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Linearization of A/D converters using interpolation of samples

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***Abstract*-** In this paper a novel approach to A/D conversion is introduced. After significant oversampling data points can be selected of which we precisely know the value – these are the samples at transition level crosses. The levels are determined beforehand using histogram testing. To have sufficient number of transition level crosses, dither can be added. Finally, to obtain uniformly sampled data, interpolation is performed. Using this method, the error of AD conversion can be significantly reduced still using the same low-bit converter.
Keywords-analog-digital conversion, linearization, dither, oversampling, interpolation[1]

I. Introduction

Analog-digital converters yield imprecise output due to integral and differential nonlinearities. The distortion due to the finite resolution of the converter is also a nonlinearity, although this is more regular. The smaller are these errors, the better is the ADC. In this paper, an approach to reduce the above errors in analog-digital conversion is described.

By conventional conversion the information available from the output of the converter is that the corresponding analog signal sample was in a given interval at the sampling instant, but nothing more is known. Although this uncertainty cannot be totally eliminated, it can be reduced appreciably by the following method. If the amplitude cannot be determined at a time instant precisely enough, let us determine for a level (comparator) the time instance when this level was crossed, similarly as in [3]. This is called *asynchronous AD conversion*. The method introduced in this paper is aimed to improve a synchronous AD converter, using a similar approach, as described in Section II.

II. Description of the method

II.1. Non-uniform sampling

This paper aims to reduce the nonlinearity of conventional AD converters. Similarly to sigma-delta converters, significant oversampling can be used, only keeping data that were sampled when a code transition level crossing occurred (Fig. 1). The advantage of this approach is that certain points of the input signal are precisely determined:

- because of oversampling, timing of the level crossings is determined with acceptable accuracy,
- amplitudes at these instants are quite precisely known, since these are equal to the transition levels of the A/D converter and can be precisely known from calibration.

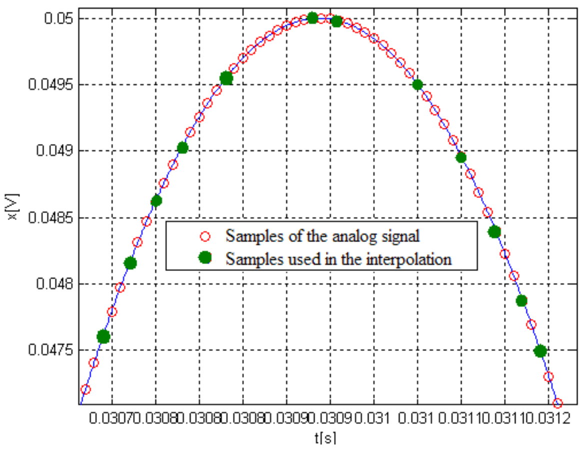

Figure 1 Illustration of non-uniform sampling

[1] This work has been supported by grant TÁMOP - 4.2.2.B-10/1--2010-0009. Special thanks to National Instruments Hungary for providing the ADC hardware.

Since we utilize these precise values instead of using the non-ideal (i.e. non-linear) output of the converter, the nonlinearities of the converter can be compensated.

With this approach, the samples are not equally spaced, because the crossings of the transition levels occur at non-uniform time instants. Therefore not only the digitized data must be saved, but also the time instants of the sampling. Furthermore, computers usually require equally spaced samples, thus *interpolation* has to be used to restore equidistant sampling.

For this, transition levels have to be determined beforehand, using histogram test. The method is described in [1], and an implementation in [2].

II.2. Interpolation

The samples are sampled by a traditional AD converter. Fig. 1 shows that the samples at transition level crosses are building a subset of all the samples which are located on an equally spaced time grid. Using interpolation, samples on this original time grid (or a similar uniform grid) can be regenerated and after that these data can be stored and processed in the conventional way.

The simplest way of interpolation is linear interpolation, but the result of this is too much distorted. To decrease this distortion effect, a more complicated but still executable interpolation method, e.g. Lagrange interpolation can be utilized. This method fits a polynomial of order *(n-1)* to *n* data points. The formula of Lagrange interpolation is as follows:

$$I_{Lagr}(x) = \sum_{j=1}^{n} y_j \prod_{k \neq j} \frac{x - x_k}{x_j - x_k}, \quad (1)$$

where x_j are the data points. Using this method the derivative of the interpolated function is continuous, but the disadvantage is that an increase in the number of data points causes an increased degree of the interpolating function. Furthermore, having noisy samples this interpolation method forces the function through these data points, distorting the result (see Fig. 2), thus in such cases smoothing is desirable.

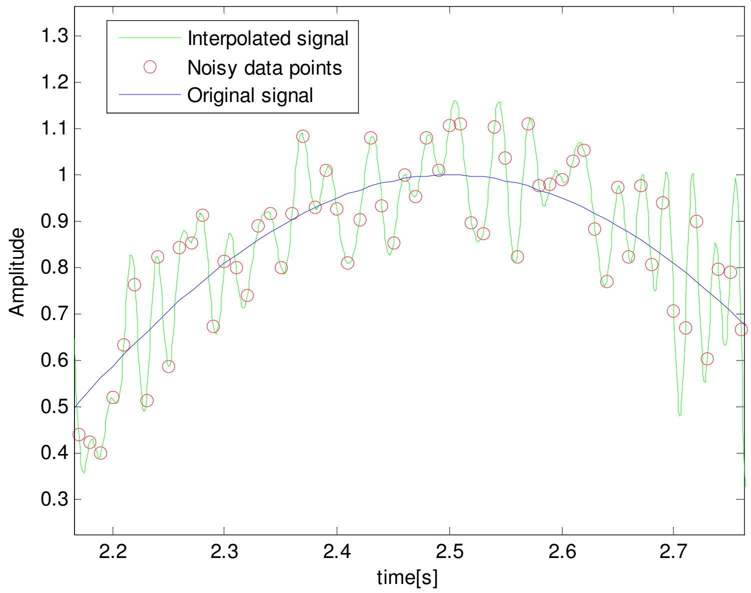

Figure 2 Effect of Lagrange interpolation using noisy samples

Another possibility would be to use splines for interpolation of the samples.

Throughout this paper, noise is assumed to be small.

Although Lagrange interpolation is a global method, it can be executed as a local interpolation using windowing technique. In this case e.g. a 4-point window is used, and after the execution of the local interpolation the middle part is kept, shifting the window forward. The global interpolated function is obtained as the sum of these parts. This method is useful because the computational demand of a 4 degree Lagrange polynomial is much lower than that of the whole sample set.

In the windowed case the derivatives are not continuous because at the joining points the function may have breaks, but the interpolated samples are still good representatives of the original signal.

II.3. Adding dither

The presented method assumes that the input signal has a sufficient number of transition level crosses. When this is not the case (the input is of low frequency or is a DC signal), not enough information can be obtained to execute the interpolation. Furthermore, for these signals the given points might not even satisfy the Shannon theorem. To ensure an adequate number of level crosses, dither can added to the signal. This pseudorandom noise "pushes the signal through" the transition levels.

On the other hand, when the signal is digitized with added dither, significant error would occur. To avoid this, the dither needs to be subtracted at the digital side. A possible method for this is to generate the dither using a D/A converter. In this case we know the value of it quite precisely.

II.4. Choosing appropriate inputs

To test the method, first of all a convenient input signal is to be chosen. The aspect of the choice is to find a signal that has the largest slew rate. This case supplies the worst case situation for the examination because when the signal changes too fast, it may cross more than one transition levels, resulting that the amplitude cannot be accurately determined.

Considering band-limited signals, a sine wave with the frequency of the bandlimit was chosen.

When choosing the type of the dither the aim is to find a relatively slow signal to be able to determine the time instant of the level crossing. The most convenient signal for this purpose is a signal with constant slew rate. A triangular signal can be chosen as dither.

II.5. Error analysis

The purpose of the method is to reduce the MSE (mean-square error) value as much as possible. As reference the PQN-model (Pseudo Quantization Noise model, [6]) can be used, in which the quantization noise is uniformly distributed between $-q/2$ and $q/2$ (q denotes the size of the quantization box). In this model the MSE equals to $\frac{q^2}{12}$.

We should try to create the digital signal with lowest possible MSE. Supposing that the signal had crossed a transition level at t_{cross}, this can be noticed only at the next sampling moment (Fig. 3). Let us denote this with $k_i T_{CLK}$ (in Fig. 3 $k_i = 5$).

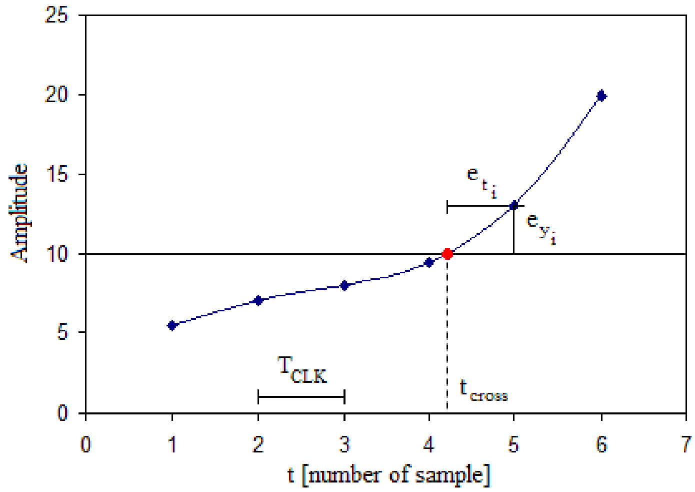

Figure 3 Illustration of the amplitude error, Figure after [3]

In this case the available information is that the signal code level changed in interval $[k_{i-1}T_{CLK}; k_i T_{CLK}]$, thus the timing error is

$$e_{t_i} = k_i T_{CLK} - t_{cross}. \tag{2}$$

If the signal changed during this time, the amplitude could be determined only with an error:

$$e_{y_i} = y(k_i T_{CLK}) - y(t_{cross}). \tag{3}$$

Since the sampling frequency is much higher than the frequency of the signal, the amplitude error can be derived from the current slew rate and the timing error using following equation:

$$e_{y_i} = y'(t_{cross})e_{t_i}. \tag{4}$$

Assuming that the input of the A/D converter is statistically independent of e_{t_i}, furthermore the expected value of the derivative $y'(t_{cross})$ equals to 0, and that e_{t_i} is uniformly distributed between 0 and T_{CLK}, the mean square error can be computed as follows:

$$MSE = E[y^2] = E[y'(t_{LX})^2]\cdot E[e_t^2] \tag{5}$$

Further derivation can be found in [3], and the MSE can be calculated (using sine input and triangular dither) by:

$$MSE = \frac{2}{3}\left(\pi \frac{f_x}{f_{CLK}} A_x\right)^2 + \frac{16}{3}\left(\frac{f_d}{f_{CLK}} A_d\right)^2. \tag{6}$$

This equation shows that MSE decreases with increasing the sampling frequency. On the other hand, the use of dither slightly increases this value. Nevertheless, adding dither is necessary to ensure adequate number of transition level crosses.

To reduce the conversion error, the knowledge can be used that the input signal is significantly oversampled. Assuming that the power density function of the quantization noise is white between 0 and the half of the sampling rate, a lowpass filter can considerably reduce MSE. Besides, this can also reduce the effect of the breaks at the joining points of the interpolated function, caused by the windowing technique described in II. 3.

II.6. Determining the number of level crosses

Assuming that the slew rate of the input signal is known (slew rate of the sum of the dither and the sine wave), the number of transition levels crossed between two sampling instants can be determined. The change is highest when the derivative of the sine is maximal and the sign of it equals to the instantaneous sign of the dither. In this case during a sampling period the following change will occur at the input of the converter (assuming that the sampling frequency is much higher than that of y_{in}):

$$\Delta y_{max} = \left(\frac{\partial y_{in}(t)}{\partial t}\right)_{max} T_{CLK} = 2\pi A_x f_x T_{CLK} + 4A_d f_d T_{CLK} = 2\pi A_x \frac{f_x}{f_{CLK}} + 4A_d \frac{f_d}{f_{CLK}}. \tag{7}$$

Knowing the LSB, the number of level crosses during a sampling period equals to

$$n_{max} = \frac{\Delta y_{max}}{LSB}. \tag{8}$$

Keeping this value low would be advantageous, because the instant of the code level change could be determined more exactly. But having too rare transition changes is also inappropriate because of the insufficient number of data points for the interpolation.

II.7. Determining the parameters of the signal

Considering a 8 bit A/D converter with 4 V_{pp} and with sampling frequency of 200 kHz, LSB = 15.625 mV. After the analysis, the frequencies and amplitudes of the input signal and the dither are to be determined.

Choosing the parameters in a way that the slew rate of the dither is much higher than that of the input signal seems to be an appropriate choice, considering the followings. When the slew rate of the dither is much lower than the maximum slew rate of the sine, the method can work for a sine, but may fail with other low-frequency signals. When the derivatives are of the same order, the slew rate at some sampling points will be high, at others it will be much lower. This is problem, because for signals with the same slew rate the instant of the transition time cannot be determined with the same accuracy (see Fig. 4).

A reasonable setting can be that after maximum 4 samples, at least one code level transition should occur. It follows that n_{max}=0.25. Let the dither have an amplitude of 3.5 LSB (7 LSB peak to peak). In this case

$$n_{max} = \frac{\Delta y_{max}}{LSB} = \frac{4\cdot A_d \cdot \frac{f_d}{f_{CLK}}}{LSB} = 0.25 \text{ and } f_d = \frac{n\cdot LSB}{4\cdot 3.5\cdot LSB}\cdot f_{CLK} = 3571.43\,\text{Hz}. \tag{9}$$

Let us choose the frequency of the dither as 3.6 kHz. Now we can set the values of the sine. Choosing the frequency $f_x = 100$ Hz, and the amplitude $A_x = 150$ mV, the slew rate of the dither is 8 times greater than that of the sine, causing that after maximum 4.51 and minimum 3.54 samples a code transition occurs.

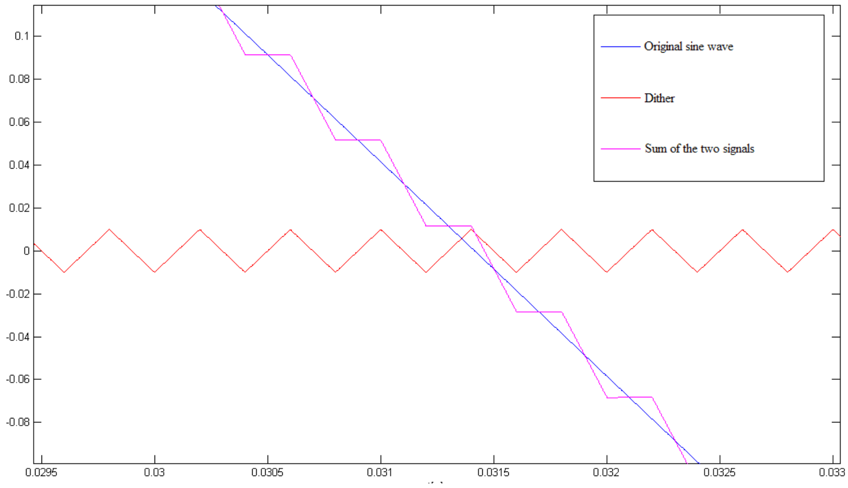

Figure 4 Illustration for sine wave and dither having the same slew rate

It has to be mentioned that setting the parameters this way does not really assure that a code transition occurs after each 4 or 5 samples. When the sign of the dither changes just before the code transition, even 8 sampling intervals can elapse until one transition, but this is rare and does not spoil the method.

III. Verification of the method

III.1. Verification by simulation

To illustrate the more complicated functionality of the method also including using dither, simulation can be set in. In this section the values determined in Subsection II.7. are utilized. With conventional conversion the MSE is $\frac{q^2}{10.59}$, which is higher than the MSE of the PQN model $\left(\frac{q^2}{12}\right)$. With the new approach this value is $\frac{q^2}{44.22}$ which is really close to the theoretical value $\left(\frac{q^2}{46.90}\right)$ given by Eq. (6).

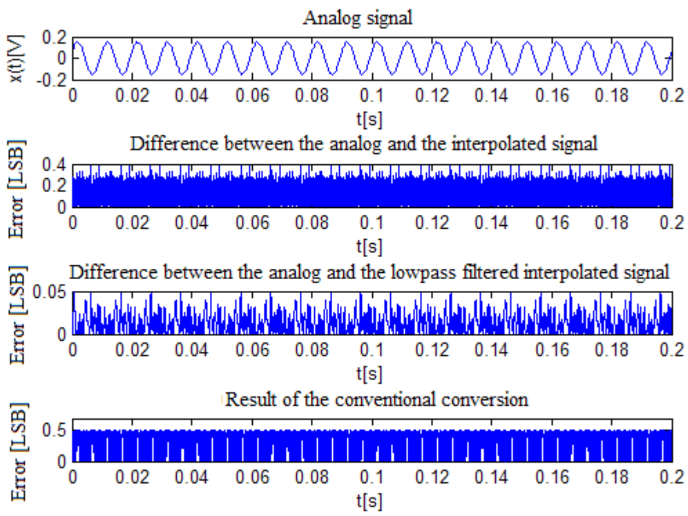

Figure 5 Simulation results

When a lowpass filter is also set in with stopband frequency of 3.5 kHz and a suppression of 80 dB, the MSE falls to $\left(\frac{q^2}{3065}\right)$ which is a major decrease compared to conventional conversion and proves that lowpass filtering have significant reducing effect on conversion error. These data show that the effective number of bits of the A/D has increased. The results of the simulations can be seen in Fig. 5.

III.2. Verification by measurement

Using an A/D converter of National Instruments myDAQ (resolution 16 bit, input range $\pm 2V$, sampling frequency 200 kHz), let us compare how much error is made during the conversion using the conventional method and by the novel approach.

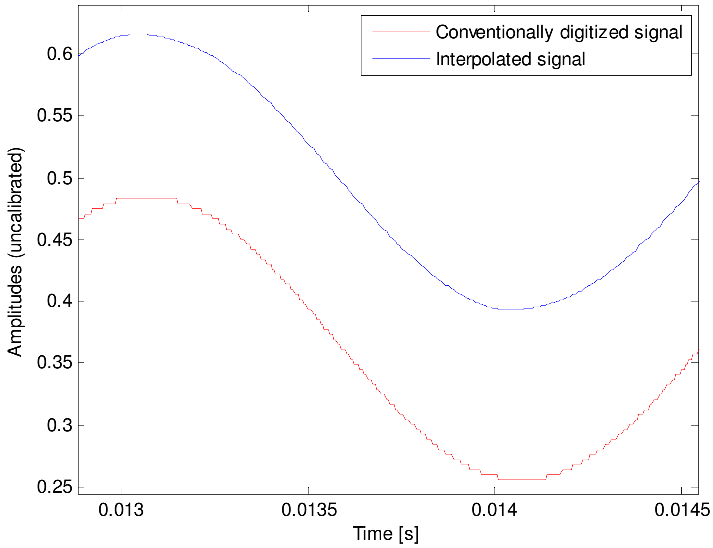

Figure 6 Illustration by measurement

The calibration of the A/D converter was not a part of this work, so the transition levels were determined relatively to each other [2]. To keep the measurement simple, no dither was added because it is only needed to ensure transition level crosses for very low frequency signals. As input, sine wave was used with parameters 1 V_{pp} and 500 Hz. To keep n_{max} in Eq. (8) low, only the higher 8 bits of the A/D were considered. Results can be seen in Fig. 6. Since the amplitudes and offsets of the two signals were different, these parameters of the conventionally digitized signal were normalized to illustrate that the interpolated signal follows the required form. The FFTs of the two signals are shown in Fig. 7. The two figures illustrate clearly that the interpolated signal follows the sinusoidal form much better. The high-frequency FFT components of the interpolated signal are about 6 dB lower.

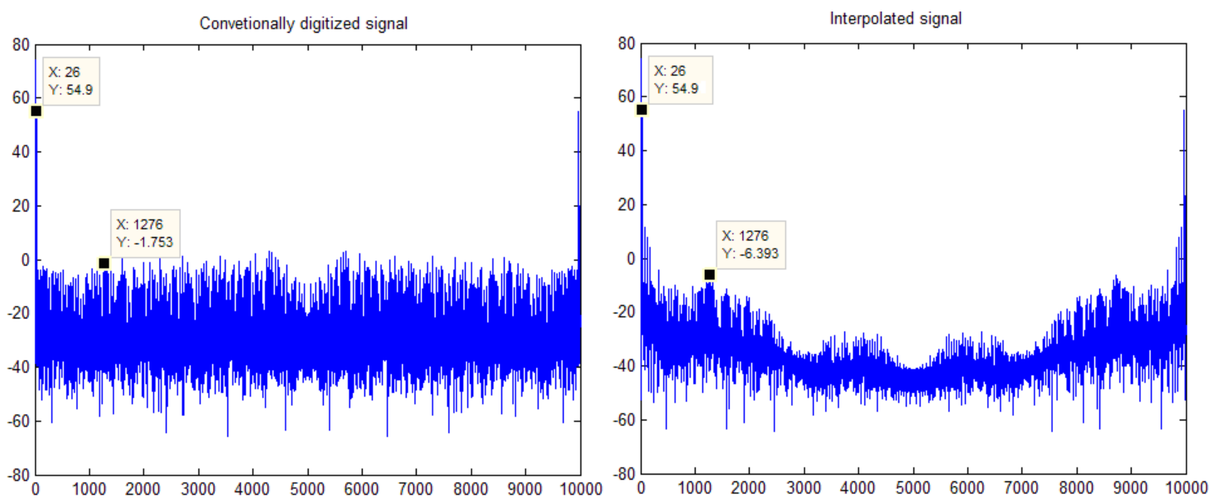

Figure 7 FFT of the conventionally digitized and the interpolated signal

Knowing that we have sine waves, LS fit can be executed. For the conventionally digitized signal $MSE_{conv.} = 0.0826\ LSB^2$, while for the interpolated signal $MSE_{interp} = 0.0403\ LSB^2$, and the effective number of bits $EOB_{conv.} = 8.006$ and $EOB_{interp.} = 8.524$. The results show that the resolution of the A/D was improved. It can be noticed that using Eq. (6) theoretical $MSE = 0.0421\ LSB^2$ which is close to the measured $MSE_{interp.}$. With higher clock frequency or lower signal slew rate the resolution could be even more enhanced.

IV. Conclusions

The aim of this paper was to prove that the nonlinearities of an A/D converter can be reduced by oversampling and interpolation. Instead of using each sample only data points are kept which were taken at transition crossings, because in this case the value is much more precisely known than for a randomly selected data point. After data processing (interpolation) the original signal can be restored with much higher accuracy than by conventional methods. In this way not only non-linearity but also the resolution of existing A/D converters can be improved (improved resolution by oversampling). The improvement of the resolution is much more than by usual averaging.

V. References

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