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Extracting Full Information from Measured ADC Data

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Abstract

ADC testing is often done using sine wave excitation (see e.g. IEEE standard 1241). The measured data are fitted in least squares sense by a sine wave, and the residuals can be analyzed further. In recent papers, it has been recognized that even more (and more precise) information can be extracted by the solution of the maximum likelihood equations. This can be considered as an improvement to the three-parameter and four-parameter fits. Further investigations lead to the statement that the same principle can be extended to any measurement which uses an excitation signal which can be described with a few parameters. A candidate for this is using an exponential signal, with 3 parameters: start value, end (steady-state) value, and time constant. The maximum likelihood (ML) equations yield a solution for these more accurate than least squares (LS) fitting.

1 Introduction

[1] One of the most general principles applied in calibration is to use high-precision excitation signals and/or high-precision instrumentation. The error of the calibration measurement is desirable to be by an order of magnitude smaller than that of the device under test. However, this is often a too strict requirement even when a medium-precision analog-to-digital converter is tested. Therefore, the problem is usually circumvented in ADC testing by applying a sine wave as an excitation signal to the ADC. Although parameters of the sine wave are still cumbersome to be precisely measured, the accepted procedure executes a least-squares fit to the output of the ADC. The estimated parameters of the sine wave are then used to evaluate the error samples and characterise the ADC [1].

This procedure works well, however, it is still not optimal in the sense that

- least squares is not optimal for quantization errors,
- ADC nonlinearities are not properly handled with least squares fit,
- eventual overload of the ADC is not modelled by the customary LS fit,
- sine wave is not the only possibility for the excitation signal,
- the sine wave has an excess weight and improper error form for the samples close to the peaks.

We are going to tackle a part of these problems in the following sections.

1.1 Excitation Signal

The sine wave is very popular because it can be described by a few parameters only (amplitude, phase, frequency and maybe a dc component), furthermore it can be produced with the desired purity. Nevertheless, in ADC testing its probability density function (PDF) is not always desirable because of the large peaks at its edges:

$$f(x) = \frac{1}{\sqrt{\pi(A^2 - (x-\mu)^2)}}, \text{ where } \mu - A < x < \mu + A. \tag{1}$$

These peaks correspond to the peaks of the sine wave which are very flat and therefore around these the signal does not excite the ADC with a sufficient variation. Moreover, the amplitude range is strictly limited to $(-A, A)$. A possibility to circumvent these difficulties is a systematic overload of the ADC at both ends. The overloaded samples

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need to be neglected in the fit, along with the samples which, after recovery, represent biased response of the circuitry due to the previous overload. This is possible in the LS fit, as well as in maximum likelihood, to be discussed below.

An important question is whether the use of a sine wave is the ultimate solution, or some other signals are still possible. Here is what we make use from the properties of the sine wave:

- it can be represented by a few parameters,
- it is rather smooth and excites the ADC at many amplitudes,
- its local behavior is different form time instant to time instant,
- the form is precisely defined by 3 parameters, and eventual changes in the parameters do not mask usual DNL/INL patterns,
- it can be generated quite simply with small error,
- usual spectral analysis can be used to supervise purity of the sine wave,
- its PDF is close to uniform (at least at the central part),
- its PDF has a known closed form, using the signal parameters.

According to the above arguments, an attractive signal form would be piecewise linear (e.g. triangle wave, generated using a simple integrator) which corresponds to many requirements above, except for small error, easy quality check, and changing behavior. Therefore, a piecewise linear signal is not really advisable. Instead, a possibility which allows to fulfil most above criteria is an exponential signal [7], especially when two exponential signals are applied consecutively in the two directions [12]. We will see in the following that this is a valid alternative of the sine wave, and fits into the same estimation framework.

1.2 Maximum Likelihood Estimation - A Common Framework

According to the standards, the parameters of the sine wave are determined from the least squares fit of the ADC output data [8]. This is a very simple and robust method, but it certainly does not utilize all information present in the signal. According to estimation theory, whenever quite general conditions are fulfilled, maximum likelihood estimation is the "best" in the sense that it is asymptotically unbiased, and asymptotically efficient. It coincides with least squares when the samples from the model (the excitation signal of which the parameters are determined) are corrupted by additive white Gaussian observation noise. Least squares generally also has favourable properties [2], but without the above conditions fulfilled, maximum likelihood outperforms it. Therefore, it is reasonable to look also at ML at least to see the ultimate performance we might want to achive or approximate.

It is obvious that even in the case of an ideal quantizer, the observation error due to quantization is not Gaussian. Moreover, we need to handle ADC nonlinearity - this is what we would like to characterize by the test.

The signal is described in parametric form. Cos and sin are used to make the equation linear in the parameters A, B:

$$x(t) = A\cos(\omega t) + B\sin(\omega t). \tag{2}$$

For a single exponential:

$$x(t) = (F_0 - F_\infty)e^{-t/\tau} + F_\infty. \tag{3}$$

Let us denote in the following the parameters of the signal to be determined by p: $= [A, B, C]^T$, or $= [A, B, C, \omega]^T$, or $= [F_0, F_\infty, \tau]^T$.

We can measure the output of the analog-to-digital converter. This has the consequence that the observations have discrete distribution. Therefore, we need to formulate the ML criterion in the discrete domain, using the quantizer characteristic. Before doing that, we also need to find a way to describe the uncertainty around the comparison levels. The usual way to describe this is noise added to the sample of the input signal before quantization. Thus, the observed discrete signal can be described as

$$z_i = Q(x(t_k) + n_k) \tag{4}$$

where Q denotes the quantizer characteristic (this is a deterministic function), and n_k is a Gaussian noise sample, with zero mean and variance σ: $n_k = N(0, \sigma)$. By this, we handle quantization error properly by Q.

For simplicity, let us denote the vector of the comparison levels of the quantizer (T_l, $l = 1, \ldots, M-1$) by T. The number of the possible ADC outputs is $M = 2^B$.

Now we are ready to formulate the likelihood function for independent noise samples:

$$L(p, \sigma, T) = \prod_{k=1}^{N} P(z_k = s_k | p, \sigma, T), \tag{5}$$

where $P(.)$ denotes the probability of the given event, and $|$ denotes the condition. For a given set of parameters, the probability $P(z_k = s_k | p, \sigma, T)$ can be evaluated by integration of the normal distribution, between the relevant comparison levels. It cannot be given in a closed form, but numerically it is treatable. Since (5) can be evaluated, it can also be maximized via the parameters, and thus the ML estimates can be obtained. This can be done for a sine wave [4], [3], or for an exponential signal, or for a combination of positive-slope and negative-slope (rising and falling) exponentials:

$$\begin{aligned} x(t) &= (F_{0,1} - F_{\infty,1})e^{-t/\tau} + F_{\infty,1}, \quad F_{0,1} < F_{\infty,1}, \text{ for } t_{1,1} \leq t \leq t_{1,2} \\ x(t) &= (F_{0,2} - F_{\infty,2})e^{-t/\tau} + F_{\infty,2}, \quad F_{0,2} > F_{\infty,2}, \text{ for } t_{2,1} \leq t \leq t_{2,2} \end{aligned} \tag{6}$$

Notice that τ is the same in the two equations. This can be exploited in the fit, writing (5) [13].

2 Fast calculations

If the number of bits in the ADC is B, there are $2^B - 1$ comparison levels to be estimated. Thus the maximization of (5) involves a large number of parameters. While in theory this is possible, in practice the value of B is limited to about 6-8. This is often not enough. Moreover, the likelihood function is a nonlinear function of the parameters, thus starting values are needed to start optimizing iteration. Consequently, we need an effective way to determine good enough starting values.

In general, histogram test is used to characterize the differential nonlinearities of the ADC [5]. This seems to be possible even in our case. The knowledge of the differential nonlinearity is equivalent to the knowledge of the comparison levels. The histogram test is a very robust procedure to obtain the transition levels. Moreover, in the case of sine waves, the so-called normalized transition levels can be calculated without the actual parameters of the sine wave [5], [6].

However, there is a problem: the measured histogram is modified by the PDF of the signal, thus DNL can be evaluated from the histogram only by using the knowledge of the signal. On the other hand, the signal parameters (or at least a part of them) can be estimated from (5) only by knowing the DNL values T. Therefore, only an iterative procedure is possible. If the following steps converge (as they do in practical cases), they lead to a reasonable maximizer parameter set:

1. determine the histogram,
2. determine the signal parameters as well as possible,
3. correct the histogram if it is necessary, using the PDF of the signal+noise,
4. minimize L by the signal parameters and σ in (5),
5. if stop criterion does not meet, go to 3 and continue.

It is worth noting that in the step 3 if a sine wave is applied, and the noise is not very large (e.g. $\sigma < 2$LSB, see [5]), the comparison levels can be directly obtained from the cumulative histogram by the transformation $-A_X \cos(\pi H_c(k)) + C$, where A_x is the amplitude and C is the dc level, see [5], [6], thus the iteration cycle is not necessary. This is a consequence of the parametric form of the sine wave: its CDF does not change with changing the frequency ω or the phase ϕ.

For the signal parameters, the two cases (sine or exponential) need to be discussed separately.

2.1 Sine Wave

The starting values can be obtained from an LS fit to the non-overloaded samples (maybe also excluding the samples around the peaks, see [8], and the ones with recovery transients after overload). If the frequency is known, this is a linear LS problem, if not, this is nonlinear LS. The starting value for ω can be determined by using Interpolated FFT [10], [11].

An alternative possibility is to make a least squares fit to the measured histogram. Start from the above values and fit Eq. (1) in LS sense numerically, to the histogram. This is certainly faster than the ML method, however, if σ is not very small, this needs numerical adjustment of the PDF to make a proper fit, since the signal PDF needs to be convolved first with the noise PDF. Moreover, although LS fit seems to be "logical", there is no guarantee how well it will perform compared to the theoretically optimal maximum likelihood estimate.

2.2 Exponential Signal

The time constant is the only parameter which nonlinearly appears in the likelihood function. As we will see later (e.g. (10) and (11)), knowledge of the end value F_∞ allows simple determination of τ, which offers an alternative (but equivalent) calculation. Its starting value can be determined in a few simple ways.

2.2.1 Calculation from Samples

Take 4 equidistant samples at $t_1 \leq t_2 \leq t_3 < t_4$, respectively, which are reasonably apart from each other (for the evaluation of (7)), and $t_2 - t_1 = t_4 - t_3 = \Delta t$. From the corresponding samples (see (3)), simple algebra gives

$$\hat{\tau} = (t_3 - t_1)\frac{1}{\ln\left(\frac{x_2 - x_1}{x_4 - x_3}\right)}. \tag{7}$$

For a double exponential, this can be done for both parts and the results can be averaged.

This is an elementary solution, however, it is sensitive to noise, since only 4 samples are used. The effect of the noise can be decreased by averaging several estimates by shifting the time instants, e.g. like:

$$\hat{\tau} = \frac{1}{K}\sum_{n=0}^{K-1} T_d \frac{1}{\ln\left(\frac{x_{t_n+\Delta t} - x_{t_n}}{x_{t_n+\Delta t+T_d} - x_{t_n+T_d}}\right)}. \tag{8}$$

Knowing the time constant, the samples can be fitted by using linear LS (see (3)), and the value of the time constant can be refined by using nonlinear LS.

2.2.2 Calculation from the histogram

An alternative solution can be built on the form of the CDF. Since the cumulative histogram is the running sum of the regular (code) histogram, its values contain the sum of the noise/DNL values, left from the actual point, thus these are averages. Therefore, samples of the CDF can be used for reasonable calculations. E.g. for a negative slope, one can calculate:

$$F(x) = \int_{-\infty}^{x} f(z)dz = \int_{x_{\min}}^{x} \frac{C_{\text{tr}}}{z - F_\infty}dz = \left[C_{\text{tr}}\ln(z - F_\infty)\right]_{x_{\min}}^{x} = C_{\text{tr}}(\ln(x - F_\infty) - \ln(x_{\min} - F_\infty)) \tag{9}$$

for $x_{\min} \leq x \leq x_{\max}$. C_{tr} can be calculated from the condition $F(x_{\max}) = 1$:

$$C_{\text{tr}} = \frac{1}{\ln(x_{\max} - F_\infty) - \ln(x_{\min} - F_\infty)}, \quad F(x) = \frac{\ln(x - F_\infty) - \ln(x_{\min} - F_\infty)}{\ln(x_{\max} - F_\infty) - \ln(x_{\min} - F_\infty)}. \tag{10}$$

$F(x_{\min})$ clearly equals 0. The only unknown is F_∞ which can be numerically determined e.g. from the equation $F(x_1) = P_1$, where P_1 can be choosen e.g. $P_1 = 0.5$, and the corresponding value of x_1 is taken from the cumulative histogram.

The solution can be made even more accurate by fitting (10) to the histogram in LS sense [9]. However, the "best" (albeit slower) solution is to return for the last refinement to the time samples, and solve the ML problem. The starting value of τ can be calculated for this as [9]

$$\tau = C_{\text{tr}} t_1, \tag{11}$$

where t_1 is the observation length from which the histogram is obtained. Since C_{tr} is a function of F_∞, see (10), determinations of F_∞ and τ are essentially equivalent.

This is again a question of sufficient statistics: the histogram contains somewhat less information with respect to the signal parameters than the samples themselves, thus the ML solution is somewhat more accurate than LS fit of the PDF to the histogram.

3 Numerical results

This section is devoted to present the numerical results obtained by running the introduced method. First, a simulation result is presented to illustrate the improved accuracy of the suggested method. After this, measured data are used as input of the algorithm.

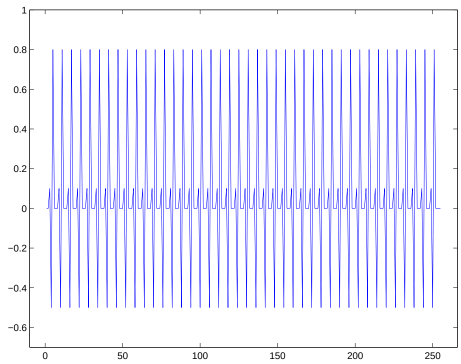

Figure 1: DNL of the simulated ADC as a function of the transition level vector.

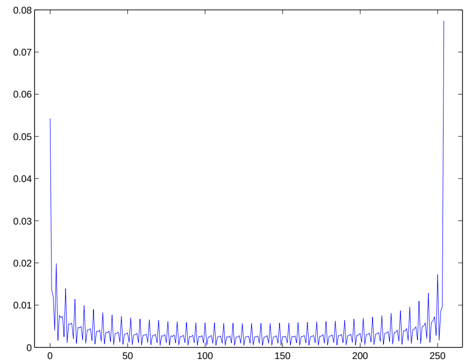

Figure 2: Normalized histogram of the ADC output data.

3.0.3 Simulated Data

A sine wave is generated with amplitude 2.021V, dc level 0V and frequency $f = 1$Hz, $f_s = 1$kHz. $M = 512000$ samples are generated. The ADC has 8 bits, with input range ±2.0V. The DNL of the ADC is set to the pattern shown in Fig. 1.

In the case of least squares fitting the error of the amplitude estimation is $A_{\text{est}} - A = -0.28$LSB, and the DC error is -0.11LSB. The amplitude error of the presented method is $A_{\text{est}} - A = 0.0015$LSB and dc error is -0.014LSB. Both errors are significantly decreased by using the maximum likelihood method.

3.0.4 Measured Data

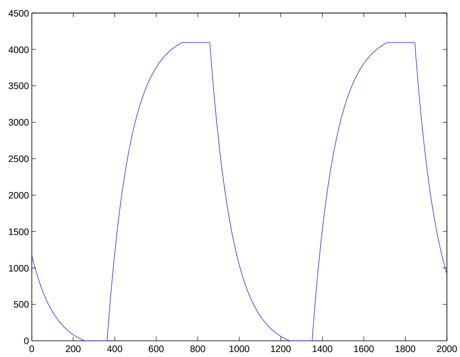

Figure 3: Exponential excitation signal. It contains alternating falling and rising parts.

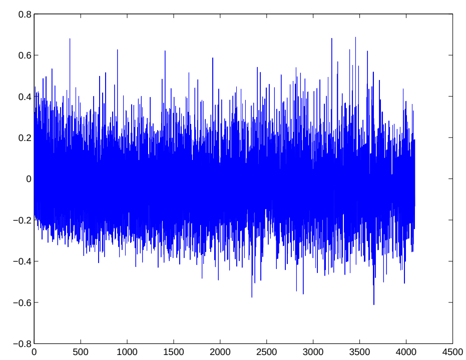

Figure 4: Estimated DNL of the ADC using exponential signal as excitation.

The second example is a measured data in which exponential excitation signal is applied. A 12-bit AD converter collected $M = 1000000$ samples ($f_s = 10$kHz). Part of the measured signal is plotted in Fig. 3.

Following the algorithm in [13] fitting the histogram results in $F_\infty = -1.038668$V. The estimated DNL can be seen in Fig. 4.

LS fitting of parametric model in time domain was performed and the algorithm returned with the following values (given for the first transient only): $\tau = 0.011$s, $F_0 = 1.0042$V and $F_\infty = -1.032$V. Using the estimated DNL values minimization of the ML cost function results in $\tau = 0.011$s, $F_0 = 1.0087$V and $F_\infty = -1.037$V. The ML solution contains a small improvement.

4 Conclusions

In this paper it is shown how is it possible to extract all information from the ADC response to sinusoidal or exponential excitation test signals. The results are significantly improved when compared to the least squares fit. By using the maximum likelihood principle, ADC testing can be made more accurate. Using fast algorithms the result can be obtained quickly.

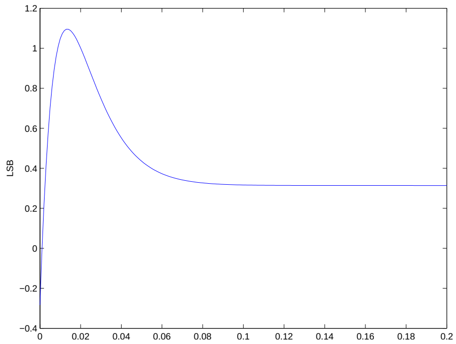

Figure 5: Difference of the results of the LS fit and ML estimation in the reconstructed signal.

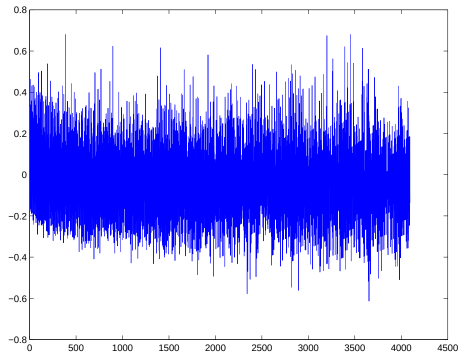

Figure 6: Estimated DNL of the ADC using the result of the ML estimation.

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