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A novel frequency domain periodic signal estimator

V Pálfi[1] and B Renczes[1]

[1] Budapest University of Technology and Economics,
   Department of measurement and Information Systems,
   Budapest, Hungary

E-mail: {palfi, renczes}@mit.bme.hu

Abstract. In this paper a frequency domain multiharmonic least squares estimator is presented. The least squares minimization can be slow for long measurement records. It is shown that performing the fit in the frequency domain using Blackman-Harris window function, the number of samples used during the calculations can be reduced significantly. The statistical properties and computational demand of the algorithm is compared to the time domain least squares method.

1. Introduction

Estimation of parameters of multiharmonic signals based on the measurement record is an important task in measurement technology. The problem arises in the identification of dynamic systems, the analysis of harmonic distortion in power measurements and in impedance measurement, too. Several solutions have been suggested in the literature. Some methods are based on the four parameters least squares fit defined in the IEEE 1241 standard [1] (in case of one sine component and unknown signal frequency). This iterative method determines the signal parameters by minimizing the least squares cost function. The method was generalized in [2] for the multiharmonic case. The resulting procedure is able to characterize any periodic signal that meets the Dirichlet conditions. The convergence of the method was deeply studied in [3]. Due to the harmonic components on higher frequencies, the algorithm is very sensitive to the frequency parameter, especially for long records. Two suggestions were proposed to increase the convergence of the algorithm and reduce the number of iterations. A similar nonlinear least squares method was presented in [4] where the multiharmonic signal was described by its complex Fourier series.

Some methods separated the determination of the signal frequency and the amplitudes of the harmonic components. The algorithm presented in [5] aims to determine the Fourier coefficients of the measured signal. First, the period is determined in two steps, and then this is used to estimate coefficients of the Fourier series and their variance also. Further developments of the method lead to an approximate maximum likelihood estimator [6].

Different approaches also exist to characterize multiharmonic signals. In [7] and [8] a genetic algorithm is used to estimate the signal parameters. [9] proposes an adaptive notch filter based least squares solution.

A common property of the above methods that the determination of the signal parameters is done using the time domain samples of the signal (fully or partially). This paper presents a frequency domain approach that allows much less computational cost compared to time domain solutions. The presented method is the generalization of [10] for multiharmonic signals.

2. Background and notation

2.1. Signal model and computational costs in the time domain

A periodic signal that consists of H harmonic components can be modelled as the following:

$$y(n) = C + \sum_{i=1}^{H} \left[A_i \cos\left(\frac{2\pi n i J}{N}\right) + B_i \sin\left(\frac{2\pi n i J}{N}\right) \right] \tag{1}$$

where C is the DC offset, A_i and B_i are the amplitudes of the ith cosine and sine, J is the number of periods and N is the number of samples in the record. Since the model is nonlinear in J, an iterative method is needed to determine the parameters. The computational burden of an iterative, time

domain least squares method (e.g. [2]) is $N \cdot (4H^2 + 10H + 6)$ operations in every iteration. In addition, the IpDFT based initial estimation of the frequency requires $N \log_2 N$ operations.

2.2. Information compression in the frequency domain

To reduce the number of samples during the calculations and decrease the computational complexity of the estimation algorithm, it is worth describing the signal in the frequency domain. This is done by performing FFT on the measurement. If the signal is not windowed, the harmonic components are convolved with the discrete sinc() function. The highest sidelobe level of the discrete sinc is -13 dB. Consequently, the information about a sinusoidal component is not concentrated around its frequency in case of noncoherent sampling. However, the application of a proper window function can help to reduce the spectral leakage and concentrate almost every information of the sine components around their peaks. Blackman-Harris window functions [11] are an ideal choice for this purpose due to their low sidelobe level. The function separates the information of the harmonic components in the frequency domain. A further advantage of windowing is that it minimizes the negative effects of not modelled harmonic components on the precision of the estimation. Fig. 1 shows a multiharmonic signal of three harmonic components in the time and frequency domains. In the time domain representation of the signal, the information about the signal parameters is distributed among the samples. However, in the frequency domain the information about the components are separated and concentrated in a few samples. This property of the frequency domain representation will be exploited in the proposed method.
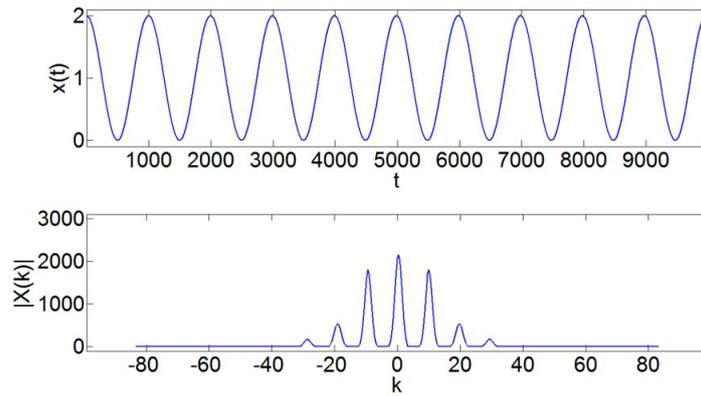

Figure 1. Time and frequency domain representation of a multiharmonic signal

To define the cost function, first the frequency domain representation of a sinusoidal component is required:

$$y(n) = A \cos\left(\frac{2\pi n J}{N}\right) + B \sin\left(\frac{2\pi n J}{N}\right),$$

$$Y(k) = FFT\{y(n)\} = e^{j\pi(k-J)\frac{N-1}{N}} \cdot \left(\frac{A+jB}{2}\right) \cdot \frac{\sin(\pi(k-J))}{\sin\left(\frac{\pi(k-J)}{N}\right)} + \cdots \tag{2}$$

$$e^{j\pi(k+J)\frac{N-1}{N}} \cdot \left(\frac{A-jB}{2}\right) \cdot \frac{\sin(\pi(k+J))}{\sin\left(\frac{\pi(k+J)}{N}\right)}.$$

Let $Y_{BH}(k)$ be the kth sample of the windowed sinewave using the three-term Blackman-Harris window function. $Y_{BH}(k)$ can be expressed using the samples of $Y(k)$:

$$Y_{BH}(k) = \frac{a_2}{2}Y(k-2) + \frac{a_1}{2}Y(k-1) + a_0 Y(k) + \frac{a_1}{2}Y(k+1) + \frac{a_2}{2}Y(k+2). \tag{3}$$

The above expressions can be generalized for a periodic signal of H components:

$$Y_m(k) = FFT\left\{C + \sum_{i=1}^{H}\left[A_i\cos\left(\frac{2\pi niJ}{N}\right) + B_i\sin\left(\frac{2\pi niJ}{N}\right)\right]\right\} =$$

$$C \cdot N + \sum_{i=1}^{H}\left\{e^{j\pi(k-iJ)\frac{N-1}{N}}\cdot\left(\frac{A_i + jB_i}{2}\right)\cdot\frac{\sin\big(\pi(k-iJ)\big)}{\sin\left(\frac{\pi(k-iJ)}{N}\right)} + \ldots \right. \tag{4}$$

$$\left. e^{j\pi(k+iJ)\frac{N-1}{N}}\cdot\left(\frac{A_i - jB_i}{2}\right)\cdot\frac{\sin\big(\pi(k+iJ)\big)}{\sin\left(\frac{\pi(k+iJ)}{N}\right)}\right\}.$$

The windowed case can be computed using (3). Similar to the time domain case, the above model is nonlinear in the number periods parameter (J). Hence, an iterative method is needed to minimize the cost function. In this paper, a least squares cost function is used to determine the parameters. We assume that $H \cdot J < \frac{N}{2}$ holds true for the measurement, thus the Nyquist condition is fulfilled.

2.3. The estimation method and its computational burden

Let $K(\boldsymbol{p})$ be the least squares cost function of the estimation method:

$$K(\boldsymbol{p}) = \boldsymbol{e}(\boldsymbol{p})^H\boldsymbol{e}(\boldsymbol{p}). \tag{5}$$

where $\boldsymbol{p}$ is the vector of the unknown parameters and $\boldsymbol{e}$ is the residual vector. Applying Taylor's expansion, the Gauss-Newton method can be derived [12]:

$$\Delta\boldsymbol{p} = (\boldsymbol{X}\boldsymbol{C}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^H\boldsymbol{C}^{-1}\boldsymbol{e}. \tag{6}$$

Here $\boldsymbol{X}$ is the Jacobian matrix (the matrix containing the derivatives of (4) with respect to the unknown parameters) and $\boldsymbol{C}$ is the covariance matrix (it is needed due to the application of the window function). Evaluation of the (6) step iteratively provides the unknown parameters. The algorithm can be divided into two steps:

- Calculation of the FFT and initial estimation of J using IpDFT,
- Minimizing the least squares cost function using the Gauss-Newton method.

The computational burden of the first step of the algorithm depends on the number of samples ($N\log_2 N$ operations). However, the second, iterative part can be done by using only 5 samples for each harmonic component around its frequency [10]. This way the computational costs can be significantly reduced, since it depends only on the number of harmonic components. In case of H components, the least-squares fit requires $20H^3 + 70H^2 + 80H + 30$ operations per iteration, which is a notable decrease compared to the time domain method ($H \ll N$).

3. Simulation results

The statistical properties of the proposed method were compared to the time domain multiharmonic fitting algorithm, presented in [2]. In these tests both algorithms were executed on the same noisy multiharmonic input signal, the parameters were estimated and the statistical properties of each estimator were determined based on the estimation errors. The additive noise had the following three sources: additive Gaussian noise, quantization noise and harmonic distortion (a non-modelled harmonic component). The algorithms estimated the parameters of the following signal:

$$x(k) = C + A_1\cos\left(\frac{2\pi kJ}{N}\right) + B_1\sin\left(\frac{2\pi kJ}{N}\right) + A_2\cos\left(\frac{2\pi h_2 kJ}{N}\right) + B_2\sin\left(\frac{2\pi h_2 kJ}{N}\right) +$$
$$A_3\cos\left(\frac{2\pi h_3 kJ}{N}\right) + B_3\sin\left(\frac{2\pi h_3 kJ}{N}\right). \tag{7}$$

Parameters C, A_i and B_i were uniformly distributed random variables in the $[0,1]$ domain. The number of periods, J, was also a random variable in $[5, N/30]$, so at least 5 periods were measured

from the periodic signal. The number of samples was set to $N = 2^{16}$, and the simulations were repeated $M = 1000$ times. After the 1000 simulations, the mean value and standard deviation of the estimation error of each parameter were calculated. The initial value of the nonlinear number of periods parameter (J) was determined using interpolated DFT [13]. The statistical properties of the methods were compared in two cases. In the first case, first Gaussian noise with $\sigma = 0.01$ standard deviation was added to the original signal, then it was quantized using a 10 bits ideal quantizer. The results can be seen in Table I.

Parameter	C	A_1	B_1	A_2	B_2	A_3	B_3	J
Original method	$4.01 \cdot 10^{-5}$	$5.57 \cdot 10^{-5}$	$6.49 \cdot 10^{-5}$	$6.31 \cdot 10^{-5}$	$6.23 \cdot 10^{-5}$	$8.06 \cdot 10^{-5}$	$8.21 \cdot 10^{-5}$	$3.14 \cdot 10^{-6}$
Proposed method	$4.61 \cdot 10^{-5}$	$5.92 \cdot 10^{-5}$	$8.50 \cdot 10^{-5}$	$8.22 \cdot 10^{-5}$	$8.11 \cdot 10^{-5}$	$1.37 \cdot 10^{-4}$	$1.34 \cdot 10^{-4}$	$6.38 \cdot 10^{-6}$

Table I. Standard deviations of the parameters, no harmonic distortion

Results show that despite the significantly reduced computational costs, the precision of the proposed method is close to the precision of the time domain algorithm. In the second case, a harmonic component was also added to the original signal with $A_h = 0.01$ amplitude and $J_h = J + 10$ periods. Table II. shows the results. Due to the compression of information in the frequency domain and the selection of samples around the harmonic components, the non-modelled sinewave had much less effect on the precision of the frequency domain estimator, compared to the original method.

Parameter	C	A_1	B_1	A_2	B_2	A_3	B_3	J
Original method	$3.89 \cdot 10^{-5}$	$5.94 \cdot 10^{-5}$	$1.03 \cdot 10^{-4}$	$9.53 \cdot 10^{-5}$	$9.35 \cdot 10^{-5}$	$1.35 \cdot 10^{-4}$	$1.41 \cdot 10^{-4}$	$9.18 \cdot 10^{-6}$
Proposed method	$4.45 \cdot 10^{-5}$	$6.24 \cdot 10^{-5}$	$8.77 \cdot 10^{-5}$	$8.18 \cdot 10^{-5}$	$8.29 \cdot 10^{-5}$	$1.31 \cdot 10^{-4}$	$1.38 \cdot 10^{-4}$	$6.52 \cdot 10^{-6}$

Table II. Standard deviations of the parameters, harmonic distortion is present

4. Conclusion

In this paper, a novel periodic signal estimation method was presented. It was shown that performing the fit in the frequency domain with the application of the Blackman-Harris window function, the computational costs can be notably reduced without significant loss in the precision.

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