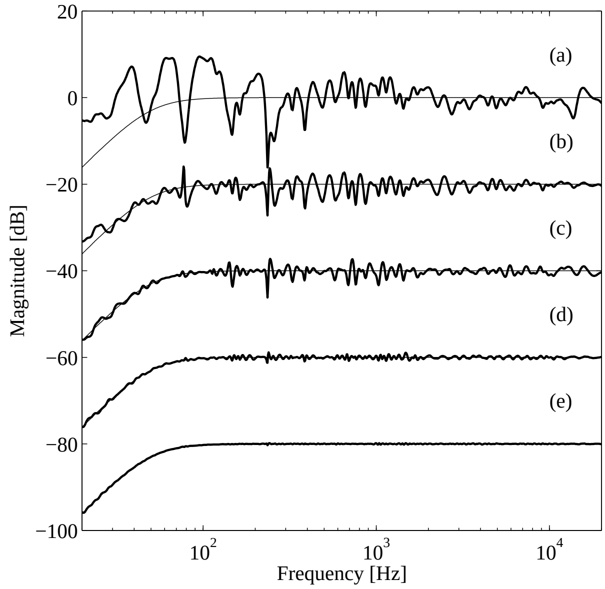

Fig. 7: Minimum-phase loudspeaker–room response equalization by the custom warping method with increasing filter order: (a) non-equalized transfer function, equalized by (b) a 32nd order filter, (c) a 64th order filter, (d) a 96th order filter, and (e) a 128th order filter. The thin lines display the desired response.

opposed to traditional warped filters. The method starts with mapping the frequency specification by the custom warping profile and designing an IIR filter based on this warped specification by any of the traditional IIR filter design techniques. Then, the poles and zeros of the filter are found and dewarped to the original frequency scale. Due to approximations during conversion, the resulting filter shows a spectral tilt, which is overcome by the optimization of the zeros. The filter is implemented as a parallel set of second-order filters, thus, the design method can also be considered as a new pole-positioning strategy for fixed-pole parallel filters [8]. The new method has been compared to various earlier filter design techniques. Compared to traditional warped IIR filters [2] and parallel filters with a logarithmic pole set [8], a significantly better fit is achieved. The new method provides a similar (actually, slightly better) performance compared to the recent multi-band method developed for parallel filters [12], while it can be easily adapted for any desired (e.g. non-logarithmic) warping profiles, resulting in a completely flexible allocation of frequency resolution. The method has been demonstrated by loudspeaker–room response modeling and equalization examples.

Future research may include the implementation of the post-optimization of the poles and zeros mentioned in Sec. 3.3 and comparison to the simpler solution used in this paper, which optimizes only the zeros instead. For pole-zero dewarping another possible method is to find the frequencies and zeros, compute the equivallent λ values as in Fig. 2 and use these λ values in Eq. (3) so that every pole and zero is dewarped by a different λ value. This variant should be compared to the method proposed in Sec. 3.2. Finally, the technique could be applied to other fields where the flexible allocation of frequency resolution is desirable.

6. ACKNOWLEDGEMENT

This work has been supported by the EEA Norway Grants and the Zoltán Magyary Higher Education Foundation.

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