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PII: S0263-2241(97)00027-4

Measurement Vol. 20, No. 2, pp. 135–141, 1997

Printed in The Netherlands
0263-2241/97 $17.00 + 0.00

Signal model based periodic noise controller design

László Sujbert *, Gábor Péceli

Department of Measurement and Instrument Engineering, TU Budapest Müegyetem rkp. 9, H-1521 Budapest, Hungary

Abstract

Active cancellation of acoustic noise seems to be a real alternative to passive isolation in the low frequency region. There are already many methods to solve this control problem, typically based on adaptive filter techniques. If the noise is periodic, a simple signal model can be integrated into the adaptive controller, the parameters of which are set using the error signal. This control loop behaves like an observer of the signal to be suppressed. The authors have developed this method which is already proved to work efficiently in practice, as well. The paper describes the design of this observer and gives the relations to the conventional structures and adaptation mechanisms. © 1997 Elsevier Science Ltd.

Keywords: Periodic noise control; Resonator based observer; Signal model

1. Nomenclature

$A_1(z)$	transfer function of primary acoustic path
$A_2(z)$	transfer function of secondary acoustic path
$C(z)$	corrector transfer function
$\mathbf{c}_n$	vector of basis functions at time sample n
$c_{n,k}$	the kth basis function at time sample n
cot	cotangent function
d	desired signal
e	Euler's number, $e = 2.71828\ldots$
f	relative frequency
f_1	relative fundamental frequency
$\mathbf{g}_n$	corrector vector at time sample n
$g_{n,k}$	the kth corrector element at time sample n
$H_k(z)$	transfer function of kth channel
j	imaginary unit, $j = \sqrt{-1}$
k	integer number
L	integer number
$L(z)$	open loop transfer function
N	integer number
n	integer number (for time step)
$R(z)$	resonator transfer function
$\mathbf{r}$	input vector of the controller
r_k	kth element of the input vector
$W(z)$	filter transfer function
w_k	complex parameter
x	reference signal
$\mathbf{x}_n$	state vector at time sample n
$x_{1,n}$	first element of the state vector at time sample n
y	output signal
y_n	output signal at time sample n
z	variable of the Z-transform
z_k	resonator pole
α	convergence parameter
ϵ	error signal
π	Ludolf's number, $\pi = 3.14159\ldots$
$\hat{}$ (caret)	denotes the estimator of the designated variable
$\lvert . \rvert$	absolute value operator
$\bar{}$ (overbar)	complex conjugate operator
$.^{\mathrm{T}}$	transpose operator

* Corresponding author. Fax: +36 1 463 4112; e-mail: sujbert@mmt.bme.hu

2. Introduction

Nowadays the idea of active compensation of low-frequency acoustic noise or vibration is very popular [1]. Active noise cancellation is based on the phenomenon of destructive interference. A "secondary" noise has to be generated, which suppresses the "primary" (i.e. the original) noise at the properly situated microphones. These control systems are usually multiple input–multiple output systems, but their functionality and performance can be investigated on the corresponding single input–single output systems, as well.

The conventional control systems have the form of Figs 1 and 2 [1]. In these figures d represents the primary signal (desired signal for the algorithm); x is a reference signal which is well-correlated with the primary source; $A_1(z)$ and $A_2(z)$ are the discrete-time equivalents of the primary and the secondary acoustic paths, respectively; y is the output of the controller; ϵ is the error signal derived by the microphone; and $W(z)$ denotes the adaptive filter. The adaptation mechanism is usually a kind of LMS algorithm, in order to minimize the power at the microphones.

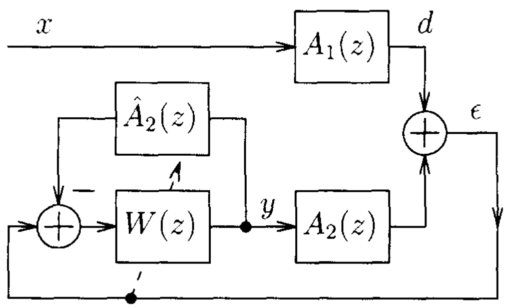

Fig. 1. Adaptive feedback control.

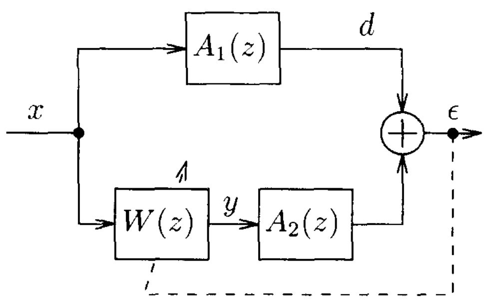

Fig. 2. Adaptive feedforward control.

Figure 1 shows the feedback controller. It has no separated reference input. $\hat{A}_2(z)$ is the model on $A_2(z)$ which provides the successful control even in the case of complicated secondary transfer function. However, the algorithm needs the identification of $A_2(z)$ in advance.

Figure 2 shows the feedforward controller. The controller also receives a reference signal. In this case $W(z)$ performs the estimated version of $A_1(z)/A_2(z)$. $\hat{A}_2(z)$ is used for the adaptation (e.g. filtered-X LMS algorithm). The implementations of such controllers were more successful than that of the feedback ones [1]. The main disadvantage of the feedback controller is that it must have a large gain because of the small error signal (in the case of successful control), but this gain cannot be increased enough because of the delay due to the acoustic path. To reduce this drawback feedback controllers are implemented with secondary sources near to the error microphones.

The systems described above are used to compensate broadband noise. If the noise is periodic, a different approach can be used. We should focus on the feedforward structure, because of its advantages against the feedback one mentioned above and the easily available reference signal. The arrangement for the feedforward control of periodic noise can be seen in Fig. 3.

There is a new element in the figure, $R(z)$, which is a virtual signal generator producing the reference signal. It is detailed in Fig. 5. The idea of this generator is based on the Fourier-decomposition of periodic signals. It consists of individual complex oscillators (resonators) at the frequencies to be controlled, and each oscillator has its own

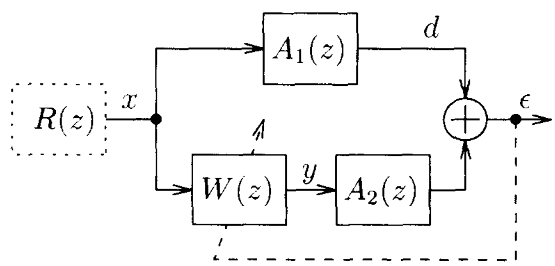

Fig. 3. Feedforward control of periodic noise.

complex amplitude (magnitude and phase). This generator can be represented by a signal model, where the fundamental frequency and the complex amplitudes are its free parameters. $A_1(z)$ performs only a transform on the complex amplitudes. Indeed, the secondary signal could be also produced by the same generator with the proper transform on the complex amplitudes. Figure 4 shows such a system. Against the previous version, there is a signal model built in the controller, $\hat{R}(z)$, where '^' refers the estimated version of the model. The controller uses the reference signal to estimate the fundamental frequency of the primary signal. $C(z)$ is a corrector to set the complex amplitudes in $\hat{R}(z)$. The whole control system is an observer on the signal model $R(z)$.

The task is to design $C(z)$, according to $\hat{R}(z)$ and $A_2(z)$. Since $A_2(z)$ can be considered to be linear [2], $C(z)$ can be designed on the basis of the classical linear system theory [3,4]. Consequently, at a given fundamental frequency, the control system remains linear, i.e. time-invariant. Due to the resonators in the controller, this approach provides the total suppression of signal components, if the frequencies of $\hat{R}(z)$ exactly coincide with those of $R(z)$. There is usually exact frequency information from the primary source (e.g. tachometer), otherwise the adaptive Fourier analyzer (AFA) [5] can be applied to find the fundamental frequency from the reference signal.

The aim of this paper is to describe the observer design emphasizing the role of the acoustic path, and to characterize the derived system. It will be also demonstrate how it relates to existing methods.

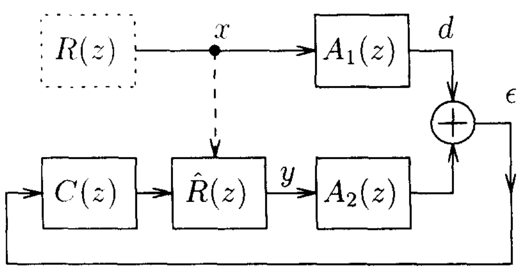

Fig. 4. Resonator based control.

3. Observer design

In this section we will consider the Luenberger type linear state observers, and it is supposed that the system to be observed is observable. In our case the system to be observed is a discrete time signal generator introduced in the previous section [3]. The signal model can be described as follows:

$$y_n = \mathbf{c}_n^{\mathrm{T}} \mathbf{x}_n \quad (1)$$

$$\mathbf{c}_n = [c_{n,k}] = e^{j2\pi f_1 kn},\ k = -L \ldots L,\ N = 2L+1 \quad (2)$$

$$Lf_1 < 0.5 < (L+1)f_1 \quad (3)$$

where $\mathbf{x}_n$ is the state vector of the signal model, y_n is its output and the input of the observer, and $\mathbf{c}_n$ represents the basis of the Fourier-expansion and f_1 is the relative fundamental frequency. The state vector consists of outputs of unexcited integrators (Fig. 5). The last equation means that all of the harmonics up to the half of the sampling frequency are represented.

Ignoring the details on the possible ways of observer design, here the observer for periodic signals will be introduced as follows. It is a control loop where the plant is the signal model itself. Its output is fed back and subtracted from the input signal (which is the output of the signal model to be observed). Thus the description of the observer is:

$$\hat{\mathbf{x}}_{n+1} = \hat{\mathbf{x}}_n + \mathbf{g}_n (y_n - \mathbf{c}_n^{\mathrm{T}} \hat{\mathbf{x}}_n) \quad (4)$$

$$\mathbf{g}_n = [g_{n,k}] = r_k \bar{c}_{n,k} \quad (5)$$

where $\hat{\mathbf{x}}_n$ is the estimated state vector, $\mathbf{g}_n$ is a vector

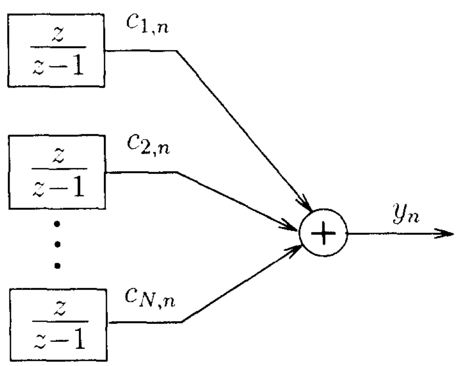

Fig. 5. The signal model.

called corrector in the previous section to set the poles of the observer by the r_k parameters. '$^-$' denotes the complex conjugate operator. The observer can be seen in Fig. 6. Due to the complex exponentials, the channels of the observer can be considered as time-invariant systems with a pole on the unit circle (z_k; $k=1\ldots N$). This is why they are called resonators. Although by r_k any pole placement (of the overall system) can be achieved, it has an important role if the poles are in the origin, i.e. the system has finite impulse response [3]. If the resonator poles are arranged uniformly on the unit circle, and $r_k=1/N$; $k=1\ldots N$, the observer performs a recursive Fourier transformer of N points. The result of the transformation is the state vector. In practical applications [5] where the fundamental frequency changes, the resonators cannot be placed uniformly, and the calculation of the corresponding r_k parameters would be time consuming. But, if $r_k=1/N$; $k=1\ldots N$ and Eqs (2) and (3) hold, the system proved to be fairly fast.

If the input signal is periodic, consisting of only components of resonator frequencies then the input of the resonators (i.e. the feedback error) equals zero. Furthermore, in this case the corresponding state variables (as a complex vector) do not change. However, if the frequency of the input signal changes, the state variables will rotate. The speed of this rotation at each resonator is proportional to the corresponding frequency difference. This is the basic idea for the frequency adaptation

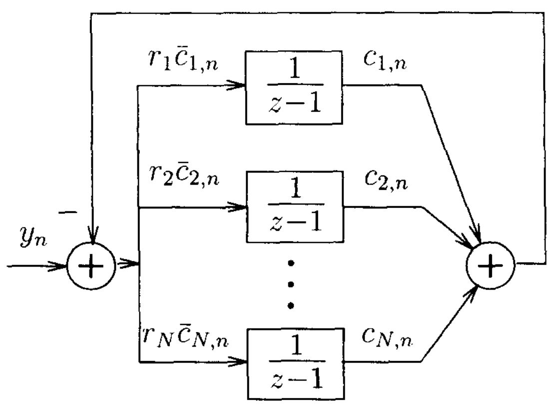

Fig. 6. Observer for periodic signals.

in the AFA [5]. The exact formula is the following:

$$f_{1,n+1} = f_{1,n} + \frac{1}{2\pi N}\,\text{angle}\,(\hat{x}_{1,n+1}, \hat{x}_{1,n}) \qquad (6)$$

where $\hat{x}_{i,1}$ is the state variable belonging to the positive fundamental frequency, and "angle" gives the angle between two complex numbers.

The noise controller can be a resonator based observer, the feedback path of which is implemented by the secondary path. It can be seen in Fig. 7a. The input of resonator channels comes from the microphone (via an ADC), the sum of the channels is connected to the loudspeaker (via DAC). The $\mathbf{c}_n$ vector is taken from an AFA. The reference signal can be any signal, relatively free of noise, with the same frequency as the primary source. The simplified arrangement can be seen in Fig. 7b where $C(z)$ and $R(z)$ denote the corrector and the signal model, respectively, and $A_2(z)$ represents the acoustic path.

Here, a both theoretically and practically important problem arises. The original task was to design a controller for the acoustic path as a plant. On the other hand, the decision was to design an observer for the periodic signal. The above

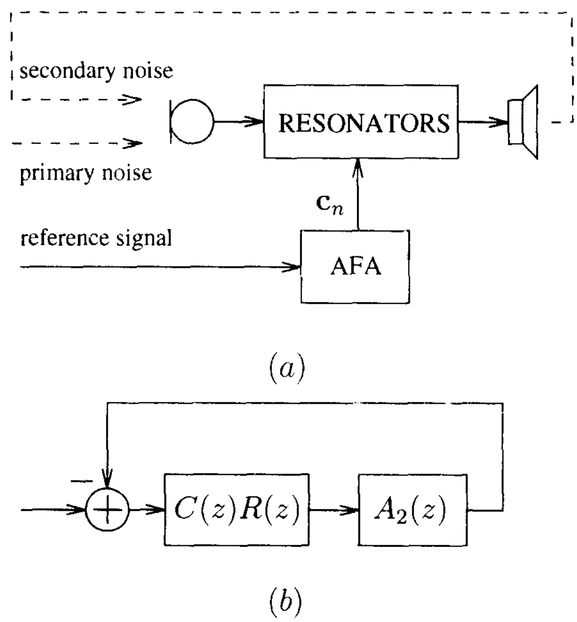

Fig. 7. Periodic noise control: (a) physical arrangement, (b) block diagram of the control loop.

derivations considered the signal model also as a plant. Therefore, our control loop can be seen from two points of view: (1) the plant is the acoustics ($A_2(z)$), and the controller is the signal model and the corrector (Fig. 4); (2) the plant is the signal model, and the acoustic path is an inevitable part of the controller. Although the signal model does not appear directly, the methods based on adaptive filters prefer the first approach. As was mentioned in the introduction, these systems need the identification of $A_2(z)$, which has lots of difficulties, regarding the high complexity of such transfer functions, and, in addition, the usage of the model needs time-consuming on-line calculations. The resonator based approach offers a simple solution by the appropriate choice of the $\mathbf{r}$ ($\mathbf{r}=[r_1...r_N]$).

The resonators in the loop provide the zero error at their frequencies. The only, but important, task of the $\mathbf{r}$ is to make the overall system stable and fast enough. It must be also emphasized that the feedback is no longer state feedback, because of the state variables in the acoustics which are not available. Consequently, there is no possibility to place the poles of the whole system to any position. It will be shown below that the best choice of $\mathbf{r}$ is the following:

$$r_k = \alpha w_k; w_k = \frac{1}{A_2(z_k)} \tag{7}$$

where α is a convergence parameter. The stability of the system is investigated by the Nyquist stability criterion (see e.g. Ref. [6]). The open loop transfer function of the system is:

$$L(z) = C(z)R(z)A_2(z) = A_2(z)\sum_{k=1}^{N} H_k(z) \tag{8}$$

where $H_k(z)$ is the transfer function of one channel of the controller:

$$H_k(z) = r_k \frac{z_k}{z-z_k} = \alpha w_k \frac{z_k}{z-z_k} \tag{9}$$

This can be written as follows:

$$H_k(f) = w_k \frac{\alpha}{2}[1+j\cot\pi(f-f_k)];\ z=e^{j2\pi f} \tag{10}$$

where f is the running frequency and f_k is the resonator frequency. By decreasing the convergence parameter α the encirclement of the point $z=-1$ can be avoided everywhere (independently of the phase of r_k or $A_2(f)$) with the exception of the neighborhood of the resonator frequencies, because here $|H_k(f_k)| \approx \infty$ for some k therefore $|W(f_k)| \approx \infty$. At $f=f_k$ the phase of the loop gain is determined by the corresponding resonator channel and $A_2(f_k)$. Hence, here it is enough to investigate the Nyquist curve of one channel given by Eq. (10). Supposing, $w_k=1$, this Nyquist curve is shown in Fig. 8. It does not encircle the point $z=-1$ if $\alpha<2$. Applying a complex $w_k \neq 1$ (which performs also a rotation on the curve), the encirclement of the point $z=-1$ can be avoided by an appropriate α if and only if:

$$-\pi/2 < \text{angle}(w_k) < \pi/2 \tag{11}$$

and this inequality should be satisfied for each channel. If $A_2(z)$ is also applied, Eq. (11) should be rewritten as follows:

$$-\pi/2 < \text{angle}(w_k) + \text{angle}(A_2(f_k)) < \pi/2;\ k=1...N \tag{12}$$

which is trivially satisfied if Eq. (7) holds.

Now it is proven that for stability only the phases of the r_k values are important. The settling time of the system is minimal, if the greatest eigenvalue is minimal. It can be achieved by

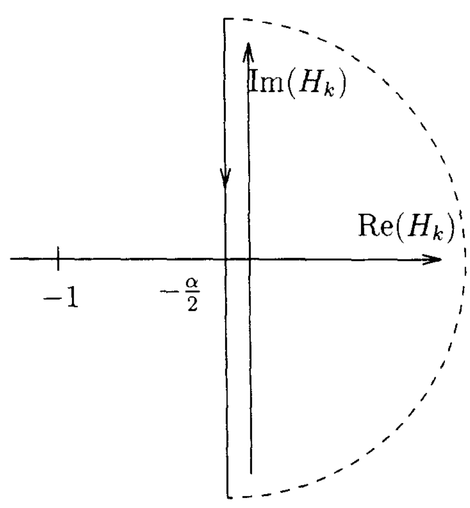

Fig. 8. The Nyquist curve of a resonator channel.

Eq. (7). If Eqs (2) and (3) are satisfied and $A_2(z) \equiv 1$, the system is very fast, the poles lie close to the origin. This feature would hold if $1/A_2(z)$ could be applied in the loop. In general this inverse filter cannot be implemented. By the choice of the parameters $w_k = 1/A_2(z_k)$ this inverse filter is approximated. Because of the error of the approximation between the resonator poles the loop gain must be decreased by a positive convergence parameter as Eq. (7) shows otherwise the system is unstable.

In order to set the $\mathbf{r}$, $A_2(z)$ is needed, but in practical cases it is not analytically known. This means that the $\mathbf{r}$ cannot be calculated for an arbitrary chosen frequency. Fortunately, however, the phase shift at a resonator frequency can change in the range of $(-\pi/2, \pi/2)$. If the complex transfer values of $A_2(z)$ are known (e.g. they are measured) "densely enough" over the whole frequency range, this set can be used to calculate the actual $\mathbf{r}$. It depends on $A_2(z)$, what "densely enough" means. If the phase changes rapidly, lots of measurement points are needed. The calculation of the actual $\mathbf{r}$ could be very simple: for a certain frequency the nearest measurement result could be used. The measurement procedure described above gives only a finite number of the samples of the frequency response of $A_2(z)$. It means that the algorithm does not require one to solve the identification problem exactly.

The suppression of signal components at resonator frequencies is complete, so the observation error is zero, but the state variables of the signal model within the observer are usually distorted, because of the measurement error of $A_2(z)$ and the usage of the "nearest" measurement results.

4. Advantages of the resonator based approach

The above sections have already described some advantages regarding $A_2(z)$: its identification problem need not be solved exactly, and its usage is very simple. In this section the advantages against the conventional adaptive filters are demonstrated. The resonator based observer can be seen as an adaptive filter bank, the reference signals of which are the harmonic components, and the filter coefficients are adapted by the error signal using the LMS algorithm [7]. This duality can be applied for our case, the only difference is that there is a filter in the auxiliary path, therefore the filter coefficients are adapted by the filtered-X LMS (XLMS) algorithm [8]. As a result of the investigations, the filter bank for noise control can be seen as a resonator based controller, with:

$$w_k = \bar{A}_2(z_k) \tag{13}$$

where '$^-$' denotes the complex conjugate operator. The phase shifts caused by the filter in the XLMS algorithm and $\mathbf{r}$ in the resonator based observer are obviously the same, thus from a stability point of view the systems are identical. However, as was mentioned, the system is the fastest if the multipliers in each channel are set by [7]. A heuristic explanation can be given: while in the resonator based observer the gain between the system output and the resonator input is unity, that of the XLMS algorithm is $|A_2|^2$. If the secondary path suppresses the signal, in the loop the square of this suppression occurs, so the system will be considerable slow. It is a possibility, of course, to set the convergence parameter in the XLMS algorithm to get the fastest system. But, in this case, this convergence parameter must be a function of the frequency. It means that the elements of the $\mathbf{r}$ vector are implemented in two parts: first the complex conjugate part by the filter, second the absolute value by the adaptive convergence parameters. There is a further difference between the algorithms: the above comparison was made supposing separated channels for each complex exponential (which is here identical with supposing sinusoids), but the usual adaptive systems have only one reference signal, consisting of all components to be suppressed.

The differences mentioned above are practically relevant. In the case of usual secondary transfer functions this speed-difference could be high. The speed of the system has an important role if the noise signal is not stationary (which is a real case), so the control system must follow the changes.

5. Conclusion

The above derivations led from the "feedforward" approach to a "feedback" controller, because of the structure of $C(z)$ and $R(z)$. However, there is feedback of the error signal in all methods, only their usages are different. The classical feedforward control uses the error signal to adapt many coefficients of a filter, while the resonator based controller "adapts" the parameters of the signal model. However, this "adaptation" is performed by a linear circuit, the only input of which is the error signal. Due to the built-in signal model, the observer based design provides advantages against the conventional adaptive control systems, regarding the identification of the secondary path and the speed of the control, without other disadvantages.

References

1. Elliott, S. J. and Nelson, P. A., Active noise control. *IEEE Signal Processing Magazine*, 1993, **10**(4), 12–35.
2. Doelman, N. J., Design of systems for active sound control. Ph.D. thesis, Delft, Netherlands, 1993.
3. Péceli, G., A common structure for recursive discrete transforms. *IEEE Transactions on Circuits and Systems*, 1986, **33**, 1035–1036.
4. Sujbert, L. and Dunay, R., Resonator based periodic noise cancellation, Technical Report, (TPD-HAG-95-RPT-0044), TPD-TNO Delft, Netherlands, 1995.
5. Nagy, F., Measurement of signal parameters using nonlinear observers. *IEEE Transactions on Instrumentation and Measurement*, 1992, **41**(1), 152–155.
6. Van de Vegte, J., *Feedback Control Systems*. Prentice–Hall, Inc., Englewood Cliffs, NJ, 1990.
7. Widrow, B., Fundamental relations between the LMS algorithm and the DFT. *IEEE Transactions on Circuits and Systems*, 1987, **CAS-34**, 814–819.
8. Sujbert, L. and Péceli, G., Periodic noise control – the interrelation of different techniques, presented at the TEMPUS Workshop on Advanced Control Systems, June 13–15, 1996, Vienna, Austria. In workshop proceedings pp. 15–23.