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Acceleration of ADC test with sine wave fit

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Abstract— **Sine wave fitting is usually done with least squares minimization in the time domain. This can be slow when the number of samples is large (10^5-10^6 or more). It is shown that the fit can be done more effectively in the frequency domain, using the Fourier transform of windowed data. This paper shows that using a Blackman-Harris window, it is enough to process just a few samples around the sine peak. To obtain accurate results in ADC characterization the input signal has to meet strict conditions: coherent sampling, uniform distribution of phases. It will be shown that the precision of the estimator is enough to determine if the signal meets the two conditions above and sometimes it provides even better results than the original time domain least squares estimator.**

Keywords— Analog-to-digital conversion; histogram test; sine fit; sine frequency estimation;

I. INTRODUCTION

Possibly accurate characterization of analog-to-digital converters (ADC) is important in practice. A widely used method for characterizing ADC's is the so-called histogram test [1]. In this method first the ADC is excited with a predefined input (e.g. sine wave), then the integral nonlinearity (INL) and differential nonlinearity (DNL) can be identified from the histogram (the number of samples in each of the code bins). Obviously, the parameters of the applied sine wave and the sampling frequency are important for proper calculations. The IEEE standard for ADC testing [2] defines that sampling should be coherent (a number of whole periods need to be measured) and the number of periods has to be relative prime to the number of samples, in order to achieve the most accurate results (uniform distribution of phase values). However, the satisfaction of these conditions can only be checked from the measured signal, since both the frequencies of the signal generator and of the sampling have limited precision. With the increase of the number of samples to obtain more precise measurement of the characteristics of the device under test, even a small deviation from coherence in the sampling may cause significant errors in the characterization with false INL and DNL values, and also causes a significant increase in the time required to execute the least-squares four parameter sine wave fit algorithm proposed by the IEEE standard. In this paper an increased-speed algorithm will be presented with the extra capability to decide from the samples if the test signal is suitable to test the ADC or not, this is presented in Section III. In Section IV this estimator is compared to the time domain least squares estimator proposed by the IEEE standard, both from the point of view of statistical properties and speed. Finally, in Section V it is shown that the precision of the estimator is enough to decide the suitability of the measured signal for the histogram test.

II. BACKGROUND AND NOTATION

A. Standards for choosing signal and sampling frequencies

In our histogram test the ADC is excited with a sine wave input. The form of the input can be represented in the so-called trigonometric form:

$$x(t) = R \cdot \cos(2\pi f t + \phi) + C, \quad (1)$$

where R, f, φ and C are the amplitude, frequency, initial phase and dc component of the signal, respectively. Let $H[i]$ be the total number of samples received in the code bin i, and the cumulative histogram values be

$$H_c[j] = \sum_{i=0}^{j} H[i]. \quad (2)$$

If the amplitude and the dc offset of the sine wave are known, the nth transition level can be estimated as

$$\hat{T}[n] = C - R \cdot \cos\left(\pi \frac{H_c[n-1]}{N}\right). \quad (3)$$

For more details about the histogram test, see [1], [2].

B. Standards for choosing signal and sampling frequencies

The sine wave input is optimal if the sampling is coherent, so a number of full periods are sampled, and all the samples represent different phases. If the sampling frequency is f_s, the number of samples is N, than the signal frequency f_x is optimal if

$$f_x = \left(\frac{J}{N}\right) \cdot f_s, \quad (4)$$

where J is an integer relatively prime to N. In this case the sampling is coherent and the phase difference between arbitrary two adjacent samples is $2\pi/N$, which is the ideal phase distance. In addition, this also means that the phases of the samples are uniformly distributed between $-\pi$ and π. However, such a condition can never be perfectly met since f_x and f_s can never be set (or even known) exactly. To take the possible error in the frequency ratio into account the following model is used [1]:

$$\frac{f_x}{f_s} = \frac{J}{N} + \Delta\rho, \quad (5)$$

where Δρ is the relative frequency deviation. In [1] it is shown that none of the sampling points deviates from their ideal phase position by more than ±1/4 part of the ideal phase distance if

$$\frac{N}{J} \cdot |\Delta\rho| < \frac{1}{4 \cdot N \cdot (J-1)}, \; J > 0. \quad (6)$$

In [3], Carbone and Chiorboli found this too conservative, and showed that

$$\frac{N}{J} \cdot |\Delta\rho| < \frac{1}{2 \cdot N \cdot J} \quad (7)$$

is enough to keep the distribution of the samples close to uniform.

Based on [1], [3], the effect of Δρ on the variance of the transition levels can also be determined. In [3] the relation between Δρ and $\mathrm{Var}\{H_c[k]\}$ is defined where the latter is the variance of the kth element of the cumulative histogram. If this is known, the variance of the kth transition level can be written as [1]

$$\mathrm{Var}\{\hat{T}_k\} \cong Var\{H_c[k]\}\left(\frac{A\pi}{N}\sin\psi_k\right)^2, \qquad (8)$$

where

$$\psi_k = \frac{H_c[k]}{N}\pi. \qquad (9)$$

Based on the previous formulas, if the error (e.g. worst case error) of a frequency estimator is known (the satisfaction of the above two conditions depends on the sine wave's relative frequency to the sampling frequency), one can determine its effect on the result of the histogram test. Later we will use the formulas to answer if the precision of the sine wave estimator is enough to ensure accurate results for the histogram test or not.

Above conditions show the importance of precise frequency estimation to guarantee accurate results in ADC characterization using the histogram test.

The standard also recommends that the measured signal should contain at least five periods (to ensure the convergence of the sine fitting algorithm).

C. Estimation of sine parameters

The previous chapter shows the way how to ensure the best test results. Condition (7) needs to be checked which requires accurate knowledge of the frequency of the input sine wave. Generally, its value is unknown with the required accuracy, so it has to be estimated. The proposed method in the standard terminology for ADC testing is the four parameters least squares fit on the measured data, which is an iterative Newton-Gauss algorithm since the model of the input is nonlinear in the frequency. The properties of this algorithm have been analyzed in details in several earlier papers.

In [4] Deyst studied the effect of harmonic distortion and noise on the frequency estimation of the sine wave using the four parameters least squares fit. It was shown that if the signal contains a harmonic component the maximum error on the estimation of the number of periods in the measured data is bounded by

$$\max|\Delta p| = \frac{0.9}{p\cdot h^{1.2}}\cdot\frac{A_h}{A_f}, \qquad (10)$$

for $p \geq 2$, where Δp, p, h, A_h, A_f are the error of the estimation of the number of periods in the signal, the number of periods in the signal, the order of the harmonic distortion, the amplitude of the harmonic component and the amplitude of the sine wave, respectively. It was also shown (with measurements) that for sine waves corrupted by noise the variance of the frequency estimator normalized by the variance of the noise is the function of the number of periods and the fundamental phase.

Handel in [5] studied the error of the frequency estimator of the four parameters sine fit algorithm when a noisy sine wave is quantized and found that the variance of the frequency estimation highly increases for signals which contain a small number of periods, and recommends a one-dimensional grid search for the frequency instead of the nonlinear fit algorithm.

Due to the limitations of the least squares fit for a small number of periods (e.g. less than one) a few other algorithms were developed. In [6] Max, in [7] Fonseca, Ramos and Serra present a new algorithm which can handle the situation of very small number of samples.

A further factor on which the precision of the frequency estimator depends is the starting value of the frequency. Generally the initial value of the frequency is estimated with interpolated FFT (IpFFT). In [21] Giaquinto and Trotta shows a method where the IpFFT is done on the DFT of the Hanning-windowed input sine wave, using the highest peak and its two neighbours. Bilau, Megyeri, Sárhegyi, Márkus and Kollár in [8] suggest a different method (based on [13]) which provides the analytic solution for the rectangular window. The main advantage of the IpFFT methods is that they provide a better estimate than counting zero crossings in the sine wave or using the highest point of the DFT for initial guess, thus the quality of the estimator is increased.

The presence of nonlinear distortions, timebase errors and pathological bins (overdrive) are also likely to have a harmful effect on the estimation of the parameters of the sine wave. For methods which can handle such problems, see [9], [10].

The larger the number of samples is, the better the test results of the histogram test are. However, the computational burden for the four-parameter sine wave fit strongly depends on the number of samples since the whole sample record is used in every iteration of the Newton-Gauss algorithm to obtain a better estimation of the parameters. Alternative methods for frequency estimation of the sine wave were developed which can speed up the estimation process, such as [22] where Hejn, Morling introduces a method based on the Householder orthogonalization, [11] where Chen and Xue suggest Gram-Schmidt orthogonalization to speed up the solving of the matrix equations, or [12] where Zhang, Xinmin, Xiao and Jinwei present a total least squares method which can identify the parameters of the sine wave much faster than the original least squares algorithm.

Previous methods used the time-domain samples of the sine wave. In [20] Holm recommends an algorithm which fits Gaussian function to the peak in the DFT and its two neighbors, and shows that this method provides an estimator which is approximately Maximum Likelihood. In the next chapter an alternative, also frequency domain method will be introduced which provides the parameters of the sine wave much faster than the original least squares fit but with similar (sometimes increased) precision.

III. Four-Parameter sine wave estimation in the frequency domain

A. *Implementation*

Since the original four parameter least squares method in the time domain (TDLS) can be very time consuming for large number of samples, the goal was to create an estimator which identifies the parameters of the sine wave with less computational burden. For this purpose, the frequency domain form of the sine wave is used instead of the time domain form during the estimation. The discrete Fourier transform (DFT) of an N length sine has a -13 dB side lobe level because it is the convolution result of the DFT of a rectangular window and of the DFT of an infinitely long sine wave. This is bad since then most samples are needed for a proper fit. To compress the data in the frequency domain into a few points the 3-term Blackman-Harris window is used which has a side lobe level of 71.5 dB ([15], [16]). Then, the very small side lobe samples need not to be fitted, only the main lobe samples. The time domain expression of the window function is the following based on [14], [15]:

$$w(k) = a_0 + a_1 \cdot \cos\left(\frac{2\pi k}{N}\right) + a_2 \cdot \cos\left(\frac{4\pi k}{N}\right), \qquad (11)$$

where a_0, a_1, a_2 are the coefficients of the 3-term window and their exact values are [15]:

$$a_0 = +0.4243801, a_1 = -0.4973406, a_2 = +0.0782793. \qquad (12)$$

To use the formula of a sine wave windowed with the 3-term Blackman-Harris window first the expression of the non-windowed N length sine wave is needed in the frequency domain:

$$X_{sin}(k) = X_{sin}^{-}(k) + X_{sin}^{+}(k), \qquad (13)$$

$$X_{sin}^{-}(k) = e^{-j\pi(k-f)\frac{N-1}{N}} \cdot \frac{A+jB}{2} \cdot \frac{\sin(\pi(k-f))}{\sin\left(\pi(k-f)\frac{1}{N}\right)}, \qquad (14)$$

$$X_{sin}^{+}(k) = e^{-j\pi(k+f)\frac{N-1}{N}} \cdot \frac{A-jB}{2} \cdot \frac{\sin(\pi(k+f))}{\sin\left(\pi(k+f)\frac{1}{N}\right)}, \qquad (15)$$

where

$$A = R \cdot \cos(\varphi)\,, B = R \cdot \sin(\varphi), \qquad (16)$$

and the expression of the sine wave in the time domain is

$$x(t) = R \cdot \cos\left(\frac{2\pi f t}{N} + \varphi\right), \qquad (17)$$

and *R*, *f*, ϕ is the amplitude, frequency and initial phase of the sine wave, respectively. During the algorithm the following formulas are used to avoid large arguments in the input of the trigonometric functions:

$$f = \lfloor f \rfloor + \langle f \rangle, f_i = \lfloor f \rfloor, f_f = \langle f \rangle, \qquad (18)$$

$$X_{sin}^{-} = e^{j\pi\left(f_f + \frac{k-f}{N}\right)} \cdot \frac{A+jB}{2} \cdot \frac{\sin(-f_f\pi)}{\sin\left(\frac{k-f}{N}\pi\right)}, \qquad (19)$$

$$X_{sin}^{+} = e^{j\pi\left(-f_f + \frac{k+f}{N}\right)} \cdot \frac{A-jB}{2} \cdot \frac{\sin(f_f\pi)}{\sin\left(\frac{k+f}{N}\pi\right)}, \qquad (20)$$

where f_i is the integer part and f_f is the fractional part of *f*. One should note that the last terms of the expressions (19) and (20) which has the general form

$$\frac{\sin(N\omega)}{\sin(\omega)}, \qquad (21)$$

may cause serious roundoff errors when $\omega \rightarrow 0$ and results in NaN (not a number) instead of *N* when ω=0. So in the implementation the fraction was replaced by a 6th order polynomial to avoid such errors.

If the measured signal contains also a DC component in addition to the sine wave, its mathematical model becomes the following:

$$x(t) = R \cdot \cos\left(\frac{2\pi f t}{N} + \varphi\right) + C, \qquad (22)$$

where *C* is the level of the DC component. This means that an X_{dc} part has to be added to the X_{sin} part in the frequency domain to represent this signal correctly:

$$X(k) = DFT\left\{R \cdot \cos\left(\frac{2\pi f t}{N} + \varphi\right) + C\right\}, \qquad (23)$$

$$X(k) = X_{dc}(k) + X_{sin}(k) \qquad (24)$$

where $X_{dc}(k) = C \cdot N$ for $k = 0 \pm N \pm 2 \cdot N \pm 3 \cdot N \pm \cdots$, and $X_{dc}(k) = 0$ otherwise. If the signal $x(t)$ is windowed with the samples of the 3-term Blackman-Harris window $w(t)$ (11), the multiplication in the time domain becomes a convolution in the frequency domain, so the kth sample of the sine wave in the frequency domain is

$$X_{BH}(k) = DFT\{x(t) \cdot w(t)\} = \underline{a}^T \cdot \underline{y}, \qquad (25)$$

$$\underline{a} = \frac{1}{2}[a_2\ a_1\ 2a_0\ a_1\ a_2]^T, \qquad (26)$$

$$\underline{y}^T = [X(k-2), X(k-1), X(k), X(k+1), X(k+2)]. \qquad (27)$$

The (22), (23) expressions above are linear in the amplitude, initial phase (more precisely in the terms A, B related to R and the initial phase) and the DC component, and nonlinear in the frequency. This means that the estimation has to be done using an iterative numerical method. The Newton-Gauss method was used to perform this operation on the cost function

$$K = \frac{1}{2}\underline{e}^T\underline{e}, \qquad (28)$$

where $\underline{e}$ is the vector of residuals. The definition of the Newton-Gauss step is

$$\Delta\underline{p} = (\boldsymbol{J}^T\boldsymbol{C}^{-1}\boldsymbol{J})\boldsymbol{J}^T\boldsymbol{\Sigma}^{-1}\underline{e}, \qquad (29)$$

where $\underline{p}$, $\boldsymbol{J}$, $\boldsymbol{\Sigma}$ is the vector of parameters, the Jacobian matrix (see Appendix A) and the covariance matrix, respectively.

Since the (24) model is nonlinear in parameter f, at the beginning of the Newton-Gauss algorithm it is important use an appropriate initial guess for the frequency, otherwise the minimization of the cost function may got stuck in a local stationary point. For this purpose, IpFFT (the analytic solution for the rectangular window) is used (described in [8],[13]).

The increase of speed is achieved as follows. The Fourier transform of a continuous sine wave with a DC component consists of Dirac delta functions at the sine frequency (-f and f) and at the DC frequency. If the continuous signal is sampled and windowed with the 3-term Blackman-Harris window, its DFT becomes periodic with f_s and the Dirac delta functions are convolved with the DFT of the window function. Due to the 71.5 dB side lobe level, the information about the sine wave is compressed into a few points in the frequency domain (if no window function was used, the Dirac delta functions would be convolved with the discrete sinc function which has a side lobe of 13 dB, thus it is much wider). When the FFT is performed on the windowed samples, a few points are collected from the result around the DC and the sine wave's frequency (the width of the 3-term Blackman-Harris window is 5 bins in the frequency domain). If the frequency of the sine wave is closest to the kth bin, then during the fit the following 8 samples are used:

$$[X(0), X(1), X(2), X(k-2) \ldots X(k+2)] \qquad (30)$$

where $X(0)$, $X(1)$, $X(2)$ represent the DC part of the input signal and $X(k\text{-}2)\ldots X(k+2)$ are the points of the sine wave. The use of the points around $N\text{-}k$ (where the second peak of the sine wave appears) is unnecessary since these are the complex conjugates of the points around k, thus they do not provide new information about the parameters of the input sine wave[1] (and $X(\text{-}1)$ and $X(\text{-}2)$ are the complex conjugates of $X(1)$ and $X(2)$, so the DC part can be represented with 3 bins instead of 5). The number of the used samples is constant and independent from the length of the measured record. These points represent the sine wave and contain enough information to identify the parameters of the input. The original TDLS estimator uses every sample of the sine in every iteration of the Newton-Gauss algorithm. In our alternative method, only the time required to perform the FFT depends on the length of the input. As it will be shown later, from the computational burden's point of view, it is worth to perform the FFT and then do the fit on a few samples instead of using every time domain sample during the calculation of the parameters.

B. *Properties*

In this subsection the statistical properties of the estimator will be discussed from a theoretical approach. Since there are many possible sources of noise on the measured signal (random noise, harmonic distortion, quantization, etc.) the distribution of the noise on the frequency domain samples is hard to determine. For this reason, we use Jennrich's theorem [19] on nonlinear least squares estimators which states that these are asymptotically normally distributed. However, the mean value and variance of each normally distributed estimator ($\hat{f}, \hat{A}, \hat{B}, \hat{C}$) has to be determined experimentally, this is done in the next section.

IV. Comparison of the New and the Original Algorithm

A. *Comparison of precision and accuracy*

In this subsection the statistical properties of the estimation errors of all the algorithms are compared. We also want to justify why the 3-term Blackman-Harris window is used in the estimation algorithm instead of Blackman-Harris windows with more terms. For this purpose, algorithms with 3-term, 4-term and 5-term Blackman-Harris windows were compared to the TDLS estimator. In this section, based on [18], we use the term accuracy as the quantification of the bias of the estimator, and similarly, precision is the quantification of the standard deviation of the estimator.

In the following tests all the estimators were executed on the same input record, and then the error of the frequency estimation was analyzed. Although both estimators provide every parameter about the sine wave, we study only the frequency because our goal is to decide from the measurement if the sampling is coherent and the phases are distributed uniformly (properties of the other estimated parameters are studied in [17]).

To simulate real-like circumstances in the examinations, a nonideal 14 bit quantizer was used to digitize the input record. Fig. 1 shows the integral and differential nonlinearity of the converter. To model the imperfection of the signal generator, independent Gaussian noise was added to the samples with zero mean and $LSB/6$ standard deviation (this assures the additive random noise is less than a half LSB with 99.7% probability). A harmonic component of the input was also added with the double value of the signal frequency and $LSB/2$ amplitude, so this way the input signal contained a noise which was only seldom larger than LSB. The noisy input was quantized with the nonideal converter and then the estimation algorithms were executed to identify the parameters. In the tests we took into account that every estimator's behavior change on various frequencies and this was also analyzed. The

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[1] If the roundoff errors are not negligible compared to other error sources, it is worth to include the complex conjugateg into the measurement points.

amplitude and the dc offset of the input signal were constants with *R*=*FS* where *FS* is the full scale range of the AD converter, and *C*=0 (see (1)).

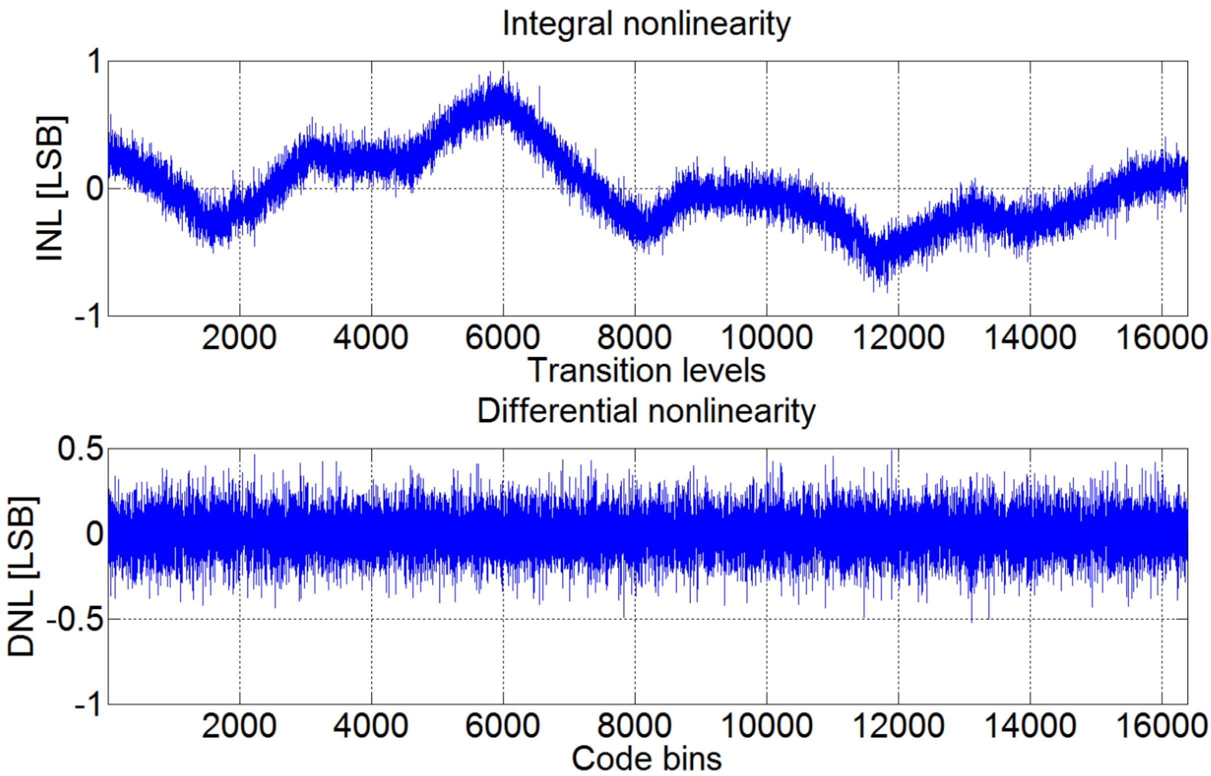

Figure 1. INL and DNL characteristics of the 14 bit quantizer

The initial phase and the frequency were random variables, where φ was uniformly distributed between –π and π. In the following tables the frequency of the input signal is given by the mean value of the number of periods it contained. Generally, if the number of periods is given as *M*, it means that the number of periods in the test case was a uniformly distributed random variable in the [*M*-0.05, *M*+0.05] interval. This is modeling that the user who want to perform a histogram test is trying to meet the conditions defined by the IEEE standard but due to the error in the sampling frequency and signal frequency he will not meet them exactly (but will be close). The initial phase of the harmonic component is also a uniformly distributed random variable between –π and π, and the record length in every case is $N=2^{18}$. On every frequency (every given *M* value) 750 tests were run. The minimum number of periods studied is 5 (based on the standard), the greatest is 78125 (this means very few, only 3.35 samples in a period). One should note that the frequency estimator estimates the value of $N \cdot f_x/f_s$, so it is a value without dimension (practically the number of periods the signal contained).

TABLE I. PRECISION OF ESTIMATORS

	3-term BH window	4-term BH window	5-term BH window	Time-domain LS
M	σ_3 (×10^{-8})	σ_4 (×10^{-8})	σ_5 (×10^{-8})	σ_t (×10^{-8})
5	13.2	288.6	250.5	282.3
6	9.53	38.7	363.9	241.3
7	9.75	7.52	166.3	197.4
8	9.63	8.27	12.6	171.1
9	10.38	9.49	6.70	158.1
10	9.58	9.12	7.59	142.7
11	10.15	9.85	9.07	129.8
12	10.11	9.78	9.46	120.07
13	9.79	9.48	9.26	110.35
14	9.96	9.46	9.38	100.64
15	9.69	9.47	9.25	93.8
25	10.05	9.91	9.49	57.05
75	10.00	9.57	9.41	19.09
125	10.58	10.20	9.97	12.94
625	10.14	9.79	9.61	6.50
3125	10.29	9.97	9.68	6.47
15625	11.11	10.61	10.21	6.56
78125	10.01	9.73	9.56	6.37

TABLE II. ACCURACY OF ESTIMATORS

	3-term BH window	4-term BH window	5-term BH window	Time-domain LS
M	μ_3 (×10^{-8})	μ_4 (×10^{-8})	μ_5 (×10^{-8})	μ_t (×10^{-8})
5	2.29	-64.63	-59.72	97.26
6	-3.11	-8.40	42.11	108.44
7	-0.34	3.00	-20.95	91.91
8	7.07	3.45	-0.86	158.03
9	1.20	1.02	0.32	58.38
10	-9.15	-5.42	-2.31	16.19
11	-0.07	-0.11	1.47	3.97
12	3.36	2.96	2.90	19.63
13	-4.14	-1.73	-2.73	-9.56
14	5.18	5.37	4.04	-5.72
15	-0.42	2.29	1.81	8.93
25	-0.74	-3.51	-3.15	33.33
75	0.16	1.42	1.81	12.48
125	-2.27	-0.68	-1.17	3.61
625	-0.35	2.01	1.89	-2.81
3125	-0.11	1.83	-0.05	2.68
15625	-6.14	-4.41	-5.53	-0.52
78125	-0.97	0.95	0.93	0.86

First we compared the precision of the estimators. Table II. shows the standard deviation of each on different frequencies. In the case of the 3-term window, the standard deviation oscillates around $10 \cdot 10^{-8}$ with small amplitude, so it gives balanced performance on every studied frequency. The estimators with 4-term and 5-term windows reach their "stationary value" when the number of measured periods become greater than seven. We can see that $\sigma_3 > \sigma_4 > \sigma_5$ when $M>6$, where σ_3, σ_4, σ_5 are the standard deviations of the estimation errors of the frequency domain least squares (FDLS) estimators with 3-term, 4-term and 5-term Blackman-Harris window, respectively. The explanation for the less variance is that the 4-term and 5-term windows are wider (in the 4-term case the fit is done on 4+7 points (bins around the DC and the sine, see (12)) and in the 5-term case 5+9 points are used), so more measurement points provide more information. The original, TDLS estimator provides high standard deviation (σ_t) on low frequencies. As the frequency increases σ_t decreases and when it reaches its "stationary value", it provides the estimation with the best precision. Behave of σ_4 and σ_5 are similar, on small number of periods their value are both higher than σ_3. To understand this behavior of the estimators we need the frequency domain approach. In the frequency domain, the 3-term Blackman-Harris window is the narrowest (5 points), and the DFT of the “unwindowed” (actually it is the DFT of the sampled rectangular window, the discrete sinc function) is the widest. These windows appear also on the frequency of the harmonic component, and on low frequencies the window of the harmonic component disturbs the window of the sine wave, it can be considered as a noise on the points of the window of the sine, and this results in estimation with higher variance. As the frequency increases and the signal contains more and more periods, the distance between the window on the signal’s frequency and the window on the frequency of the harmonic component increases, and they will not contain common bins in the frequency domain. The discrete sinc is the widest window, so its variance will decrease at the very latest. The value of σ_4 and σ_5 decreases later then the value of σ_3 because the 4-term and 5-term Blackman-Harris windows are wider than the 3-term. Table II. also shows that the 3-term window estimator can estimate the frequency with the least variance when the input signal contains 5 periods, the minimum number recommended by the standard.

Next, the accuracy of the frequency estimators is studied. We describe this with the mean value of the estimation error (μ). Table III. shows the results on different frequencies. The FDLS estimator which uses the 3-term Blackman-Harris window provides balanced results again. The value of μ_4 and μ_5 decreases when the number of periods is greater than seven. However, μ_t decreases only on greater number of periods when the effect of the overlap of the rectangular windows on the sine and on the harmonic component decrease. These results also confirm that on small frequencies it is worth to use the FDLS estimator.

So, the main reason for choosing the 3-term window for the fit algorithm is that it provides balanced performance equally on high and low frequencies. When its standard deviation and mean value are worse compared to other estimators, the difference is not significant (naturally, from the aspect of our goal). However, it is an interesting result that the original time domain estimator provides better results only when the number of periods is greater than 125.

The behavior of the frequency estimator of the TDLS method coincides with the results published in [4], [5]. Namely, in [4] Deyst with the (10) formula predicts that as the number of periods in the measurement increases (so distance between the windows on the frequency of the sine and the frequency of the harmonic component increases), the maximum error of the frequency estimator decreases. Furthermore, Handel in [5] showed that the standard deviation of the frequency estimator is higher for less number of periods.

B. Comparison of speed

In this subsection the required total computational times for the FDLS estimator with the 3-term Blackman-Harris window (time requirement of the FFT is included) and the time domain estimator are compared. Both algorithms were executed on a computer with Intel® Core™ i3 530 2.93 GHz processor and 4 GB system memory. The algorithms were implemented in Matlab. Fig. 2 shows the implementation of each algorithm.

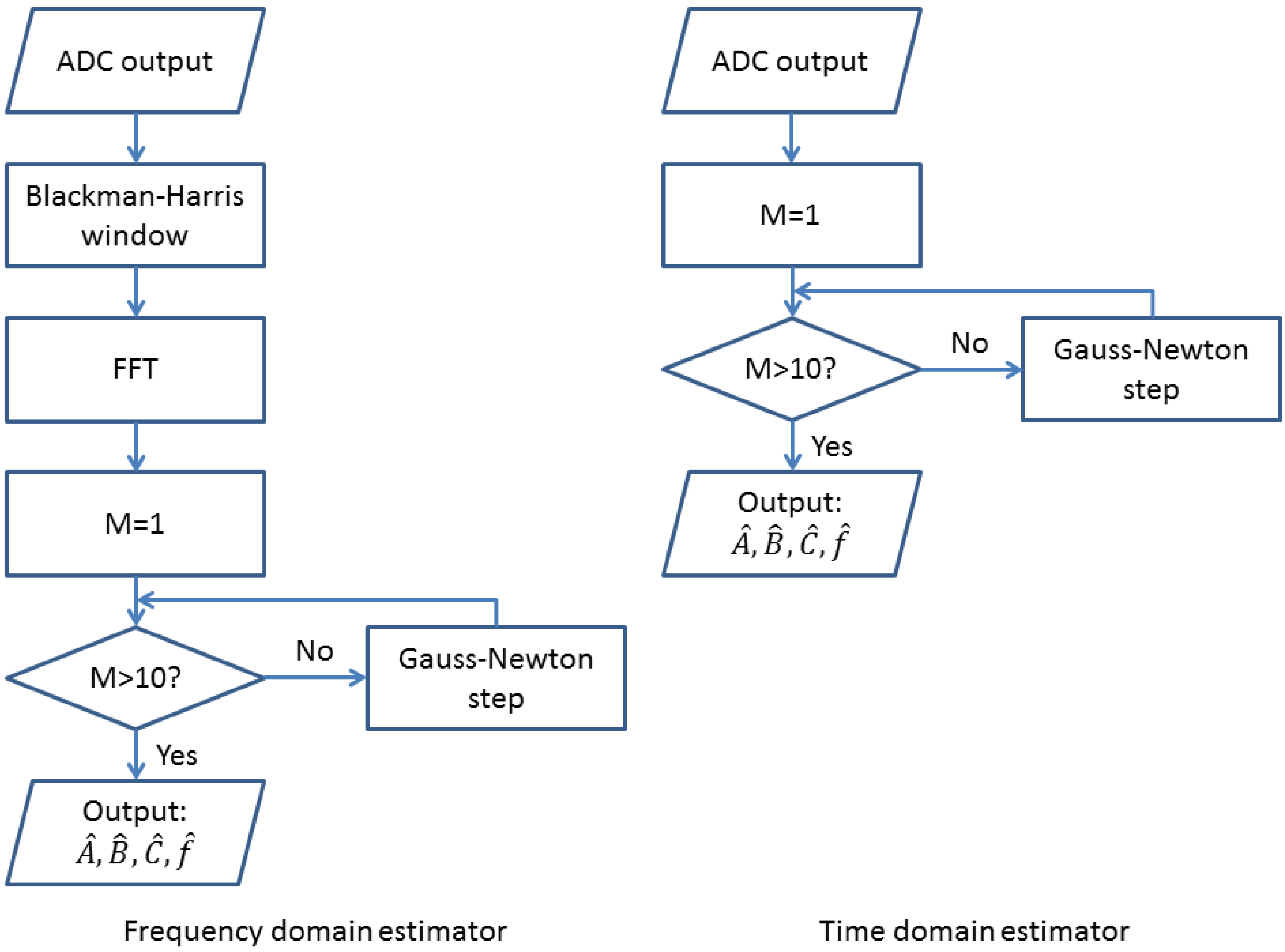

Figure 2. Algorithm of the frequency domain and the time domain estimator

The number of iterations was chosen to 10, this was enough to ensure convergence in the studied cases. Table IV shows the required computation times.

TABLE III. COMPARISON OF REQUIRED COMPUTATIONAL TIME

N	Original method	Frequency domain method
2^{14}	0.051 s	0.025 s
2^{16}	0.192 s	0.032 s
2^{18}	0.778 s	0.063 s
2^{20}	2.954 s	0.206 s
2^{22}	46.131 s	1.078 s

The reason for the significant increase of speed is that after performing the FFT the fit is done using 3+5 samples in the iterations of the Newton-Gauss algorithm instead of the original algorithm which uses every sample. This can be done since the 3-term Blackman-Harris window compresses the information about the sine wave and the dc component into two narrow bands in the frequency domain. After determining approximately the frequency of the sine wave and choosing the adequate 3+5 points to perform the fit the other points of the FFT result are neglected.

C. Convergence

In this subsection the convergence of the FDLS estimator is studied in the case of frequency estimation. As it was mentioned in Section III, the initial guess for the frequency is determined with interpolated FFT. The analytic solution for the rectangular window is used (see [10], [13]) as initial guess, using the highest peak and its neighbor of the FFT of the unwindowed sine wave. Fig. 3 shows the logarithm of the absolute value of the frequency estimator's step size during the iterations.

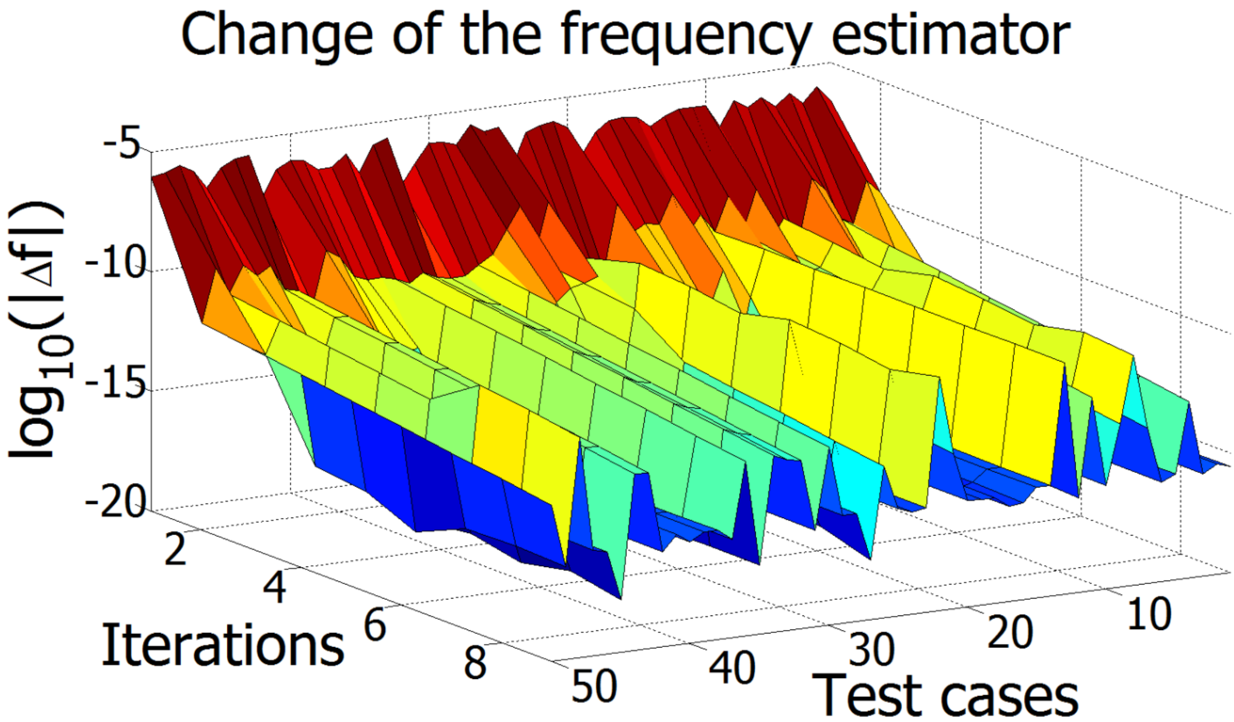

Figure 3. Change of the frequency estimator during the Gauss-Newton algorithm

In this test 50 different cases were studied, the test environment was similar to the one used in Section IV. In every case the size of the first step is above 10^{-6} which proofs that the IpFFT itself does not provide an estimation of the frequency with the required precision. After 3 iterations the step size decreases under 10^{-12} (sometimes under 10^{-18}), so the frequency estimator does not change significantly in the remaining iterations.

V. EFFECT OF THE ERROR OF THE FREQUENCY ESTIMATION

In this section the goal is to confirm that the precision of the estimator is enough to decide if the sampling is coherent and the distribution of phases is uniform in the measured data. This way it can be used to check the signal before the histogram test is done to ensure accurate results.

As it was mentioned earlier, the error of the frequency estimator, e_f is a random variable which is asymptotically normally distributed. Its standard deviation has been measured in many cases in the previous section.

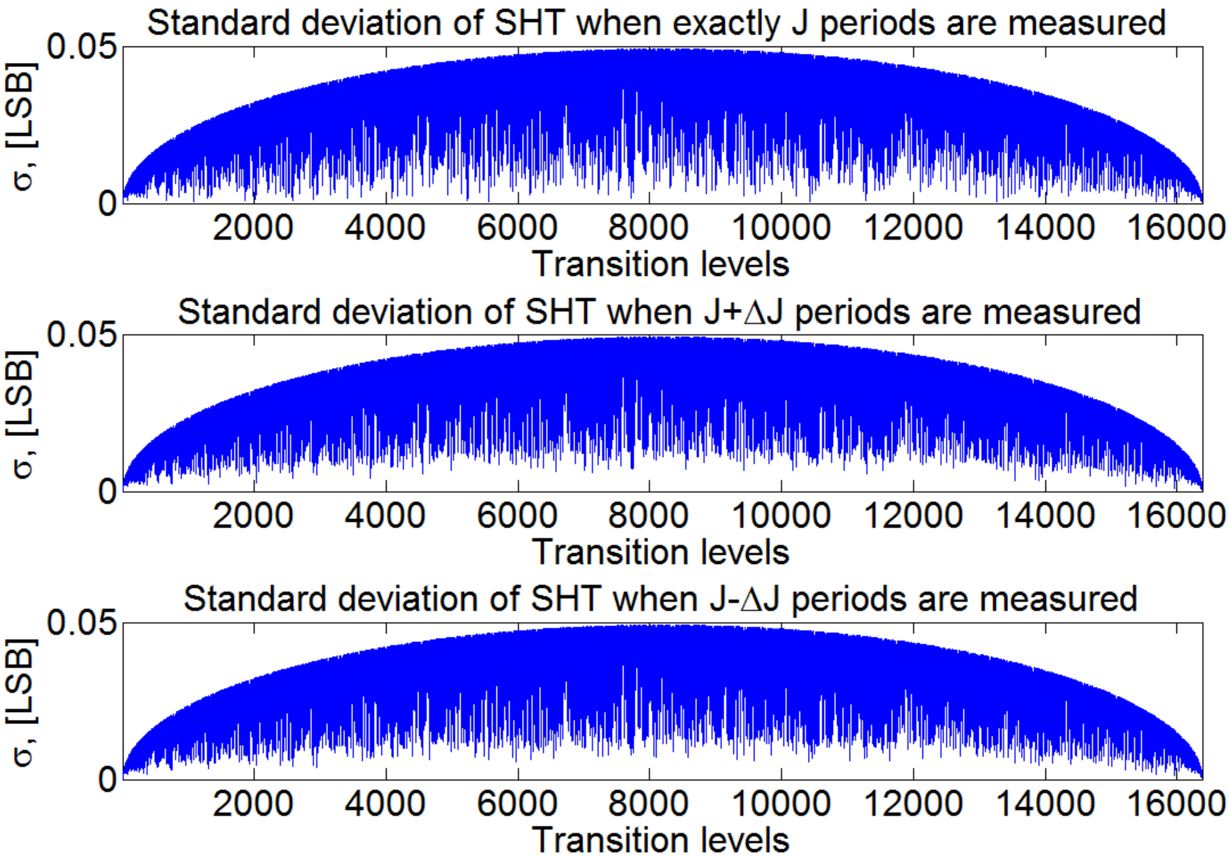

Figure 4. Standard deviations for J, J+ΔJ and J-ΔJ measured periods, respectively.

In view of σ, we can say that 99.7% is the probability that ($-3\sigma < e_f < 3\sigma$). In the next examinations we will assume that the value of e_f is -3σ and 3σ. The frequency estimator estimates the value of $N\cdot f_x/f_s$, which is the number of periods in the measured signal. This will be denoted by J, the error of J will be denoted by ΔJ, so we will study the cases when ΔJ=-3σ or ΔJ=3σ. These cases model the situation when the estimator founds the signal to contain exactly J periods while in fact it contains J-ΔJ or J+ΔJ periods.

A. Distribution of the samples

We will use the (7) condition of Carbone and Chiorboli to decide if the samples are uniformly distributed in a record which contains $J\pm\Delta J$ periods. The formula can be converted:

$$|\Delta J| < \frac{1}{2N}. \qquad (31)$$

Any ΔJ value calculated from the measured standard deviations of the frequency estimator using the 3-term Blackman-Harris window (see Table I.) meets the above condition. This means that it can be decided based on the estimation if the samples are uniformly distributed or not.

B. Standard deviation of transition levels

In this subchapter the effect of a ±Δ*J* error will be studied on the result of the histogram test. Using the (8) equation, we will determine the standard deviation of the estimation of transition levels in three cases. The fluctuations characterized by the standard deviation have two main sources. The first is the initial phase of the signal and the second is the error in the number of periods, ±Δ*J*. The transition levels of the ADC are estimated from the cumulative histogram which is based on the histogram of the measured sine wave (this shows how many code hits occurred in each bins). If the initial phase of the signal changes, some of the samples may occur in a neighbor code bin, which causes an essential uncertainty in the result of the histogram test. The second source means that we have *J*±Δ*J* periods instead of *J*, which also affects the histogram because samples from the fractional period at the end of the sine wave (this can be either a very little part of a period or an almost full period) occur in some code bins. But if this effect is negligible compared to the first one (because with the maximum error of the frequency estimator the fractional period contains only few samples at the end of the signal, or only a few samples are missing from the end to be complete a complete period), we can say that the estimator's error does not affect significantly the result of the histogram test. So, in the following tests the standard deviations of the histogram tests will be compared when the signal contains *J*, *J*+Δ*J* and *J*-Δ*J* periods. The tests were run with those values of *J* from Table II. where *J* and *N* were relative primes (every odd values of *J*). The value of Δ*J* was also calculated from Table II., namely Δ*J*=3σ. Then the standard deviation of the histogram test was estimated in the three cases (when the input record contained *J*, *J*+Δ*J* and *J*-Δ*J* periods). Fig. 4 shows the typical results for the standard deviations (the results were very similar for all tested values of *J*). It can be seen that there is no significant increase in the standard deviations of the result of the histogram tests when the record contained *J*+Δ*J* or *J*-Δ*J* periods instead of *J*. So the simulation showed that the maximum error with which the frequency estimator founds the signal coherent does not raise significantly the standard deviation of the estimation of the transition levels, thus it is a negligible error source compared to the random initial phase.

C. Difference between the cumulative histograms

Next the cumulative histograms were compared of measurements which contained *J*, *J*+Δ*J* and *J*-Δ*J* periods. This was done because the transition levels are the functions of the cumulative histogram, which is a discrete probability variable. Therefore, describing them with their standard deviations may be misleading because this description may hide the case when the error is significant but its probability is very small so it occurs rarely, but when it occurs it causes huge errors in the result.

In these tests the initial phase of the signal was a uniformly distributed random variable between –π and π. 1000 tests were run with every value of *J*, and the cumulative histogram of the *J* , *J*+Δ*J* and *J*-Δ*J* cases were compared. After the comparison we found that none of the bins of the cumulative histograms in the coherent (exactly *J* periods) and almost coherent cases (J+Δ*J* and *J*-Δ*J* periods) differ by more than 1 sample (the least number of hits in the cumulative histogram is more than 10^3, the total number of samples was 2^{18}, so the 1 sample difference does not cause significant change in the result).

These tests showed that using the frequency estimator the meet of both regulations defined by the IEEE standard can be decided based on the measured data.

VI. Conclusion

In this paper an alternative fitting method was presented which provides faster estimation of the four parameters of the sine wave with no significant loss in the precision. The FDLS estimator sometimes provides better result than the original TDLS method, recommended by the standard. The coherence and relative prime condition can be determined from the measured sine wave, so a tool based on the frequency estimation method can be used to determine quickly if a measurement is appropriate to characterize an AD converter or not.

APPENDIX A

In this appendix the forms of the derivatives used in the Newton-Gauss algorithm are presented. The Jacobian matrix, ***J***, has a size *N*×4 (*N* measurements, 4 parameters) and consists of the derivatives with respect to the parameters *A*, *B*, *C*, *f*:

$$\boldsymbol{J} = \left[\frac{\partial X_{BH}}{\partial A}, \frac{\partial X_{BH}}{\partial B}, \frac{\partial X_{BH}}{\partial C}, \frac{\partial X_{BH}}{\partial f}\right]. \qquad (32)$$

This is calculated using the derivatives of the expression $X(k) = DFT\left\{R \cdot \cos\left(\frac{2\pi f t}{N} + \varphi\right) + C\right\}$:

$$\frac{\partial X(k)}{\partial A} = \frac{\partial X_{sin}^{-}(k)}{\partial A} + \frac{\partial X_{sin}^{+}(k)}{\partial A}, \qquad (33)$$

$$\frac{\partial X(k)}{\partial B} = \frac{\partial X_{sin}^{-}(k)}{\partial B} + \frac{\partial X_{sin}^{+}(k)}{\partial B}, \qquad (34)$$

$$\frac{\partial X(k)}{\partial f} = \frac{\partial X_{sin}^{-}(k)}{\partial f} + \frac{\partial X_{sin}^{+}(k)}{\partial f}, \qquad (35)$$

$$\frac{\partial X(k)}{\partial C} = \frac{\partial X_{dc}(k)}{\partial C}. \qquad (36)$$

$$\frac{\partial X_{sin}^{-}(k)}{\partial A} = e^{j\pi\left(f_f + \frac{k-f}{N}\right)} \cdot \frac{1}{2} \cdot \frac{\sin(-f_f\pi)}{\sin\left(\frac{k-f}{N}\pi\right)}, \qquad (37)$$

$$\frac{\partial X_{sin}^{+}(k)}{\partial A} = e^{j\pi\left(-f_f + \frac{k+f}{N}\right)} \cdot \frac{1}{2} \cdot \frac{\sin(f_f\pi)}{\sin\left(\frac{k+f}{N}\pi\right)}. \qquad (38)$$

$$\frac{\partial X_{sin}^{-}(k)}{\partial B} = e^{j\pi\left(f_f + \frac{k-f}{N}\right)} \cdot \frac{j}{2} \cdot \frac{\sin(-f_f\pi)}{\sin\left(\frac{k-f}{N}\pi\right)}, \qquad (39)$$

$$\frac{\partial X_{sin}^{+}(k)}{\partial B} = e^{j\pi\left(-f_f + \frac{k+f}{N}\right)} \cdot \frac{-j}{2} \cdot \frac{\sin(f_f\pi)}{\sin\left(\frac{k+f}{N}\pi\right)}. \qquad (40)$$

$$\frac{\partial X_{sin}^{-}(k)}{\partial f} = j\pi\frac{N-1}{N} \cdot X_{sin}^{-}(k) + e^{j\pi\left(f_f + \frac{k-f}{N}\right)} \cdot \frac{A+jB}{2} \cdot \frac{-\pi \cdot \cos(f_f\pi) \cdot \sin\left(\frac{k-f}{N}\pi\right) + \frac{\pi}{N}\cos\left(\frac{k-f}{N}\pi\right) . \sin(-f_f\pi)}{\frac{1-\cos\left(\frac{2\pi(k-f)}{N}\right)}{2}}, \qquad (41)$$

$$\frac{\partial X_{sin}^{+}(k)}{\partial f} = -j\pi\frac{N-1}{N} \cdot X_{sin}^{+}(k) + e^{j\pi\left(-f_f + \frac{k+f}{N}\right)} \cdot \frac{A-jB}{2} \cdot \frac{\pi \cdot \cos(f_f\pi) \cdot \sin\left(\frac{k+f}{N}\pi\right) - \frac{\pi}{N}\cos\left(\frac{k+f}{N}\pi\right) . \sin(f_f\pi)}{\frac{1-\cos\left(\frac{2\pi(k+f)}{N}\right)}{2}}. \qquad (42)$$

$$\frac{\partial X_{dc}(k)}{\partial C} = N \text{ for } k = 0 \pm N \pm 2 \cdot N \pm 3 \cdot N \pm \cdots, \frac{\partial X_{dc}(k)}{\partial C} = 0 \text{ otherwise.} \qquad (43)$$

With the above expressions the derivatives of $X_{BH}(k)$ can be calculated as:

$$\frac{\partial X_{BH}(k)}{\partial A} = \underline{a}^T \cdot \frac{\partial \underline{y}^T}{\partial A}, \frac{\partial X_{BH}(k)}{\partial B} = \underline{a}^T \cdot \frac{\partial \underline{y}^T}{\partial B}, \quad (44)$$

$$\frac{\partial X_{BH}(k)}{\partial f} = \underline{a}^T \cdot \frac{\partial \underline{y}^T}{\partial f}, \frac{\partial X_{BH}(k)}{\partial C} = \underline{a}^T \cdot \frac{\partial \underline{y}^T}{\partial C}. \quad (45)$$

When the last term in (21) is replaced by a polynomial, the derivatives with respect to *f*, *A*, *B*, *C* are also replaced by the derivatives of the polynomial used to avoid roundoff errors (similarly as in Chapter III).

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