PERIODIC NOISE CANCELATION USING RESONATOR BASED CONTROLLER

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ABSTRACT

A resonator based filter bank is proposed as controller in acoustic noise canceling loops where the noise to be suppressed is periodic. The controller consists of resonators at the frequencies of the periodic signal to be suppressed. The paper presents the design procedure for both single and multiple channel case and investigates the behavior of the noise canceling systems. The proposed method is compared to the conventional adaptive feedforward controller updated by the filtered-X LMS algorithm. The resonator based controller shows faster convergence and needs less computations than the usual methods. The advantages of the new controller are demonstrated by a theoretical and a practical example.

Keywords: active noise control, feedback control, feedforward control, resonator, experiment

INTRODUCTION

Active suppression of periodic acoustic noise is one of the simpler noise control problems: both feedback and feedforward control can be used. If only the error signal is available, feedback control is satisfactory, since a harmonic signal can be easily predicted. If reference signal is also available, the implementation of feedforward control is straightforward. There are many feedforward control applications which use adaptive filters, with the parameters updated in each sample interval, mainly on LMS basis. These systems are able to suppress both broadband and periodic noise. In the case of periodic noise cancelation the reference signal consists of all the harmonic components to be suppressed. However, in this case a more adequate controller design is possible. The theoretical background of such a controller design is the adaptive Fourier analysis. The adaptive Fourier analyzer (AFA) is a structurally adaptive system for exact measurement of band-limited periodic signals of arbitrary fundamental frequency [2]. It is an extension of the resonator

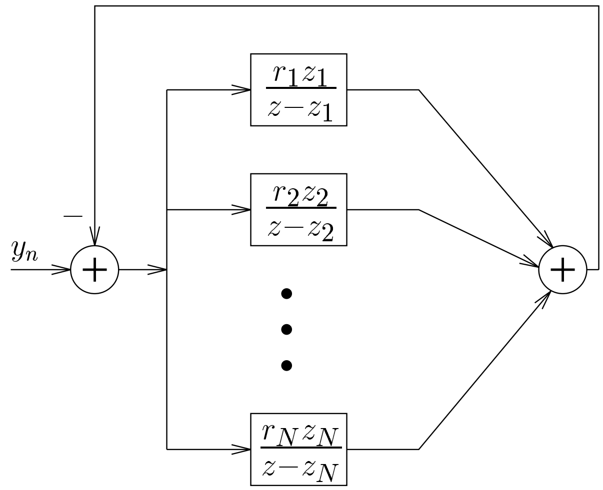

Figure 1: *Observer for periodic signals*

based observers developed earlier to perform the recursive discrete Fourier transform (RDFT) [3], [4]. In these observers the resonators work in a common feedback loop providing zero steady-state feedback error at the resonator frequencies. The AFA adapts the resonator frequencies to coincide with those in the input signal.

The proposed noise controller can be considered as an extension of the AFA mentioned above. In Section 2 the AFA is recalled and the derivation of the new structure for single channel case is given. Section 3 deal with the extension of the new method for multiple channel systems. In Section 4 the proposed method is compared to the feedforward adaptive controller extending the idea described in [9]. The advantages of the resonator based noise canceling system are illustrated in Section 5.

SINGLE CHANNEL NOISE CONTROLLER DESIGN

Adaptive Fourier Analyzer. The resonator based observer was designed to follow the state variables of the so-called conceptual signal model [2]. The signal model is described as follows:

$$y_n = \mathbf{c}_n^T \mathbf{x}_n \tag{1}$$

$$\mathbf{c}_n = [c_{n,k}] = e^{j2\pi f_1 kn},\ k = -L..L \tag{2}$$

$$Lf_1 < 0.5 < (L+1)f_1 \tag{3}$$

where $\mathbf{x}_n$ is the state vector of the signal model at step n, y_n is its output (the input of the observer), $\mathbf{c}_n$ represents the basis of the Fourier expansion, and f_1 is the fundamental frequency relative to the sampling frequency. The corresponding observer is (Fig. 1):

$$\hat{\mathbf{x}}_{n+1} = \hat{\mathbf{x}}_n + \mathbf{g}_n(y_n - \mathbf{c}_n^T \hat{\mathbf{x}}_n); \quad \mathbf{g}_n = [g_{n,k}] = r_k \bar{c}_{n,k} \tag{4}$$

where $\hat{\mathbf{x}}_n$ is the estimated state vector, $\{r_k;\ k = 1..N;\ N = 2L+1\}$ are free parameters to set the poles of the system and the overbar denotes the complex conjugate operator.

Due to the complex exponentials, the channels of the observer can be considered as time-invariant systems with a pole on the unit circle $\{z_k;\ k = 1..N\}$ (Fig. 1). This is why they are called resonators. If the resonator poles are arranged uniformly on the unit circle, and $\{r_k = 1/N;\ k = 1..N\}$, the observer performs a recursive Fourier transformer of N points. The result of the transformation is the state vector. In this case the system has finite impulse response [4]. In practical applications [2] where the fundamental frequency changes, the resonators cannot be placed uniformly, and the above setting of parameters r_k does not provide finite impulse response. But, if (2), and (3) hold, the system is fairly fast. If the estimated frequency does not coincide with that of the input signal y_n, the complex state variables will rotate, and the speed of this rotation at each resonator is proportional to the corresponding frequency difference. This is the basic idea for the frequency adaptation in the AFA [2]. The exact formula is the following:

$$f_{1,n+1} = f_{1,n} + \frac{1}{2\pi N}\text{angle}(\hat{x}_{1,n+1}, \hat{x}_{1,n}) \tag{5}$$

where $\hat{x}_{i,1}$ is the estimated state variable belonging to the positive fundamental frequency, and "angle" gives the angle between two complex numbers.

The AFA is already implemented in vector voltmeters and it is a fast and robust system. The fundamental frequency of the measured signal can be estimated precisely even when it has high harmonic distortion [2].

Derivation of the Noise Controller. In steady-state the input of the resonators (i.e. the feedback error) equals zero. This means that the feedback signal (the sum of the resonator outputs) *cancels* the input signal. If acoustic noise should be canceled, the output of the resonators should be connected to a loudspeaker and fed back using a microphone. (A multiplication by -1 is necessary in the controller.) The arrangement can be seen in Fig. 2.a. The frequency is estimated by an independent AFA and it passes the actual $\mathbf{c}_n$ to the controller. Reference signal can be any periodic signal with the same fundamental frequency as the primary noise.

Fig. 2.b shows the block diagram of the control loop, where $R(z)$ and $A(z)$ denote the resonator based controller and the acoustic transfer function between the loudspeaker and the microphone, respectively. Due to the presence of $A(z)$, the stability of the system is not obvious. The controller design will be accomplished by the appropriate choice of the parameters r_k. They can be chosen as follows:

$$r_k = \alpha w_k;\ w_k = \frac{1}{A(z_k)} \tag{6}$$

where α is a convergence parameter. The actual set w_k depends on the fundamental frequency of the primary noise. $A(z)$ is in general not analytically known and (6) cannot be calculated on-line, therefore the transfer function should be measured at a finite number of points and the inverses should be calculated off-line. Thus the actual set w_k should be a result of a mapping $\{f_k\} \to \{w_k\}$ (e.g. the nearest available one). The required number

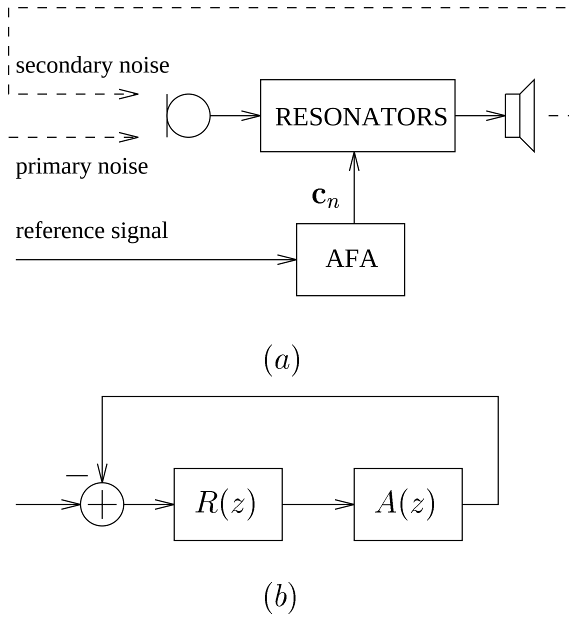

Figure 2: *Periodic noise control: (a) Physical arrangement (b) Block diagram of the control loop*

of the measurement points is determined by the following inequality:

$$-\pi/2 < \mathrm{angle}(w(f)) + \mathrm{angle}(A(f)) < \pi/2 \tag{7}$$

where f is the relative frequency and $w(f)$ denotes the above mentioned mapping. If (7) holds, the system can be stabilized with an appropriate α at any fundamental frequency.

Equation (7) is the condition of the stability. The stability can be proven by the Nyquist stability criterion [6]. From the stability point of view the phases of the parameters r_k are essential. Their amplitudes influence the available maximal speed of the convergence. If (2), and (3) are satisfied and $A(z) \equiv 1$, the system is very fast, the poles lie close to the origin. This feature would hold if $1/A(z)$ could be applied in the loop. In general this inverse filter cannot be implemented. By the choice of the parameters $w_k = 1/A(z_k)$ the $N-1$ degree numerator polynomial of $R(z)$ is set which is a Lagrange type interpolation of the transfer function of the inverse filter. This procedure is in complete correspondence with the frequency sampling method. The approximation is poor at places but it is exact at the resonator frequencies. Because of the error of the interpolation between the resonator poles the loop gain must be decreased by a positive convergence parameter as (6) shows otherwise the system is unstable.

The convergence of the control system can be characterized by the greatest eigenvalue of the system matrix: the smaller this eigenvalue the faster the system. The greatest eigenvalue depends on the choice of the set w_k and the convergence parameter α. Some simulations show that the greatest eigenvalue can be minimized by tuning α if the set w_k is chosen as (6) shows.

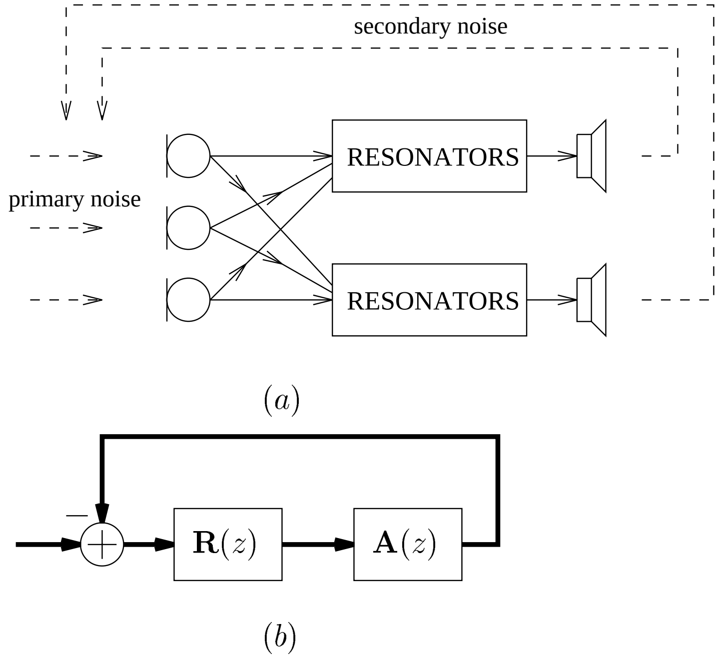

Figure 3: *Multiple channel resonator based noise canceling system. (a) Physical arrangement; (b) Block diagram*

MULTIPLE CHANNEL NOISE CONTROLLER DESIGN

The resonator based noise canceling system can be seen in Fig. 3. The figure shows an example with 3 microphones and 2 loudspeakers. In the multiple channel system to each loudspeaker belongs a resonator set, the input of which is the weighted sum of the microphone signals. The fat lines in the block diagram denote the vector signals, $\mathbf{R}(z)$ and $\mathbf{A}(z)$ are matrices. The weighting of the microphone signals corresponds to the parameters w_k. Indeed, instead of a simple parameter set, here a matrix set should be applied. Extending the idea described above they can be chosen as follows:

$$\mathbf{W}_k = \mathbf{A}^{\#}(z_k) \tag{8}$$

where $\mathbf{W}_k$ is the weighting matrix and # denotes the pseudo- (or Moore-Penrose) inverse. This set of parameters $\mathbf{W}_k$ offers similar convergence properties as (6) in the single channel system. The condition of the stability is:

$$-\pi/2 < \text{angle}\{\lambda_i[\mathbf{W}(f)\mathbf{A}(f)]\} < \pi/2;\ i = 1..I \tag{9}$$

where $\lambda_i(.)$ denotes the i-th eigenvalue of the corresponding matrix. This condition can be derived from the multiple channel Nyquist stability criterion‘[7].

Single channel noise control systems provide zero steady-state error. Unfortunately, in general, multiple channel systems cannot achieve zero error, because there are usually more microphones than loudspeakers. A multiple channel system should minimize the power of the remaining noise. The problem is the following: there are N harmonics to be controlled on M output channels and N harmonics to be suppressed on L input channels. The N output harmonics should be set to minimize the power of the error vector. Since the harmonics can be suppressed independently of each other, there are N independent tasks. The problem of the suppression of one harmonic component is identical with solving a linear equation system with M variables and L constraints:

$$\mathbf{d}_k = \mathbf{A}_k \mathbf{y}_k \tag{10}$$

where $\mathbf{d}_k$ and $\mathbf{y}_k$ denote the noise to be suppressed and the output of the noise control system belonging to the k-th harmonic component, respectively. The solution of the equation system (10) can be expressed by the pseudo-inverse as follows:

$$\mathbf{y}_k = \mathbf{A}_k^{\#} \mathbf{d}_k \tag{11}$$

This solution can be derived using the projection theorem [8]. Depending on the relation between L and M, there are three possibilities:

- if $L = M$, the ordinary inverse can be used, and only one solution exists. The error in steady-state is zero;
- if $L < M$, the error in steady-state is zero, and the norm of the output is minimal;
- if $L > M$, the error in steady-state is not zero, but its norm (i.e. its power) is minimal.

Most of the multiple channel noise control systems have more microphones than loudspeakers, which corresponds to the third case. It can be proven that the resonator based controller finds this optimal solution, if (8) holds.

THE INTER-RELATION OF DIFFERENT TECHNICS

As it is mentioned in the Introduction, for periodic noise control the implementation of the adaptive feedforward control is straightforward. The adaptive transversal filter in this controller is updated by the filtered-X LMS (XLMS) algorithm [1], [5]. The required number of the coefficients is the number of the harmonics to be suppressed. The arrangement can be seen in Fig. 4. The resonator based observer can be seen as an adaptive filter bank, the reference signals of which are the complex exponentials, and the filter coefficients are adapted by the error signal using the LMS algorithm [9]. This duality can be applied for our case, the only difference is that the XLMS algorithm is used. Thus the adaptive filter for periodic noise control is a resonator based controller, where:

$$w_k = \bar{A}(z_k) \tag{12}$$

where the overbar denotes the complex conjugate operator. The phase shifts caused by the filter in the XLMS algorithm and the parameters w_k in the resonator based observer

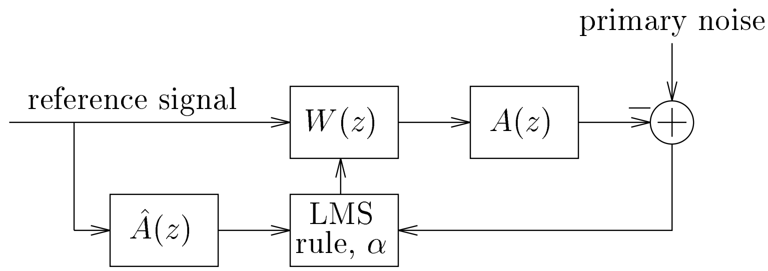

Figure 4: *Adaptive feedforward controller using the XLMS algorithm.*

are obviously the same, thus from stability point of view the systems are identical. However, the adaptive filter does not aspire to approximate the inverse filter, therefore its convergence could be very slow. A heuristic explanation can be given: while in the resonator based observer the gain between the system output and the resonator input is unity, that of the adaptive filter is $|A|^2$. If the secondary path suppresses the signal, in the loop the square of this suppression occurs, so the system will be considerable slow.

In multiple channel noise control systems the multiple-error LMS (MLMS) algorithm can be used [1], [10]. It can be represented also by the corresponding multiple channel resonator based noise controller as follows:

$$\mathbf{W}_k = \mathbf{A}^H(z_k) \tag{13}$$

where $(.)^H$ denotes the conjugate transpose operator. It can be proven that by this choice of the matrices $\mathbf{W}_k$ the steady-state error of the adaptive system can be minimized. It is not a surprise, because the LMS algorithm is designed to minimize the expected value of the square of the error signal.

It has shown that the resonator based noise controller provides faster convergence, while the other features of the control system are identical with those of the cited adaptive system. The proposed controller provides further advantages, regarding the modelling of $A(z)$. The adaptive filter updated by the XLMS algorithm needs a copy of $A(z)$ which is $\hat{A}(z)$ in Fig. 4. $\hat{A}(z)$ is usually a transversal filter and its coefficients are results of the identification of $A(z)$. The number of the coefficients depends on $A(z)$, and it is usually in the range of the number of measurement points required by the resonator based controller. This number could be some hundreds in the practice, and this filter should work on-line. However, the resonator based controller uses only N (the number of harmonic components) parameters on-line. Since the implementation of the resonators and the LMS-updated adaptive filter takes nearly the same computational burden, the computational advantages of the resonator based controller are obvious.

EXAMPLES

Comparison of Noise Controllers. In this section the adaptive feedforward controller and the resonator based controller is compared to each other. This example is result of

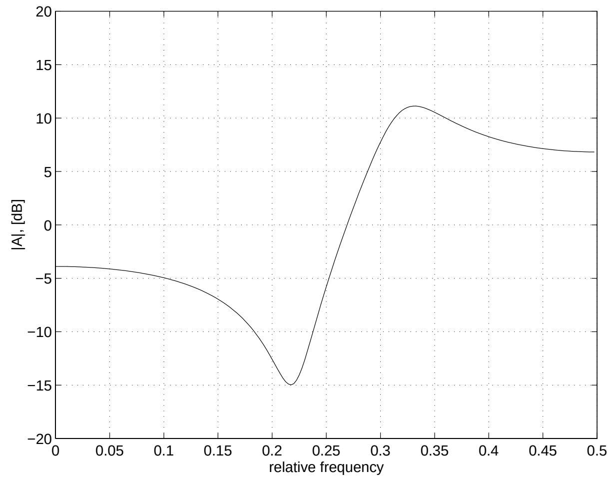

Figure 5: *Magnitude response of $A(z)$ in (14)*

mathematical analysis, where the plant is a simple second-order IIR filter:

$$A(z) = \frac{z^2 - 0.3373z + 0.8100}{z^2 + 0.6627z + 0.6414} \tag{14}$$

Its magnitude response can be seen in Fig. 5. Both algorithms suppose 4 harmonics and a DC component. It means that 9 weights are necessary in the adaptive filter, i.e. 9 resonators are required. The relative fundamental frequency is $f_1 = 0.05$. The adaptive feedforward controller receives a reference signal consisting of all the possible signal components of equal amplitudes. Both systems can cancel the noise, i.e. the microphone signal converges to zero. The algorithms differ from each other in the control speed. For simplicity it is supposed that the generation of the reference signal takes the same time as it is required by AFA to estimate the fundamental frequency. This settling time is much shorter than that of the noise canceling loop. If the fundamental frequency does not change, both controllers are time-invariant systems.The speed of the systems can be characterized by the greatest eigenvalue of the corresponding system matrices as it is written above. Fig. 6 shows the greatest eigenvalues as the function of the convergence parameter α. The upper curve belongs to the XLMS algorithm, while the lower one belongs to the resonator based controller. The minima of the greatest eigenvalues and the corresponding convergence parameters are: $\lambda_1 = 0.9995$ with $\alpha_1 = 0.16$; $\lambda_2 = 0.99$ with $\alpha_2 = 0.30$, respectively. If the initial value of the error signal is one, in worst case it decreases after 1000 steps to about 0.6, $5 \cdot 10^{-5}$, respectively. The greatest eigenvalue of the XLMS based system does not change increasing the adaptive filter length. The resonator based controller is not only faster but have a much wider range in α where the system is stable which is advantageous in practical cases when α can be set only by experiments.

Practical Example. The examined set-up is a simple model of a ventilation duct (Fig. 7). It is a circular pipe with an attached loudspeaker for the simulation of the

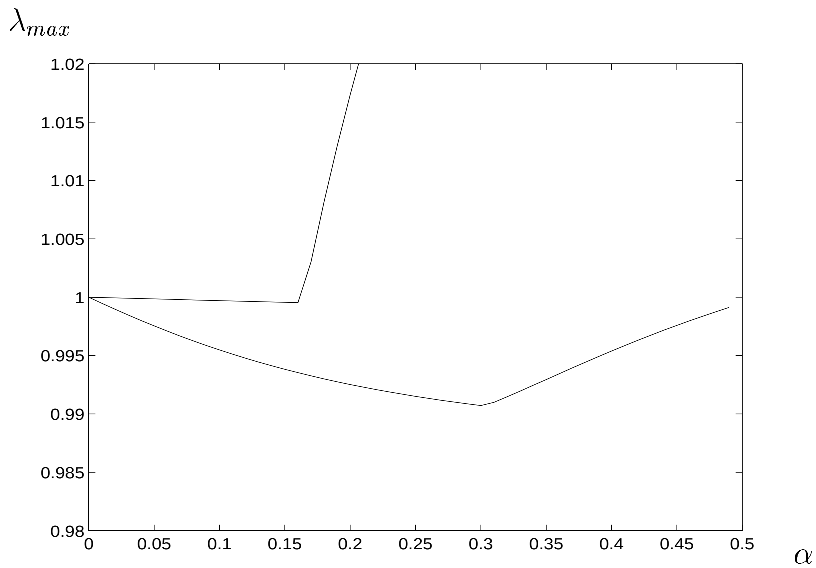

Figure 6: *Eigenvalues of different compensators. Upper curve: adaptive filter updated by the XLMS algorithm; lower curve: resonator based controller*

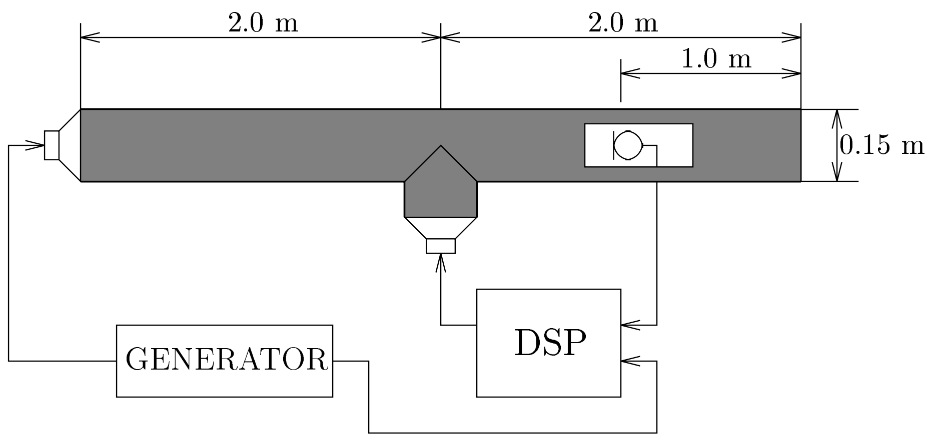

Figure 7: *The experimental set-up*

noise and another one for the secondary source. There is microphone inside for the error signal. The used microphone is quite common, its characteristic is not sensitive to the input sound direction. The noise is generated by the generator and its trigger output was used as reference signal. For measurement and control purposes we have used a MOTOROLA DSP96001 based PC card. It has two analog channels with 16 bit A/D and D/A converters. The sampling frequency was 2 kHz, the acoustic transfer function was measured in 200 points uniformly. For the evaluation of the control results we have used a spectrum analyzer and a digital storage oscilloscope.

In the following example the excitation of the loudspeaker was a triangular signal with a fundamental frequency of 105 Hz. Since the system has linear and harmonic distortion, the periodic noise to be suppressed had different harmonic contents as Fig. 8 shows. According to the fundamental frequency, 8 harmonics were controlled. Fig. 9 shows the spectrum of the error signal, when the control was on. Fig. 10 shows the transient of the noise canceling system.

The system described in [11] is already a multiple channel controller. The tests were made in a small room designed like an airplane-cabin. The program is implemented on TMS320C30 floating-point DSP and it deals with up to 4 loudspeakers and 4 microphones and 2 independent fundamental frequencies, while the sampling rate is 1300 Hz. Stationary periodic noise was suppressed the level of the system noise which means in certain cases more than 50 dB reduction. At an artificial sweep rate of 5 Hz/sec of the disturbing frequency about 20 dB reduction could be measured.

CONCLUSION

In this paper a resonator based noise controller was introduced for cancelation of periodic noise. Design method was described for both single and multiple channel systems. Although the resonator based structure seems to be a feedback controller, it is in fact a special feedforward controller. Due to the built-in signal model, this controller provides better control results, than the conventional methods: it is faster and needs less computations, but keeps all the advantages of the usual methods. Based on theoretical and practical investigations the method can be suggested for periodic noise cancelation.

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Figure 8: *Spectrum of the error signal without control*

Figure 9: *Spectrum of the error signal with control*

Figure 10: *Transient of the error signal*