A NEW FILTERED LMS ALGORITHM FOR ACTIVE NOISE CONTROL

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INTRODUCTION

The least mean square (LMS) algorithm is well-known for engineers involved in active noise control (ANC). It is proved to be a robust algorithm for adaptation of transversal digital filters used for different purposes in ANC systems: it can be applied for the adaptation of the controller, as well as for off-line or on-line estimation of the relevant acoustic transfer functions. A very good introduction to LMS adaptive filters is available in [9] or [10] written by the inventor of the algorithm.

In ANC applications the output of the adaptive filter drives the secondary path, and the error signal is derived only at the microphone. In such cases the simple LMS algorithm is unstable due to the phase shift caused by the secondary path. The problem is analyzed e.g. in [7] and the solution is the so-called filtered reference or filtered-X LMS (XLMS) algorithm [9]. This algorithm requires a model of the secondary transfer function, which the reference signal is filtered by. The identification of the secondary transfer function can be done off-line, using the simple LMS algorithm. The model shall be so accurate that its phase error does not exceed $\pi/2$, otherwise the adaptive system is unstable. The XLMS algorithm was extended also for multiple input – multiple output controllers and it is refered as multiple error LMS (MLMS) algorithm [4]. A tutorial on the utilization of the XLMS or the MLMS algorithm can be found e.g. in [3] or [6].

Although the XLMS algorithm is stable, its convergence can be very slow, depending on the secondary transfer function. In ANC experiments, the suppression of some sinusoidals by the XLMS algorithm requires tens of seconds. This means that the XLMS algorithm is practically unusable in such situations. Recognizing this drawback of the XLMS algorithm, many modifications of the original algorithm were published. (see e.g. [1], [5]), or some recursive algorithms were proposed [6]. One of the most successful improvements is the transform-domain LMS algorithm [2], [6]. The main advantage of this algorithm originates from the possibility to set the convergence coefficients independently at each frequency bin.

This paper introduces an alternative structure which can improve the convergence speed of the XLMS and the MLMS algorithm. The disadvantageous convergence properties of these algorithms originate from the high dynamics in the magnitude response of the secondary path. The convergence rate of the adaptation depends on the loop gain in the adaptation path, which is proportional to the square of the magnitude response, due to the

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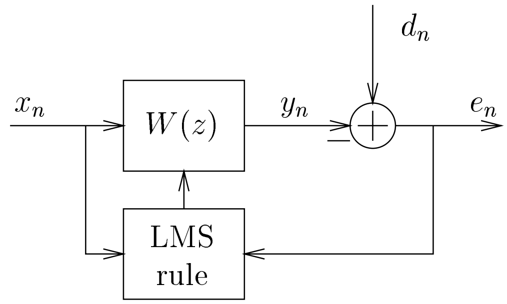


Figure 1: *The LMS adaptive filter*

LMS adaptation rule. The proposed algorithm is a modification of the original XLMS and the MLMS algorithm: the reference signal is filtered not only by the model of the secondary transfer function, but by an auxiliary filter which is designed to provide an overall magnitude response close to the unity along the whole frequency axis. In order to keep the stability, the error signal has to be also filtered by this auxiliary filter. The paper introduces this algorithm both for single channel and for multiple channel systems and illustrates their behavior.

Section 2. recalls the LMS algorithm and its extensions, and section 3. introduces the proposed structure. Section 4. investigates the main features of the novel algorithm showing examples. The paper is closed with a short conclusion.

2. THE LMS ALGORITHM AND ITS EXTENSIONS

The LMS algorithm. The LMS adaptive filter can be seen in Fig. 1. In this figure $W(z)$ denotes the adaptive transversal filter, x_n, y_n and e_n are the reference signal, the output of the filter and the error signal at time step n, respectively. d_n is the desired signal, which x_n has to be correlated with. The system is described by the following equations:

$$y_n = \mathbf{w}_n^T \mathbf{x}_n \tag{1}$$

$$e_n = d_n - y_n \tag{2}$$

where $\mathbf{w}_n$ denotes the N coefficients of the adaptive filter and $\mathbf{x}_n$ is the vector formed from the actual and delayed samples of the reference signal at time step n. The LMS adaptation rule is the following:

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu e_n \bar{\mathbf{x}}_n \tag{3}$$

where the overbar denotes the complex conjugate operator and μ is a positive constant which controls the stability and the convergence rate of the adaptation. The correlation between x_n and d_n can be represented by a discrete transfer function. After a successful adaptation, $W(z)$ approximates this transfer function in a least mean square sense. If x_n and d_n are input and output signals of an acoustic path, the LMS adaptive filter is able to identify this acoustic transfer function.

The XLMS algorithm. In ANC applications the adaptive filter is the controller. In this case $W(z)$ is updated by the XLMS algorithm. The structure can be seen in Fig. 2, where the secondary transfer function is denoted by $A_2(z)$. $\hat{A}_2(z)$ is a model of the secondary transfer function which is identified off-line. The system is described as follows:

$$e_n = d_n - A_2(z) y_n \tag{4}$$

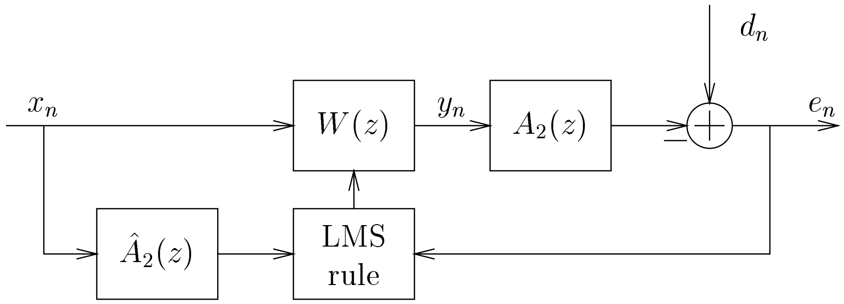


Figure 2: *The filtered-X LMS algorithm*

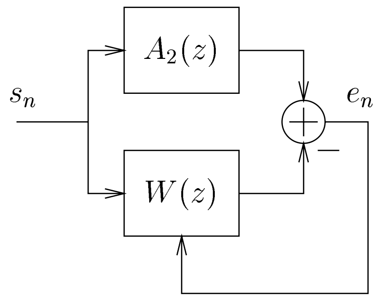


Figure 3: *Model identification for the XLMS algorithm*

where y_n is defined as in (1). (3) is modified in the following way:

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu e_n \bar{\mathbf{r}}_n \tag{5}$$

where $\mathbf{r}_n$ is the vector formed from the actual and delayed samples of the filtered reference signal r_n:

$$r_n = \hat{A}_2(z) x_n \tag{6}$$

$\hat{A}_2(z)$ can be either infinite or finite impulse response (IIR or FIR) filter, but it is usually an FIR filter. The identification of $A_2(z)$ can be done by the system depicted in Fig. 3. It is a utilization of the simple LMS adaptive filter. If the excitation s_n is white noise, $W(z)$ provides a satisfactory model of $A_2(z)$. The system is stable if the phase error of the model does not exceed $\pi/2$ [7], [10].

The MLMS algorithm. Assuming the most general case, there are K reference signals, L output (loudspeaker) signals and M error (microphone) signals [3], [6]. For simpler notations the algorithm is introduced for the $K = 1$ case, i.e. when only one reference signal is used [4]. Note that it is not a restriction, in practical cases it can be achieved that one reference signal is correlated with all the components to be canceled. The adaptive filter is now a vector, each column of which is a transversal filter as it was described for the single channel case. The system updated by the MLMS algorithm can be seen in Fig.4. and it is described as follows:

$$\mathbf{y}_n = \mathbf{W}_n^T \mathbf{x}_n \tag{7}$$

$$\mathbf{e}_n = \mathbf{d}_n - \mathbf{A}_2(z) \mathbf{y}_n \tag{8}$$

where $\mathbf{W}_n$ denotes the adaptive filter vector, $\mathbf{x}_n$ is the vector formed from the actual and delayed samples of the reference signal, $\mathbf{y}_n$ and $\mathbf{e}_n$ are the output and the error vector,

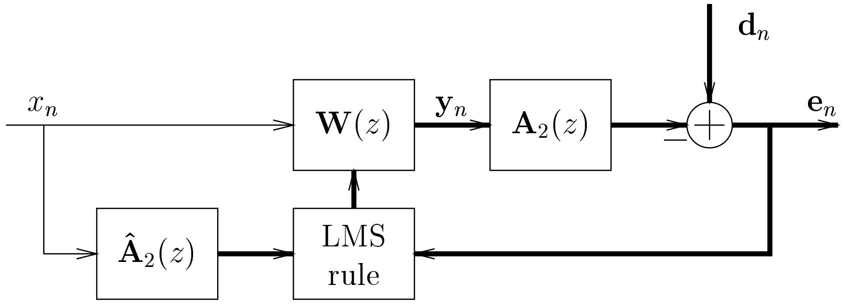


Figure 4: *The MLMS algorithm*

respectively. $\mathbf{A}_2(z)$ is the secondary transfer function matrix. The equation corresponding to (3) is the following:

$$\mathbf{w}_{i,n+1}^T = \mathbf{w}_{i,n}^T + \mu\Big(\mathbf{R}_{n-i}^H \mathbf{e}_n\Big)^T \tag{9}$$

where $\mathbf{w}_{i,n}^T$ denotes the row vector of $\mathbf{W}_n$ belonging to the i-th coefficient of the adaptive filter vector and $\mathbf{R}_{n-i}$ stands for the filtered reference signal delayed by i samples, $(i = 0 \dots N-1)$. The superscript H is the transpose conjugate operator. The filtered reference signal is defined similarly to the single channel case:

$$\mathbf{R}_n = \hat{\mathbf{A}}_2(z) x_n \tag{10}$$

In this equation $\hat{\mathbf{A}}_2(z)$ denotes the model of the secondary transfer function matrix, each element of which is a transversal filter with the same length.

The algorithm can be extended easely for the $K \neq 1$ case. The description of the algorithm can be found e.g. in [3]. The identification of the secondary transfer function matrix can be done using the LMS algorithm, too. In order to identify each element of $\mathbf{A}_2(z)$, the L secondary sources have to be excited separately.

3. THE PROPOSED NEW FILTERED LMS ALGORITHM

In this section first the single channel algorithm is described. This first subsection introduces the idea of the new algorithm and deals with the main assumptions of the system design. The introduced idea is generalized for the multiple channel case in the second subsection.

Single channel algorithm. The disadvantageous convergence properties of the XLMS algorithm originate from the high dynamics in the magnitude response of $A_2(z)$. The convergence rate of the adaptation depends on the loop gain in the adaptation path, which is proportional to $|A_2(z)|^2$, due to the XLMS adaptation rule (5). The convergence speed of the LMS algorithm is controlled by the parameter μ, which is limited due to the maximum of $|A_2(z)|$. If $|A_2(z)|$ has high dynamics, there are some frequency bands, where the loop gain is very small. For any signal appearing in this frequency range the convergence rate will be small.

The proposed new filtered LMS algorithm filters both the reference and the error signal, therefore it can be called filtered reference – filtered error LMS (EXLMS) algorithm. It tries to solve the problem caused by the dynamics of $|A_2(z)|$ keeping the filter $\hat{A}_2(z)$. An FIR

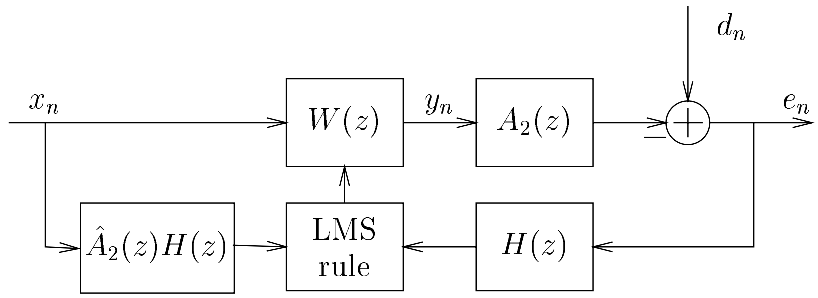


Figure 5: *The proposed filtered LMS algorithm*

filter is applied, which filters both the error and the reference signal. This filter is designed so that the resultant magnitude response oscillates around the unity. The proposed structure can be seen in Fig. 5. The system is a modification of the XLMS structure. The new element in the figure is $H(z)$, which is the filter mentioned above. The system is described by (1) and (4), while (5) is modified in the following way:

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu H(z) e_n \bar{\mathbf{r}}_n \tag{11}$$

where

$$r_n = H(z)\hat{A}_2(z)x_n \tag{12}$$

Since $H(z)$ is applied in both pathes, the system is stable. In the following the main assumptions for filter design are introduced.

In this structure $\hat{A}_2(z)$ cares of the stability, while $H(z)$ is the compensator to achieve a fairly fast convergence, therefore its magnitude response is specified so, that:

$$|H(z)| \approx \frac{1}{|\hat{A}_2(z)|} \tag{13}$$

The error of the approximation could be higher than it is usual in filter design. It involves that the required number of the coefficients of $H(z)$ can be much lower than that of $\hat{A}_2(z)$.

The magnitude response of $H(z)$ can be prescribed by the loop gain of the adaptation:

$$L(z) = \mu |H(z)A_2(z)|^2 \tag{14}$$

where it is assumed that $\hat{A}_2(z) = A_2(z)$. In ANC applications the convergence parameter μ is set experimentally to achieve the best convergence rate. In such cases smaller steps than 6 dB to change μ have no meaningful influence to the convergence rate. Since μ and the square of the resultant transfer function of the filters control directly the loop gain, $H(z)$ has to be designed so that the resultant magnitude response $|H(z)\hat{A}_2(z)|$ varies in a 3 dB range. The filter design itself can be done using the simple frequency sampling method, where $1/|\hat{A}_2(z)|$ is sampled. If $H(z)$ is designed so that the resultant magnitude response is "too smooth", i.e. it ripples very close to the unity, the convergence is slower, because of the large delay of $H(z)$.

The role of $H(z)$ can be interpreted by the frequency domain adaptive filtering [2]. Frequency domain adaptation provides the possibility to set the convergence coefficients at each channel independently, according to the power of the signal appearing in the channel. It can be shown that the usual normalization with the power of the signals appearing at the

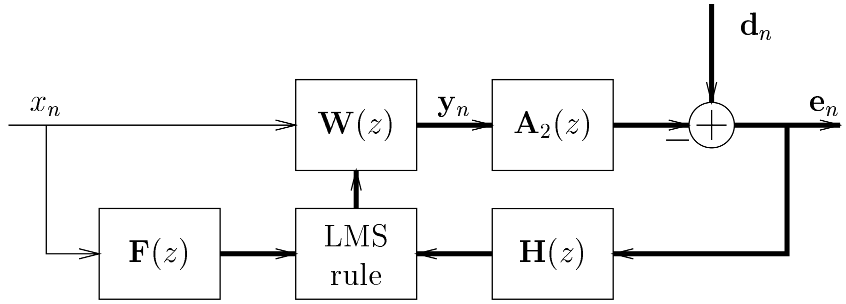


Figure 6: *The proposed multiple channel LMS algorithm*

channels leads to a similar resultant magnitude response as the proposed compensator filter $H(z)$. The proposed method ensures similar possibility, but in the time domain, decreasing the computational burden of the adaptation. The idea to compensate the magnitude response of the secondary path was successfully adapted earlier for periodic noise control [8].

Multiple channel algorithm. The multiple channel algorithm is the extension of the single channel algorithm derived above. The impact of the secondary transfer matrix ($\mathbf{A}_2(z)$, see. Fig. 4) should be compensated. The aim of $H(z)$ in the single channel system is to provide a nearly unity overall magnitude response. Now, in the multiple channel system it is straightforward to compensate the transfer function matrix so that the magnitude response of the filters corresponding to one adaptive filter coefficient is the unity. The algorithm is introduced only for the single reference case.

The proposed structure can be seen in Fig. 6. The system is described by (7) and (8), while (9) is modified in the following way:

$$\mathbf{w}_{i,n+1}^T = \mathbf{w}_{i,n}^T + \mu\Big(\mathbf{F}_{n-i}^H \mathbf{e}_n \mathbf{H}(z)\Big)^T \tag{15}$$

where

$$\mathbf{F}_n = \hat{\mathbf{A}}_2(z) < H_l(z) > x_n \tag{16}$$

and $\mathbf{H}(z) = [H_1(z) \ldots H_L(z)]$. $< H_l(z) >$ is a diagonal matrix, the elements of which are the transversal filters $H_l(z)$. Their magnitude response are specified so, that:

$$|H_l(z)| \approx \frac{1}{||\hat{\mathbf{A}}_{2,l}(z)||_2} \tag{17}$$

where $\hat{\mathbf{A}}_{2,l}(z)$ denotes the l-th column of $\hat{\mathbf{A}}_2(z)$ and $||.||_2$ is the euclidean norm operator, i.e.:

$$||\hat{\mathbf{A}}_{2,l}(z)||_2 = \sqrt{\sum_{m=1}^{M} |\hat{A}_{2,ml}(z)|^2} \tag{18}$$

The magnitude response of each $H_l(z)$ can be prescribed similarly to the single channel case, and the filters can be designed independently from each other. In the system $\hat{\mathbf{A}}_2(z)$ ensures the stability and the filter set $\mathbf{H}(z)$ compensates the magnitude response. Since in (15) the filtered error signal is multiplied by the complex conjugate of $\mathbf{F}_n$, the phase shift caused by $\mathbf{H}(z)$ is zero. This is why this algorithm provides the same stability borders as the original MLMS algorithm.

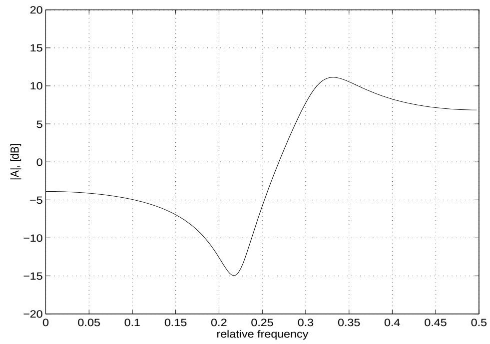


Figure 7: *Magnitude response of $A_2(z)$ in (19)*

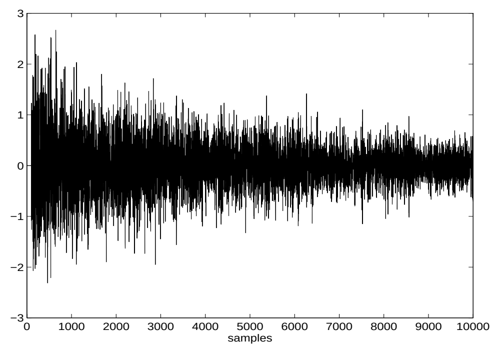


Figure 8: *Error signal of the XLMS algorithm*

The filter design described in this section needs only off-line calculations. The on-line calculations of the compensator filters do not require high computational capacity. Furthermore, there is a trade-off between the computational demand and the convergence rate. All the decisions of the designer lead to a stable ANC system.

EXAMPLES

Simulational example. In this subsection some simulation results are discussed. The plant in all examples is a simple second-order IIR filter:

$$A_2(z) = \frac{z^2 - 0.4164z + 1.2346}{z^2 + 0.6627z + 0.6414} \tag{19}$$

Its magnitude response can be seen in Fig. 7. Although it is a very simple transfer function with only about 25 dB dynamics, it can illustrate the efficiency of the proposed method in a convincing manner. In all examples the adaptive filter has 200 coefficients and the signal to be canceled d_n is defined as:

$$d_n = x_{n-100} \tag{20}$$

where x_n is the reference signal which is a white noise with the same uniform distribution in all case. The convergence parameter μ is set in all examples experimentally to achive the highest convergence rate.

First the adaptive filter is updated by the XLMS algorithm (Fig.2). In this case $\mu = 0.0002$, and it is the best one. The error signal can be seen in Fig.8. The convergence is very slow, after 10,000 steps the error signal is about one third of the initial value.

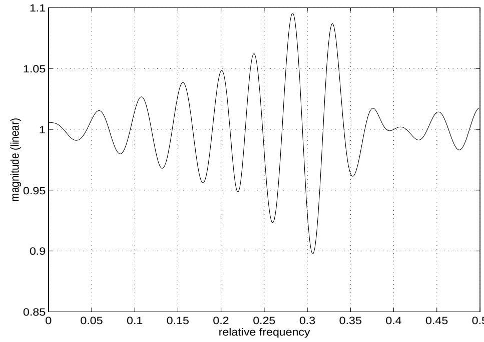


Figure 9: *Magnitude response of* $H(z)A_2(z)$

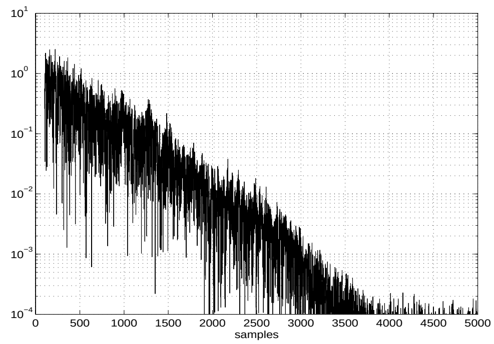


Figure 10: *Error signal of the proposed EXLMS algorithm*

In the second experiment the adaptive filter is updated by the proposed EXLMS algorithm (Fig. 5). $H(z)$ has 41 coefficients. The magnitude response of $A_2(z)$ is well compensated as it can be seen in Fig.9. In this case $|H(z)A_2(z)|$ varies approximately in a 3 dB range, and $\mu = 0.005$, the error signal can be seen in Fig.10. The amplitude of the error signal is plotted in logarithmic scale. After 4,000 steps the error signal is about 10,000 times smaller than the initial value, it means that $H(z)$ with 41 coefficients could significantly improve the convergence rate. Since the adaptation can be treated as complete, the primary signal is canceled.

Practical Example. The examined set-up is a simple model of a ventilation duct (Fig. 11). It is a circular pipe with an attached loudspeaker for the simulation of the noise and another one for the secondary source. There is microphone inside for the error signal. The used microphone is quite common, its characteristic is not sensitive to the input sound direction. The noise is generated by the generator and its output was used as reference signal. For measurement and control purposes we have used a MOTOROLA DSP96001 based PC card. It has two analog channels with 16 bit A/D and D/A converters. The sampling frequency was 2 kHz, the secondary transfer function is modelled by a 200 coefficient FIR filter, the magnitude response of which is compensated by a 64 coefficient FIR filter. For the evaluation of the control results we have used a spectrum analyzer and a digital storage oscilloscope.

The following figures show the convergence of the system when the primary noise was a periodic sound with a fundamental frequency of 120 Hz. Fig. 12.a shows the error signal when the system was adapted by the XLMS algorithm and Fig. 12.b shows the error signal when the proposed EXLMS algorithm was used. The waveform of the error signal cannot be seen,

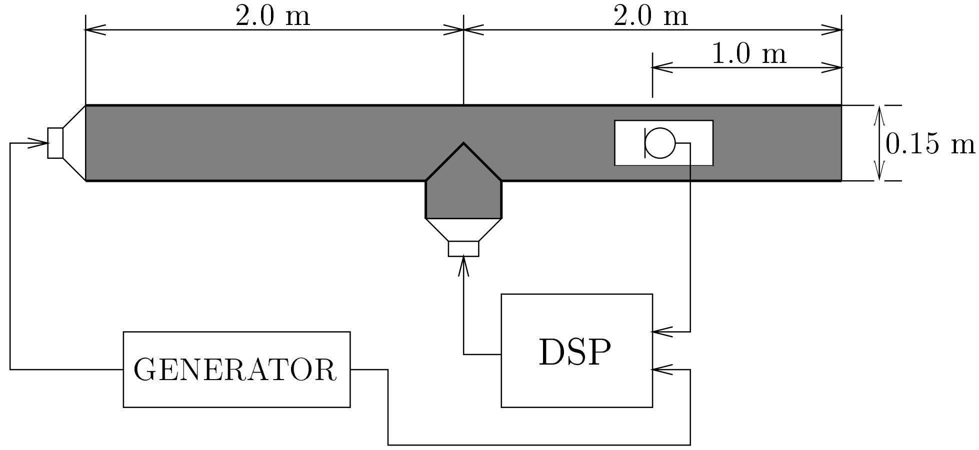
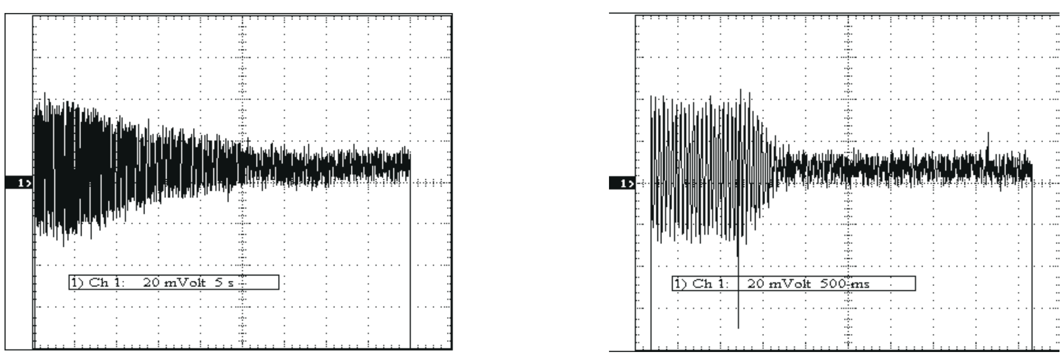


Figure 11: *The experimental set-up*



(*a*) (*b*)

Figure 12: *The error signal using (a) the XLMS and (b) the EXLMS algorithm*

since its period time is much shorter than the settling time. Both algorithms could suppress the primary noise, the remaining noise in the diagrams correspond to the measurement noise. Note that the time scales of the two figures are different: it is 5 sec/div for Fig. 12.a, and 0.5 sec/div for Fig. 12.b. It means that the settling time in the case of the XLMS algorithm is about 30 sec, while it is less than 1 sec in the case of the proposed EXLMS algorithm.

CONCLUSION

The paper presented a new filtered LMS algorithm which can improve the convergence rate of the widely used filtered reference and multiple error LMS algorithms. The proposed structure is a modification of the filtered reference LMS algorithm: in addition to the filter in the reference signal path, a secondary filter is applied, and the same filter is applied in the error signal path. This secondary filter is designed so that the resultant magnitude response in the adaptation loop oscillates around the unity. The paper described the algorithm for the multiple input – multiple output case, as well. The numerical and the pratical examples shown in the paper verify that the proposed method improves the convergence rate without high additional computational demand.

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