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Active Distortion Cancelation of Sinusoidal Sources

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Abstract – *The paper presents an on-line digital signal processing method for active cancelation of distortion of sinusoidal sources. Generation of a sinusoidal signal of several volts is a simple task, but many measurements need high voltage, current, or non-electrical excitation. This signal conversion often leads to distortion. The proposed active distortion controller is a resonator-based observer which determines the harmonic content of the signal, and the distorting components are subtracted from the output signal. The paper introduces the controller design in details, and a practical example illustrates the effectiveness of the proposed method.*

Keywords – *sinusoidal measurements, non-linear distortion, adaptive Fourier analyzer, resonator-based controller*

I. INTRODUCTION

Many measurement procedures require sinusoidal excitation. Generation of a pure sine-wave is a common task, and the resulted signal is usually a sinusoidal voltage of several volts. However, in many cases it is not appropriate for the system to be measured: the system needs high-voltage or current excitation, or the excitation is a non-electric signal. The latter is the situation if vibration analysis is performed. In this case an electromechanical actuator is used to transform the generator voltage to force. The problem is depicted in Fig. 1. The output of the generator is s_n, which is a sinusoidal signal and the driver produces the real excitation signal d_n. (n indicates the time instant.) The driver is a non-linear dynamic system, resulting possibly high non-linear distortions in d_n.

The solution of the problem is to generate such an s_n, that d_n is a pure sine-wave. In this case an arbitrary waveform generator is required which can produce the appropriate periodic signal. An off-line method is proposed in [1] to implement the above idea. The algorithm tries to produce a signal with a given spectrum at the output. A digitizer measures d_n, and by knowing the error of the desired and the realized spectra, the digital input of the waveform generator is modified in order to decrease the error. By making several attempts, the algorithm converges to produce the required spectra. This general solution can also be used to generate a sine wave with small distortions. In this case, the desired spectrum is the theoretical spectrum of a sine wave. The method is not adaptive, it is valid only for the setup used during the error minimization algorithm. A new setup calls for a new compensation.

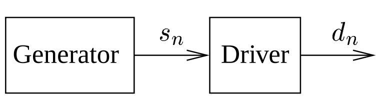

Fig. 1. Generation of the excitation signal.

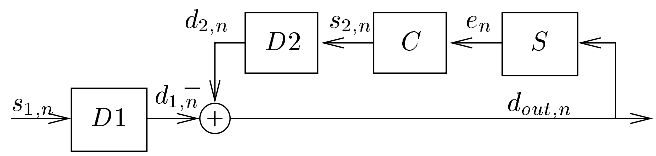

Fig. 2. Active distortion cancelation at the output of the driver.

In this paper an on-line method is proposed. It is an active method: a resonator-based observer determines the harmonic content of the signal, and the distortions are subtracted from the output signal. A digital signal processor receives the generator signal as a reference, and works as a controller canceling the higher harmonics of d_n on-line. The presented method is mainly based on previous results achieved in periodic noise control. The realized system was able to cancel acoustic noise [5].

Section II. introduces the idea of the active distortion cancelation in details and recalls the resonator-based observer, and section III. deals with the controller design. Section IV. presents a practical example, while section V. is the conclusion.

II. PRELIMINARIES

A. Active distortion cancelation

The idea is based on the recognition that distortion at the output of the driver in Fig. 1 cannot be avoided. However, it is possible to add a signal to d_n which cancels the distortion. If s_n is a sinusoidal signal, all the components above the fundamental frequency shall be canceled. A possible canceling system can be seen in Fig. 2. The input of the system is the sinusoidal excitation $s_{1,n}$. It is distorted by the primary driver $D1$, the output of which is $d_{1,n}$. The output of the system is $d_{out,n}$. A sensing circuit S is connected to the output: its role is to convert the output signal to be appropriate for the controller C. The linearity of S is essential. The input of the

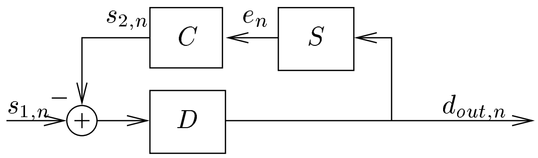

Fig. 3. Active distortion cancelation at the input of the driver.

controller, the error signal e_n should be zero in steady-state for each harmonic component of $d_{out,n}$, with the exception of the fundamental one. The output of the controller $s_{2,n}$ is led to a secondary driver $D2$, and its output is $d_{2,n}$. The difference of the two driver outputs results in the output signal: $d_{out,n} = d_{1,n} - d_{2,n}$. Note that the subtraction of the signals need special hardvare according to the type of the output signal.

Another possible distortion canceling system can be seen in Fig. 3. In this system the output of the controller $s_{2,n}$ does not led to a driver but subtracted from the primary excitation signal $s_{1,n}$. This arrangement is more advantageous than the previous one, since the secondary driver can be ignored, and the subtraction can be made by a simple circuit. Unfortunately, in some cases $s_{1,n}$ is not accessible, in these cases only the first arrangement in Fig. 2 can be used.

B. Resonator-based observer

The resonator based observer was designed to follow the state variables of the so-called conceptual signal model [3]. The signal model is described as follows:

$$y_n = \mathbf{c}_n^T \mathbf{x}_n \tag{1}$$

$$\mathbf{c}_n = [c_{k,n}] = e^{j2\pi f_k n},\ k = -L \dots L \tag{2}$$

$$f_{-k} = -f_k,\ f_0 = 0,\ k = -L \dots L \tag{3}$$

where $\mathbf{x}_n$ is the state vector of the signal model at time step n, y_n is its output (the input of the observer), $\mathbf{c}_n$ represents the basis functions. To generate a real signal, eq. (3) shall be satisfied. This restriction is not necessary, but advantageous in most cases. Obviously, in these cases the correspondig state variables shall form complex conjugate pairs. The conceptual signal model can be considered as a summed output of resonators which can generate any multisine with components up to the half of the sampling frequency. The corresponding observer is (Fig. 4):

$$\hat{\mathbf{x}}_{n+1} = \hat{\mathbf{x}}_n + \mathbf{g}_n(y_n - \mathbf{c}_n^T \hat{\mathbf{x}}_n); \quad \mathbf{g}_n = [g_{k,n}] = r_k \bar{c}_{k,n} \tag{4}$$

where $\{\hat{\mathbf{x}}_n = [\hat{x}_{k,n}];\ k = 1..N;\ N = 2L+1\}$ is the estimated state vector, $\{r_k;\ k = 1..N\}$ are free parameters to set the poles of the system, and the overbar denotes the complex conjugate operator.

Due to the complex exponentials, the channels of the observer can be considered as time-invariant systems with a pole on the unit circle. This is why they are called resonators. The

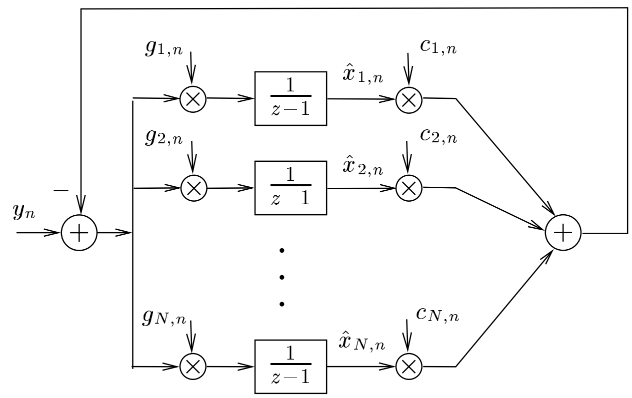

Fig. 4. The resonator based observer.

resonator frequencies can be expressed as the ratios of the consecutive samples of the basis functions [3]:

$$z_k = \frac{c_{k,n+1}}{c_{k,n}} = e^{j2\pi f_k};\ k = 1..N \tag{5}$$

and the transfer function of a channel is:

$$Q_k(z) = \frac{r_k z_k}{z - z_k} \tag{6}$$

These channels work in a common feedback loop forming a single input – multiple output filter bank, the transfer functions of which are:

$$P_k(z) = \frac{\frac{r_k z_k}{z - z_k}}{1 + \sum_{i=1}^{N} \frac{r_i z_i}{z - z_i}};\ k = 1..N \tag{7}$$

If the resonator poles are arranged uniformly on the unit circle, and $\{r_k = 1/N;\ k = 1..N\}$, the observer has finite impulse response, and the observer corresponds to the recursive discrete Fourier transform (RDFT) [3]. In that case the transfer function (7) is very simple:

$$P_k(z) = \frac{1}{N}\frac{z^N - 1}{z - z_k} z^{-N},\ k = 1..N \tag{8}$$

Its magnitude response is:

$$|P_k(f)| = \left| \frac{\sin \pi N(f - f_k)}{N \sin \pi (f - f_k)} \right|,\ k = 1..N \tag{9}$$

(9) has zeros at each resonator frequency, except when $f = f_k$, where $P_k(f) = 1$.

In practical applications where the fundamental frequency changes, the resonators cannot be placed uniformly, and the above setting of parameters r_k does not provide a finite impulse response. The adaptive Fourier Analyzer described in [4] adapts the resonator frequencies to coincide with those of the input signal, avoiding the picket-fence effect and leakage. This structure was successfully utilized e.g. in high-precision vector-voltmeters [4] or in active noise control systems [5].

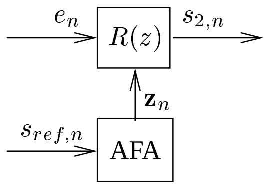

Fig. 5. The controller with the adaptive Fourier analyzer (AFA).

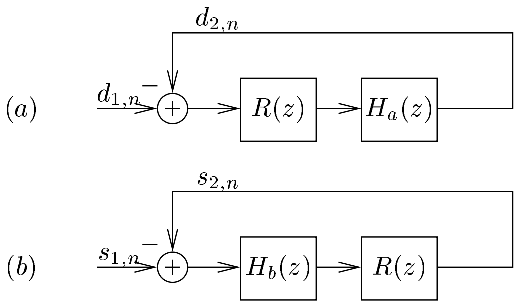

Fig. 6. Simplified block diagrams for stability analysis. Diagram (a) and (b) correspond to Fig. 2 and Fig. 3, respectively.

III. RESONATOR-BASED DISTORTION CONTROLLER

In the resonator-based observer (Fig.4) in steady-state the input of the resonators (i.e. the feedback error) equals zero. This means that the feedback signal (the sum of the resonator outputs) *cancels* the input signal. This feature can be utilized for active distortion cancelation. The control loop in Fig.2 or Fig.3 corresponds to that in Fig.4 in the following way: the controller consists of resonator channels, and the feedback is realized by the driver, the subtractor and the sensor. The input signal of the observer y_n corresponds to $d_{1,n}$ or $s_{1,n}$, while the error signal corresponds to $d_{out,n}$. The only difference is that the controller must not contain resonator at the fundamental frequency.

The frequency is estimated by an independent AFA and it passes the actual resonator positions ($\mathbf{z}_n$) to the controller as it can be seen in Fig. 5. The AFA needs a reference signal which can be any periodic signal with the same fundamental frequency as $s_{1,n}$.

Since the feedback is no more -1 as it was in the case of the observer (see Fig. 4), the stability of the control loop is not obvious. The simplified block diagrams of the control loops can be seen in Fig. 6. Diagram (a) and (b) correspond to Fig. 2 and Fig. 3, respectively. $H_a(z)$ describes $D2$ and S, while $H_b(z)$ corresponds to D and S. Note that the two diagrams are not identical since $H_a(z)$ and $H_b(z)$ are non-linear systems.

The stability of such controllers was investigated for active noise control systems [5]. In that case the feedback of the resonator-based observer is realized by an acoustic system which can be treated linear. Although our active distortion cancelation loop is non-linear, practical experiments prove that the linear approach for system stabilization works. In the following it will be supposed that the control loop contains $R(z)$ and the linear system $H(z)$.

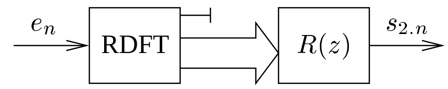

Fig. 7. Fourier decomposition of the error signal. The fundamental component is filtered out.

The stability of the control loop can be ensured by the appropriate choice of the parameters g_k. They can be chosen as follows:

$$g_k = \alpha z_k w_k;\;\; w_k = \frac{1}{H(z_k)};\;\; k = 1..N \qquad (10)$$

where α is a convergence parameter. The actual set w_k depends on the fundamental frequency of the signal to be generated. $H(z)$ is in general not analytically known and (10) cannot be calculated on-line, therefore the transfer function should be measured at a finite number of points and the inverses ($f_i;\ i = 1..M$) should be calculated off-line. Thus the actual set w_k should be a result of a mapping $\{f_i\} \rightarrow \{w_k\}$ (e.g. the nearest available one). The above parameter setting ensures $\pm\pi/2$ phase margin. The latter warrants that phase changes due to the non-linear behavior of $H(z)$ can be tolerated. Obviously, linear changes in $H(z)$ are also tolerated within the given phase margin. Note that while the system is stable, the cancelation of the components is complete.

The controller suppresses all the harmonic components with the exception of the fundamental one. It is because the resonators have infinite gain at their pole frequencies. Other signal components appear in $s_{2,n}$ according to the transfer function of the closed loop. From controller point of view, the fundamental component of the signal is one of the “other” components and appears in the control signal with a small amplitude. To avoid this, the controller can be completed by an RDFT module, according to Fig. 7. The Fourier components of the error signal e_n are determined and led to the inputs of the resonators, with the exception of the fundamental one. The RDFT can be computed by another resonator-based observer, the parameters of which are designed as it is described in section II. The transfer function of one channel of the RDFT output is given by (9). Since it has zeros at another resonator frequencies, the fundamental component cannot appear at any channel of the controller $R(z)$. Furthermore, each channel has a gain of exactly 1 at its pole frequency, therefore the parameters g_k given by (10) are valid.

All the elements of the proposed active distortion controller can be seen in Fig. 8. The design procedure can be summarized as follows:

1. build the hardware elements of the control loop according to Fig. 2 or Fig. 3,
2. measure the transfer function at finite points, and calculate the inverses off-line, according to (10),
3. implement the proposed controller as it can be seen in Fig. 8.

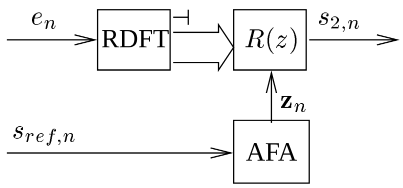

Fig. 8. All the elements of the proposed active distortion controller.

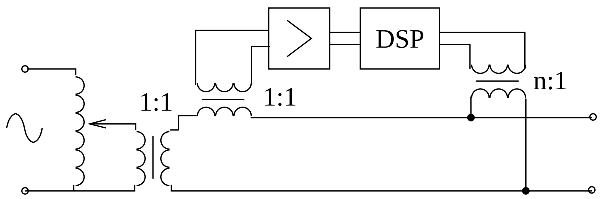

Fig. 9. Distortion cancelation of the mains.

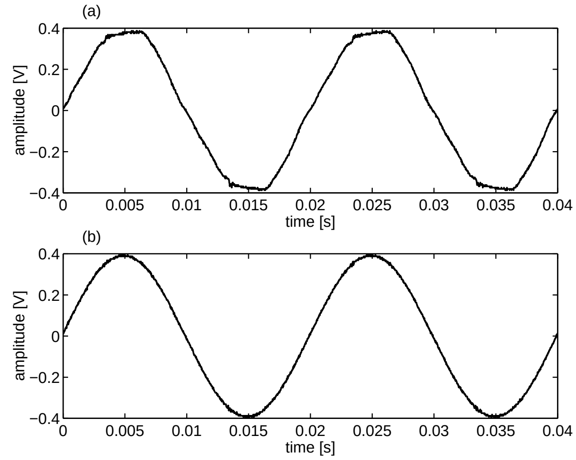

Fig. 10. Sensor voltage waveform without (a) and with (b) distortion cancelation.

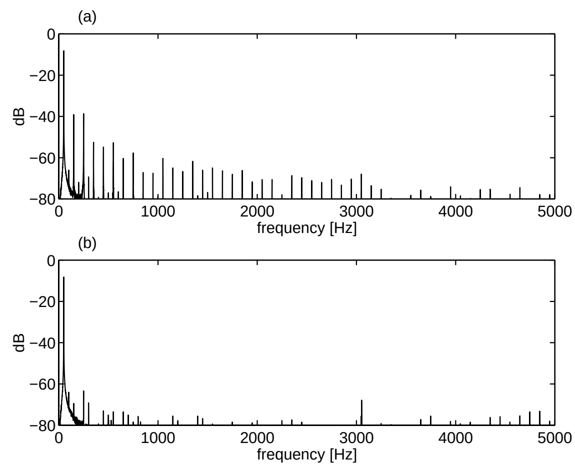

Fig. 11. Spectrum of the sensor voltage without (a) and with (b) distortion cancelation.

IV. PRACTICAL EXAMPLE

The proposed method was successfully tested for different applications. Here an experiment is cited, where the distortion of the mains was canceled. The measurement set-up can be seen in Fig. 9. The input signal was generated from the line power, and the correction signal was added by an ordinary transformer, while the sensing transformer was a precise one. The control loop corresponds to that in Fig. 2. The control algorithm was implemented on an ADSP-21065L DSP-based signal processing board [6]. The transfer function was measured at 50Hz and its harmonics. 60 AC resonator channels were used (this means a total number of 121 resonator channels), therefore the transfer function was measured from 50Hz to 3kHz. Although the line frequency is slightly different from 50 Hz, the measured transfer function samples ensure the stability of the control loop. The RMS value of the output voltage was set to 100V, and the waveform had a distortion of 4.9%. After compensation, the distortion of the output signal was about 0.3%, even if a load (filament lamp) was connected to the output. The corresponding waveforms and spectra can be seen in Fig. 10 and Fig. 11, respectively.

Measurement results clearly confirmed the expectations: the resonator-based controller can cancel the distortion of the output, even if the the frequency or the harmonic content of the mains is changing.

V. CONCLUSION, FUTURE WORK

The paper presented an on-line method for active distortion cancelation. The origin of the controller is the resonator-based observer introduced in section II. Section III. dealt with the derivation of the distortion controller. For stability analysis, all the elements of the structure were approximated by linear transfer functions. However, further research should focus on the analysis of the non-linear control loop. Possible application areas of the method are those, where a high-voltage or high-current signal is needed with increased spectral purity, e.g. power meter calibrations. Another important application field is vibration analysis, where the force signal of the shaker can be controlled.

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