Resonator-Based Nonparametric Identification of Linear Systems

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***Abstract*—A nonparametric identification method for linear systems is proposed. The identification is done via synchronized multisine measurements where the synchronization is ensured by a resonator-based generator–observer pair. The advantage of the proposed structure is that it works as a filter bank and, hence, provides the measurement results online. Exponential averaging is an option of the method and it requires no extra calculations. A further advantage is that the identification can be done over any frequency set without any loss of performance. Explicit formulas are given for noise suppression and settling time. The method is illustrated by practical examples.**

***Index Terms*—Multisine, nonparametric frequency domain identification, resonator-based observer.**

I. Introduction

FOURIER analysis is a well-known method for nonparametric frequency domain identification of linear systems [1]. Frequency domain data are often inputs for parametric identification [2]. The utilization of multisine excitation provides the possibility of the elimination of the systematic errors like leakage and picket fence (or scallop loss) (see e.g., [3]). In most cases, the output of the system is analyzed by the discrete Fourier transform (DFT), while the frequency domain parameters of the excitation are known in advance. The DFT is calculated via the fast Fourier transform (FFT). In order to suppress the measurement noise, averaging of the analysis results is also necessary.

Resonator-based observers were developed earlier to perform the recursive discrete Fourier transform (RDFT) [5], [6]. In these observers, the resonators work in a common feedback loop providing zero steady-state feedback error at the resonator frequencies. The summed output of such resonators can generate any periodic signal up to the half of the sampling frequency. [7] introduces an adaptive Fourier analyzer, where the resonator positions are set according to the signal components to be analyzed. [8] suggests a different method for the adaptation, concentrating on the computationally effective implementation of the algorithm.

It is straightforward to utilize such a generator–observer pair for frequency domain nonparametric system identification: the system to be identified has to be in between the generator and the observer and the ratio of the state variables of the observer and the generator supplies the estimated transfer function. In this paper, this procedure is described and a detailed analysis is given. The main contribution of this paper is that it provides explicit formulas for noise suppression and settling time which are of key importance in the case of frequency-response measurement. The suggested arrangement can be a competitor of the FFT-based analysis, since it works as a filter bank and provides the measurement results online over an arbitrary frequency set.

Section II recalls the identification problem and the resonator-based observer. Section III introduces the resonator-based identification and a detailed analysis is given. Real measurement data are provided in Section IV, while Section V is the conlusion.

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II. Preliminaries

A. Nonparametric Frequency Domain Identification of Linear Systems

Let $A(f)$ be a linear, time-invariant discrete time system. The nonparametric frequency domain identification of $A(f)$ is the estimation of its samples over a finite set of f_k [1], [2], and [4] as follows:

$$\hat{A}(f_k) = \frac{Q(f_k)}{P(f_k)} \tag{1}$$

where $\hat{A}(f_k)$ is the estimation of $A(f_k)$, and $P(f)$ and $Q(f)$ denote the Fourier transform of the input and the output signal, respectively. There are many possibilities to excite the system; a possible choice is the utilization of multisine excitation [4]. In this case, $P(f_k)$ is usually known in advance, and $Q(f_k)$ is calculated by the DFT. If the length of the input sequence equals the number of the DFT points [supposing the steady-state of $A(f)$], the estimation is not distorted due to the leakage and the picket-fence effect. There are different averaging techniques to reduce the random noise corrupting the measurement. If the identification should follow the changes in $A(f)$, exponential averaging can be used.

B. Resonator-Based Observer

The resonator-based observer was designed to follow the state variables of the so-called conceptual signal model [6]. The signal model is described as follows:

$$y_n = \mathbf{c}_n^T \mathbf{x}_n \tag{2}$$

$$\mathbf{c}_n = [c_{k,n}] = e^{j2\pi f_k n}, \quad k = -L, \ldots, L \tag{3}$$

$$f_{-k} = -f_k,\ f_0 = 0, \qquad k = -L, \ldots, L \tag{4}$$

where $\mathbf{x}_n$ is the state vector of the signal model at time step n, y_n is its output (the input of the observer), and $\mathbf{c}_n$ represents the

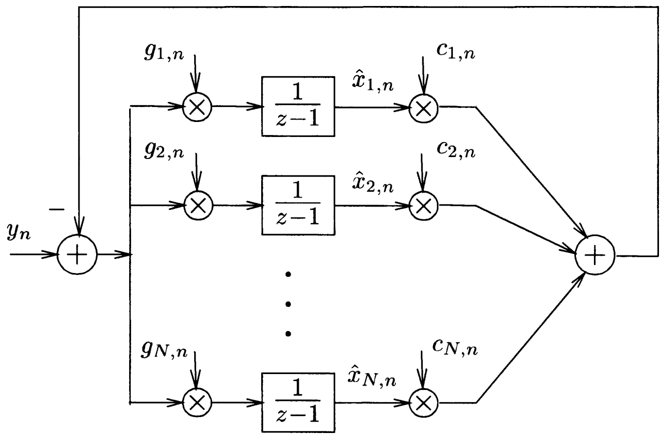

Fig. 1. Resonator-based observer.

basis functions. To generate a real signal, (4) shall be satisfied. This restriction is not necessary, but advantageous in most cases. Obviously, in these cases, the correspondig state variables shall form complex conjugate pairs. The conceptual signal model can be considered as a summed output of resonators which can generate any multisine with components up to half of the sampling frequency. The corresponding observer is (Fig. 1)

$$\hat{\mathbf{x}}_{n+1} = \hat{\mathbf{x}}_n + \mathbf{g}_n \left(y_n - \mathbf{c}_n^T \hat{\mathbf{x}}_n \right) \quad \mathbf{g}_n = [g_{k,n}] = r_k \bar{c}_{k,n} \quad (5)$$

where $\{\hat{\mathbf{x}}_n = [\hat{x}_{k,n}], k = 1, \ldots, N, N = 2L+1\}$ is the estimated state vector, $\{r_k; k = 1, \ldots, N\}$ are free parameters to set the poles of the system, and the overbar denotes the complex conjugate operator.

Due to the complex exponentials, the channels of the observer can be considered as time-invariant systems with a pole on the unit circle. This is why they are called resonators. The resonator frequencies can be expressed as the ratios of the consecutive samples of the basis functions [6]

$$z_k = \frac{c_{k,n+1}}{c_{k,n}} = e^{j2\pi f_k}, \qquad k = 1, \ldots, N \quad (6)$$

and the transfer function of a channel is

$$Q_k(z) = \frac{r_k z_k}{z - z_k}. \quad (7)$$

These channels work in a common feedback loop forming a single input-multiple output filter bank, the transfer functions of which are

$$H_k(z) = \frac{\frac{r_k z_k}{z - z_k}}{1 + \sum_{i=1}^{N} \frac{r_i z_i}{z - z_i}}, \qquad k = 1, \ldots, N. \quad (8)$$

If the resonator poles are arranged uniformly on the unit circle, and $\{r_k = 1/N, k = 1, \ldots, N\}$, the observer has finite impulse response, and the observer corresponds to the RDFT [6]. In that case, the transfer function (8) is very simple

$$H_k(z) = \frac{1}{N} \frac{z^N - 1}{z - z_k} z^{-N}, \qquad k = 1, \ldots, N \quad (9)$$

the magnitude response of which is

$$|H_k(f)| = \left| \frac{\sin \pi N (f - f_k)}{N \sin \pi (f - f_k)} \right|, \qquad k = 1, \ldots, N. \quad (10)$$

Equation (10) has zeros at each resonator frequency, except when $f = f_k$, where $H_k(f) = 1$.

In practical applications where the fundamental frequency changes, the resonators cannot be placed uniformly, and the above setting of parameters r_k does not provide a finite impulse response. The adaptive Fourier analyzer described in [7] adapts the resonator frequencies to coincide with those of the input signal, avoiding the picket-fence effect and leakage. It was successfully utilized, e.g., in high-precision vector-voltmeters [7] or in active noise control systems [9].

III. Resonator-Based Identification

A. Identification System

The above described resonator-based generator–observer pair can be used for frequency domain nonparametric system identification as is depicted in Fig. 2. The excitation is given by the state vector of the generator $(\mathbf{x}_0)$ which does not change while the identification is in progress. The system to be identified $(A(z))$ has to be in between the generator and the observer and the ratio of the corresponding state variables of the observer and the generator supply the results

$$\hat{A}(f_k) = \frac{\hat{x}_k}{x_k}, \qquad k = 1, \ldots, N. \quad (11)$$

Exponential averaging is an option of the structure, and it is controlled by the parameter α. Its role is discussed in detail in the following subsections.

The setup of Fig. 2 assumes that the input of the system is known exactly. In practical cases, where the exact input of the system could be unknown, the input state variables can be measured by another resonator-based observer.

Since the same basis functions $\mathbf{c}_n$ are applied both in the generator and the observer, no picket-fence effect and leakage occurs, even if finite wordlength effects are taken into consideration. The operation of the method can be characterized by noise suppression and measurement time. These are discussed below.

B. Identification Over a Uniform Resonator Set

The resonators are arranged uniformly on the unit circle, if $f_k = k/N$. If $\{r_k = 1/N, k = 1, \ldots, N\}$, the observer performs the RDFT [6]. It is the case when $\alpha = 1$ in Fig. 2. Each channel has an equivalent noise bandwidth of $1/N$. If the measurement noise is white, the ratio of the variances are

$$\frac{\sigma_1^2}{\sigma_0^2} = \frac{1}{N} \quad (12)$$

where σ_0^2 is the variance of the original measurement noise, and σ_1^2 is the variance of $\hat{x}_{k,n}$. The system has finite impulse response, and the measurement time is N steps.

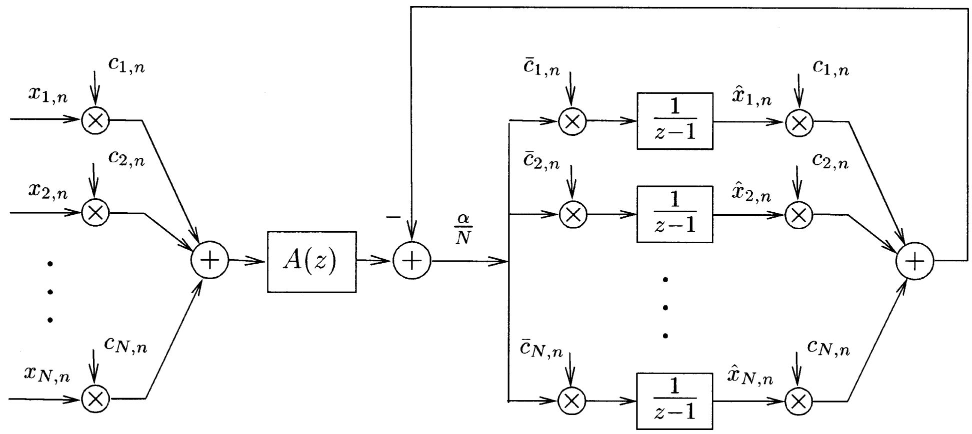

Fig. 2. Resonator-based identification.

If $0 < \alpha < 1$, the measurement results are exponentially averaged. The equivalent time constant is

$$\beta = 1 - (1-\alpha)^{\frac{1}{N}}. \tag{13}$$

In this case, the noise suppression is (see e.g., [10])

$$\frac{\sigma_2^2}{\sigma_0^2} \approx \frac{\beta}{2} \tag{14}$$

where σ_0^2 is the variance of the original measurement noise, and σ_2^2 is the variance of the averaged output. This averaging improves the noise suppression of (12) if β is small enough. However, not all α values result in a lower noise variance. For N sufficiently large, β can be approximated by

$$\beta \approx \frac{-\log(1-\alpha)}{N}. \tag{15}$$

Substituting this into (14), the noise suppression is

$$\frac{\sigma_2^2}{\sigma_0^2} \approx \frac{-\log(1-\alpha)}{2N} \tag{16}$$

where $-\log(1-\alpha)/2 > 1$ if $\alpha > 1 - e^{-2} = 0.8647$. It follows that (14) is slower than (12) for $\alpha > 0.8647$.

Since the system has infinite impulse response, the measurement time depends on the accuracy of the measurement

$$K \approx \frac{\log \varepsilon}{\log(1-\beta)} \tag{17}$$

where ε denotes the final error to be achieved. Note that in practical cases, first β is determined upon the specification of the identification task, and α is calculated by the inverse of (13)

$$\alpha = 1 - (1-\beta)^N. \tag{18}$$

C. Identification Over an Arbitrary Resonator Set

In many practical cases, the identification shall be done over a nonuniform frequency set: e.g., acoustical measurements require logarithmic frequency points. In these cases, (9) and (10) are no more valid, and the system has infinite impulse response, even if $\alpha = 1$. However, it remains that the kth transfer function has zeros at each resonator frequency, except when $f = f_k$, where it is exactly 1. It means that the structure is able to perform undistorted measurements, according to (11). Note that the identification in this case does not require extra calculations compared to the uniform resonator set case.

The calculation of the noise suppression and the measurement time is generally very complicated, since each channel has a different equivalent noise bandwidth. Fortunately, if $\beta \ll 1$, the relevant transfer functions can be well approximated as follows:

$$\begin{aligned} H_k(z) &= \frac{\frac{r_k z_k}{z-z_k}}{1+\frac{r_k z_k}{z-z_k}+\sum_{i=1,i\neq k}^{N}\frac{r_i z_i}{z-z_i}} \\ &\approx \frac{\frac{r_k z_k}{z-z_k}}{1+\frac{r_k z_k}{z-z_k}} = \frac{r_k z_k}{z-z_k(1-r_k)}, \qquad k=1,\ldots,N \end{aligned} \tag{19}$$

where $r_k = \alpha/N$ in Fig. 1. In this case, the kth channel can be approximated with another resonator-based observer output, which contains one resonator only, at the frequency of f_k, with $r_k \approx \beta$. The approximation is good along the frequency axis, with the exception of the neighborhood of the original resonator positions, since the approximating transfer function has no zeros at those places. This is demonstrated in Fig. 3. The figure shows that the magnitude response of the original and the associated structure are close to each other. It also implies that the equivalent noise bandwidth is nearly the same for the two structures, so (14) is a good estimation for the case of an arbitrary resonator set.

Due to the Parseval's theorem, the measurement time can be estimated by the corresponding transfer functions. The squared error of the approximating transfer function (19) tends to zero as β tends also to zero. Numerical simulations show that the error of the approximation can be omitted for β values frequently used in the practice ($\beta \leq 0.01$). Thus, (17) is also a good estimation for the measurement time in the case of an arbitrary resonator set.

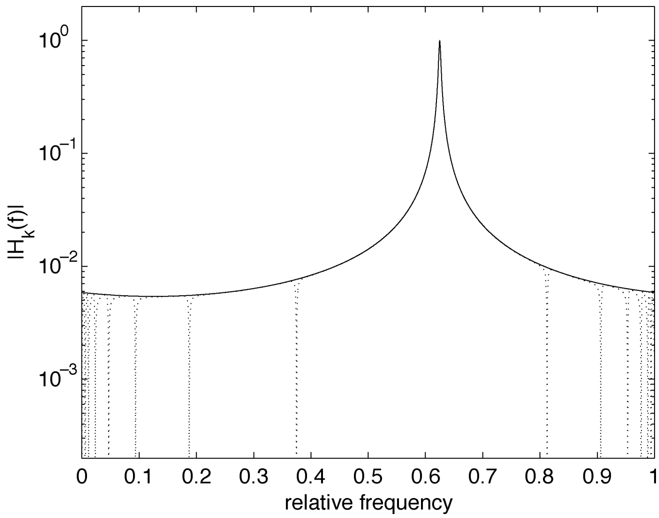

Fig. 3. Magnitude response of one resonator channel with logarithmic frequency set ($\beta = 0.01$). Solid line: magnitude response of the approximating structure; dashed line: magnitude response of the original structure.

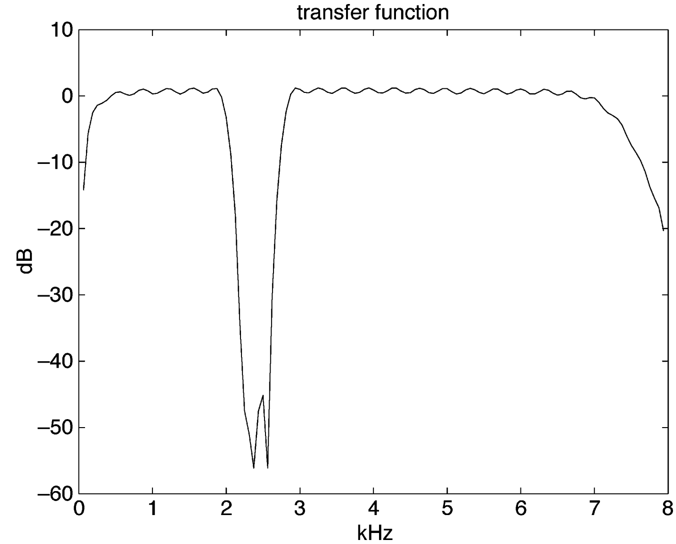

Fig. 4. Result of the identification of a bandstop FIR filter.

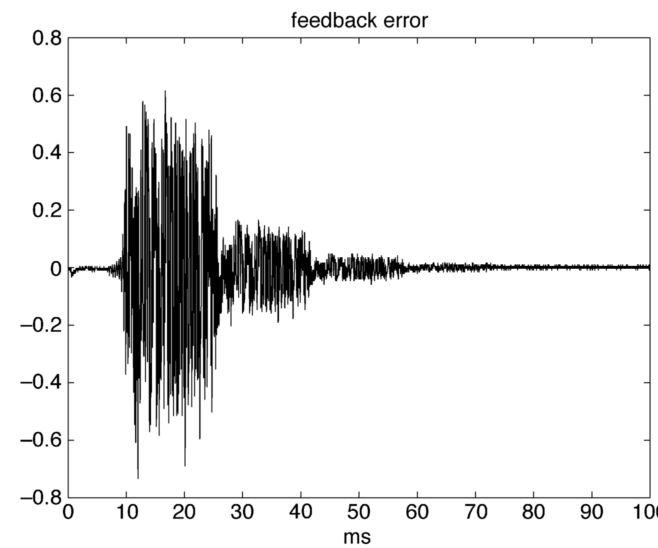

Fig. 5. Settling of the resonator-based structure in the case of identification of a bandstop filter.

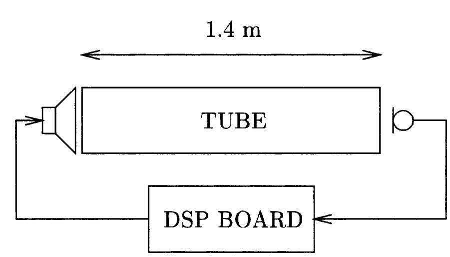

Fig. 6. Schematics of the measurement setup in example 2.

IV. MEASUREMENTS

A. Measurement System

In order to test the algorithm, a digital signal processing (DSP) program was written. The proposed identification method was implemented on an ADSP 2181-based EZ-KIT Lite development board [11]. (ADSP 2181 is a 16-b fixed-point signal processor [12].) It has two analog channels with 16-b delta–sigma A/D and D/A converters. The sampling frequency can be set from 5.5125 to 48 kHz, in several steps. The system to be identified has to be connected in between the D/A and the A/D converters. The input variables of the program are: 1) the multisine excitation sequence (thus, the number of the resonators) and 2) the time constant β for exponential averaging (see Section III). The proposed identification is an online method, so the resolution is limited by the computational complexity of the method. Up to the sampling frequency of 8 kHz, the transfer function can be measured in 512 points. At the maximal sampling frequency of 48 kHz, the measurement can be done in 64 points.

Now the proposed method is illustrated by two examples. The first one is the identification of a bandstop finite impulse response (FIR) filter, which illustrates the main features of the method transparently. The second example is more complicated: acoustic transfer function measurement of a tube.

1) Example 1: Identification of a Bandstop Filter: The bandstop filter is a FIR filter implemented on another DSP board, with a sampling frequency of 16 kHz. The measurement result can be seen in Fig. 4. The transfer function is measured in 256 points, i.e., with a resolution of 62.5 Hz. Damping near to dc and 8 kHz are due to the ac coupling and the delta–sigma A/D and D/A converters. The suppression in the stop band is about 50 dB, while the specification is 60 dB. This difference is because of the 16-b wordlength.

The settling of the system can be seen in Fig. 5. The figure shows the feedback error of the structure (see Fig. 1) in the case of the above measurement, with a time constant of $\beta = 0.005$. The settling can be considered complete if this error signal is zero. Although the settling of the system is exponential, the decay is not continuous: the magnitude of the transient changes only in every 16 ms (in every $N = 256$ steps), when it is multiplied by $1 - \alpha$. In the example $1 - \alpha \approx 0.28$, according to (18). This settling is characteristic only when the resonators are arranged uniformly.

2) Example 2: Identification of an Acoustic Transfer Function: The schematics of the measurement setup can be seen in Fig. 6. A plastic tube of a length of 1.4 m and a diameter of 0.2 m

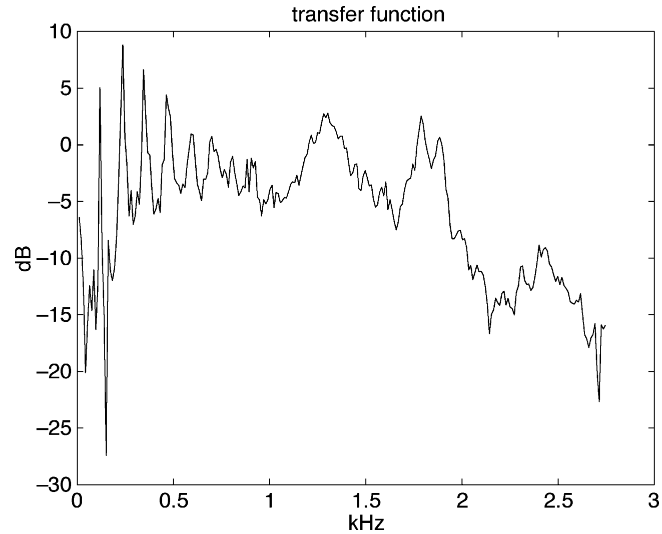

Fig. 7. Result of the identification of the acoustic transfer function of a tube.

was excited by a loudspeaker at one end of the tube, while a microphone was placed to the another end of the tube. The sampling frequency was 5.5125 kHz, and the transfer function is measured in 512 points, i.e., with a resolution of about 10 Hz. The result of the identification can be seen in Fig. 7. The time constant of the averaging was set to $\beta = 0.005$ as in the previous case. Fig. 7 clearly shows the resonances at frequencies of about 120 Hz and its multiples. At low frequencies, there are standing acoustic waves in the tube. Indeed, at 120 Hz, the half of the wavelength of the sound in air equals the length of the tube.

V. Conclusion

A nonparametric identification method for linear systems was introduced. The paper described the theoretical background of the method and it was illustrated by two practical examples. The advantages of the proposed method can be summarized as follows.

- It provides the identification result online.
- Exponential averaging can be easily implemented by changing a multiplier factor in the structure.
- The identification can be done over an arbitrary frequency set and it requires no extra calculations.

As a disadvantage, it should be mentioned that the number of frequency points is limited, compared to the conventional FFT-based methods. The computational demand of the proposed method is proportional with N for each time instant n, which corresponds to the DFT. Consequently, the highest sampling frequency (providing online analysis) for a given N is less than that of the FFT-based solutions. The resonator-based method, therefore, is an alternative of the FFT-based analysis if slowly time varying systems should be identified, especially if the frequency set is not uniform.

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