Improving histogram test by assuring uniform phase distribution with setting based on a fast sine fit algorithm

Vilmos Pálfi, István Kollár

Budapest University of Technology and Economics, Department of Measurement and Instrument Engineering
H-1521 Budapest, Pf. 91.
Phone: (+36 1) 463-2057, fax: (+36 1) 463-4112
palfi@mit.bme.hu; kollar@mit.bme.hu

Abstract-To accurately characterize an ADC using the sine-wave histogram test, the input signal has to meet strict conditions: sampling has to be coherent, and the number of periods has to be relative prime to the number of samples (see IEEE standard 1241). Due to the limited precision of the sine frequency and of the sampling frequency, such conditions should be checked from the measured signal. In this paper a new method is presented which is able to check the fulfilment of the above conditions from the output signal, and if the signal fails to fulfil the conditions, the number of samples to be neglected in the measurement can be determined to improve the quality of the histogram test result.
Keywords-ADC testing, sine-wave fit, histogram test, phase distribution [1]

I. Introduction

Determination of imperfections of analog-to-digital converters (ADCs) is a significant problem in measurement. Integral nonlinearity (INL) and differential nonlinearity (DNL) are two of the most commonly used properties to characterize an ADC. To evaluate the INL and DNL values in every code bin, accurate knowledge of the transition levels is required. Several methods exist which provide such information, one of the most commonly used procedure is the so-called sine-wave histogram test [1]. The main advantages of this test are that it is quite simple to perform and generation of a pure sine input is easy compared to other pure waveforms (e.g. triangle wave). However the result of the test is very sensible to relation of the sampling frequency and of the frequency of the sine input. Improper relation between these frequencies may cause rough errors in the estimation of the transition levels which result in false INL and DNL values. In this paper we present a method which can significantly improve the accuracy of the test results based on a fast sine fitting algorithm, and [5].

II. Fundamentals

A. Histogram test

In the sine-wave histogram test the ADC is excited with a sine wave input. A histogram is created using the quantized samples, and the histogram is compared to the theoretical histogram of the sine-wave. Based on the differences, the transition levels can be estimated. Let $H[i]$ be the total number of samples in the ith code bin, and H_c the so-called cumulative histogram:

$$H_c[j] = \sum_{i=1}^{j} H[i]. \tag{1}$$

Then the nth transition level of the ADC can be estimated as:

$$\hat{T}[n] = C - R \cdot \cos\left(\pi \frac{H_c[n-1]}{N}\right) \tag{2}$$

where C and R are the dc offset and amplitude of the input sine wave, respectively. More details about the histogram test can be found in [1] and [2].

[1] This work has been supported by foundation László Schnell, and by grant TÁMOP - 4.2.2.B-10/1--2010-0009. Special thanks are due to National Instruments Hungary for providing the ADC hardware.

B. Appropriate relation between sampling and signal frequencies

Based on the IEEE standard [2], a necessary condition of the histogram test provide the most accurate results is that sampling is coherent, so an integer number of periods of the sine wave is measured. It is also obvious that the transition levels will be known with the best accuracy if every sample at the input of the ADC has a voltage value different from the other samples, and these values cover the full scale range of the ADC, in a more or less uniformly distributed manner. This can be assured with the so called relative prime condition. If a coherently sampled sine wave contains N samples and J periods, than all the samples have different voltage values (and the distribution of the phases of the samples will be uniform in $[-\pi, \pi]$, there will be no holes in this interval) if J and N are relative primes. The relation between J, N, the signal frequency (f_x) and the sampling frequency (f_s) is:

$$\frac{f_x}{f_s} = \frac{J}{N} \tag{3}$$

Such a condition can never be perfectly met, due to the uncertainty of the frequencies. In [4], Blair, Flach and Souders have shown that none of the samples will differ from their ideal phase position with more than ±¼ part of the ideal phase distance ($2\pi/N$) if the expression holds:

$$\Delta J \leq \frac{J}{4 \cdot (J-1) \cdot N}, \qquad J > 1 \tag{4}$$

Furthermore, it was shown that if the signal meets condition (4), the variance of the cumulative histogram (σ_c^2) is bounded by ¼. Carbone and Chiorboli found this bound too conservative, and proved that σ_c^2 is still less than ¼ if:

$$\Delta J \leq \frac{1}{2 \cdot N} \tag{5}$$

The significance of these results is that they define a threshold value for the coherence of the signal: such a deviation ΔJ from an integer number of periods does not have significant effect on the result of the histogram test. Thus, (5) and the relative prime condition can be used to determine if a measured sine wave is suitable for the histogram test or not.

C. Fast sine fit algorithm

In paper [5] a very fast, frequency domain sine fit algorithm was presented. It uses the frequency domain samples of a sine-wave windowed with the 3-term Blackman-Harris window. The most important properties of this frequency estimator are summarized as:

- the frequency estimator is asymptotically normally distributed and unbiased,
- the frequency estimator is precise enough (its variance is sufficiently small) to check the fulfillment of the coherence and relative prime conditions, based on the measurement.

This means that the fitness of a measured sine-wave for the histogram test can be determined from the measured data. In this paper we will answer the question what can be done if the signal does not meet the prerequisites.

III. Main results

Here the answer is given what to do when the input signal does not meet the coherence and relative prime conditions. Since the latter has true meaning only if the signal is coherent, first the case is studied when not an integer number of periods were measured from the excitation signal, thus the measured signal contains also a fractional period.

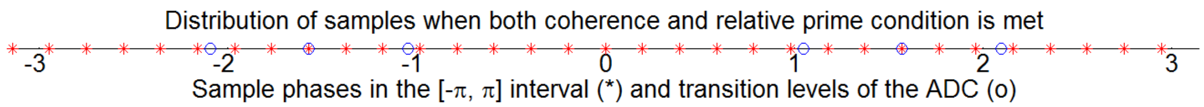

Fig. 1. Distribution of samples when both conditions are met.

The presence of a fractional period means that the histogram is distorted by the extra samples. The histogram test implicitly assumes coherent sampling because the results are derived from the comparison with the probability distribution function of the sine wave. Due to the fractional period some of the bins will contain additional samples, therefore they appear wider than they really are (see Fig. 2).

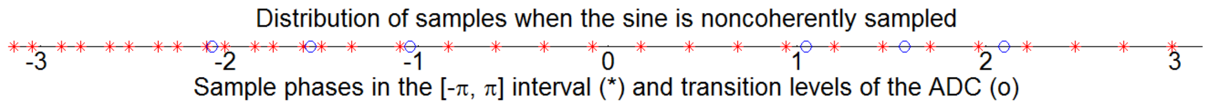

Fig. 2. Distribution of samples for a incoherently sampled sine wave

This error source leads to wrong estimation of the transition levels, and this results in false INL and DNL values. Another related problem is that the distance between two adjacent samples is not constant, so if a transition level falls between two samples with higher distance then it can be estimated only with lower accuracy. It is very important to recognize that a incoherent record will always cause errors in the estimation of the transition levels, and this justifies the importance of neglecting the fractional period in the measured record. To avoid such an error source in the measurement of the transition levels, these additional samples may be discarded from the measured data. They can be thrown away before performing the histogram test.

When sampling is coherent, the relative prime condition has to be studied. Let G be the greatest common divisor of the number of samples and number of periods. If $G>1$, the input contains G times the same sequence, that is, parts of the record, of length N/G. The samples of these parts of record excite the ADC at the same voltage levels (since every (N/G)th sample has the same phase position), thus after the first part of record the measurement does not provide any new information about the transition levels. This means that the result of the histogram test corresponds to a test result based on N/G samples, which also means that the variance of the transition levels (due to the initial phase of the input) will be also be larger, the same as in for records of length N/G.

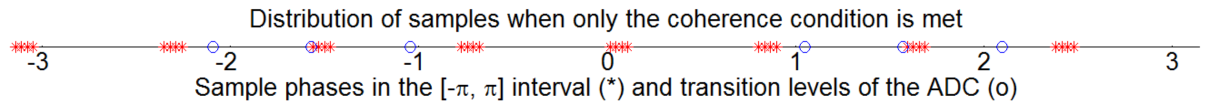

Fig. 3. When only the coherence is (almost) met the samples are distributed in node-groups

An interesting case arises when the signal is not *exactly* coherent, but the whole record meets the Carbone-Chiorboli condition (so it can be considered practically coherent), and the greatest common divisor is G (this case is illlustrated in Fig. 3). In this case the parts of length N/G contain samples which are not exactly but almost at the same phase position. In the phase diagram of the samples in $(0,2\pi)$ groups of samples containing G samples are seen, with relatively large holes between the groups (approximately G times the ideal phase distance). Although every sample in the record of length N represents a different phase position, the closeness of the groups to a transition level increases the variance of the estimator of the transition levels since depending on the value of the initial phase of the sine-wave, the whole group may be shifted to the other side of the transition level which may significantly change the cumulative histogram at this level. This case is very similar to the exactly coherent case with G greatest common divisor: since the whole node may be shifted through a transition level if the initial phase changes, and the variance of the result equals to the variance of the case of N/G samples. It is important to notice that in contrary of the case of incoherent sampling, such an input record does not provide systematic errors in the estimation of the transition levels.

Characterization of the phase distribution of the input is important because the user should be aware of the amount of information contained by the record. In the case of an incoherent record the additional samples were suggested to be thrown away to avoid the source of error. In the last two cases (exactly and almost coherent sampling, G greatest common divisor) the record does not need to be truncated, however the user should be warned that the result of the test for this input will be the same as a result for a shorter input since samples are in the same phase positions. The greatest common divisor G characterizes the record because approximately we can say that the maximal distance between two adjacent samples, Δ_{max} will be G times the optimal phase distance, $\Delta_{opt}=2\pi/N$. Thus the precision of the result for an record of length N with common divisor G will correspond to the precision of a record of length N/G.

The case of the common divisor G cannot be improved by any manipulation of the available samples, since the problem stems from the grouping (nonuniform distribution) of the phases. Therefore, if the algorithm can warn about this, only a repeating of the measurement can help. The program can however tell the amount of the necessary frequency change. Even if the frequency of the sine generator cannot be set to an extremely precise value, its small, careful change (if setting is digital) can reach the proper setting. For analog setting, the proper frequency change can be achieved using a frequency meter.

In order to characterize the input from the above aspects its relative frequency has to be known. It can be best estimated from the samples. The following three outcomes can occur:

- The record meets both the coherent and relative prime condition (Carbone-Chiorboli). In this case the histogram test will give accurate results.
- The signal does not meet the coherent condition, but after discarding a few samples (a small number of samples in comparison to the length of the whole record) both conditions are met. Discarding these samples will increase the accuracy of the histogram test result.
- The optimal number of samples (according to the Carbone-Chiorboli condition) is much less in comparison to the length of the whole record. In this case the histogram test will not give accurate results. The user should be warned, and the repetition of the measurement is recommended with a slightly modified signal frequency to meet the conditions.

In the next section a frequency estimation method is presented which was found to be accurate and precise enough to be able to characterize the input signal.

A. Determining the optimal number of samples

The frequency estimator can be used to determine the optimal number of samples, $N_{opt} \leq N$. The number of samples is considered optimal if the studied record meets the Carbone-Chiorboli condition (5), and the relative prime condition. An algorithm was implemented which removed samples from the record until both conditions are fulfilled. This way the cases shown in Fig. 2 and Fig. 3 can be avoided so the transition levels of the ADC can be estimated precisely. The less the number of samples is, the easier is to meet the condition (5), thus in the implemented algorithm the number of samples is decreased until both conditions become true. Therefore, it is usually a possibility to discard some samples. However, with less samples the variance is larger: therefore this remedy can be applied with care. If the number of samples to be removed is too high then the best is to make a new measurement with proper excitation frequency.

Since the value of N_{opt} is determined based on the frequency estimator which is a random variable (see Subsection II/C), N_{opt} depends also on the estimation error. The estimation error of the frequency is normally distributed with zero mean, so if its variance is known, its effect on N_{opt} can be simply given. For this purpose, the standard deviation of the estimation error, σ_f was measured on different frequencies in real-like circumstances. If σ_f is known, the behaviour of the optimal number of samples can be checked as determined.
As an example, a nonideal 14-bit quantizer model was used in the tests to convert the input sine wave. Its INL and DNL is given in Fig. 4.

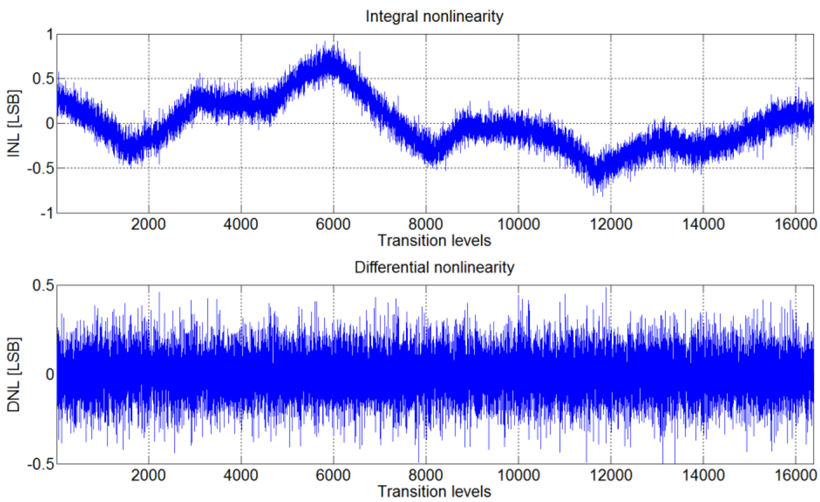

Fig. 4. ADC INL and DNL characteristics ([5])

The imperfections of the sine-wave generator were modelled by the addition of Gaussian noise and first-order harmonic distortion to the signal before quantization, see Fig. 5. The simulations were done using noisy sine wave records which contained N=2^{18} samples. The amplitude and dc component of the sine-wave both were constant during the tests, A=FS/2 (FS: full-scale range of the AD) and dc=0. The initial phase was a uniformly distributed random variable in the (0, 2π) interval. The frequency of the sine wave was also a random variable uniformly distributed around its nominal value.

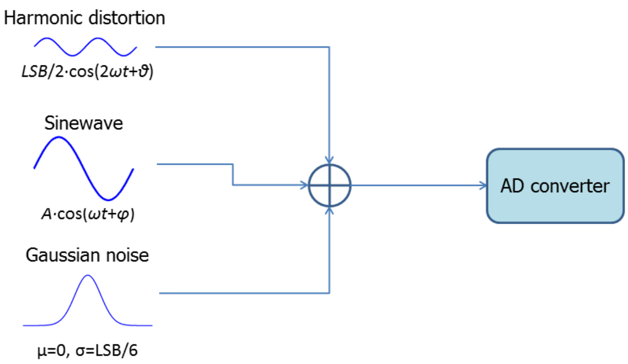

Fig. 5. The model of noise in the simulations

From this point the frequency is described with the exact number of periods (complete and fractional parts) the measured signal contains. This is denoted with J. The number of periods was a random variable in the interval (J_{nom}-0.05, J_{nom}+0.05). On every frequency 10.000 tests were run, and the following standard deviations were determined for the estimators.

J_{nom}	4.9	5	5.1	5.9	6	6.1	6.9	7	7.1
σ_f ($\cdot 10^{-8}$)	32.8014	12.3325	33.7200	9.5136	9.8678	9.8563	10.7819	9.4482	9.5390
J_{nom}	7.9	8	8.1	8.9	9	9.1	9.9	10	10.1
σ_f ($\cdot 10^{-8}$)	10.1303	9.5357	10.1502	9.7670	9.8316	9.5195	9.4676	9.7533	9.7618
J_{nom}	10.9	11	11.1	11.9	12	12.1	12.9	13	13.1
σ_f ($\cdot 10^{-8}$)	10.4359	10.4090	10.4781	9.8358	9.9037	9.7133	10.3826	9.5687	9.7638
J_{nom}	13,9	14	14,1	14,9	15	15,1	24,9	25	25,1
σ_f ($\cdot 10^{-8}$)	9.8983	9.7740	10.1007	9.6418	9.2329	9.5467	9.8465	10.1344	9.9864
J_{nom}	74.9	75	75.1	124.9	125	125.1	624.9	625	625.1
σ_f ($\cdot 10^{-8}$)	10.6139	10.0042	9.5956	10.0204	10.2512	10.6092	10.0356	10.2300	10.0137
J_{nom}	3124.9	3125	3125.1	15624.9	15625	15625.1	78124.9	78125	78125.1
σ_f ($\cdot 10^{-8}$)	10.3129	10.3047	9.6331	9.9189	10.0393	10.1047	10.2682	10.3587	9.8932

Table I. Standard deviation of the frequency estimator on different frequencies.

In the next step on every frequency – standard deviation pair of table I. the optimal number of samples and optimal number of periods were determined.

For this purpose, 10.000 normally distributed random variables with mean value J_{nom} and standard deviation σ_f were created which represented the number of periods in a record of length N=2^{18} samples. The goal of this test is to study the uniqueness of the value of N_{opt} (If N_{opt} is the same for each run with a nominal test frequency, than the standard deviation of the frequency estimation is small enough to let us determine the optimal number of samples without influencing the result). The solution was not unique in the case of the following nominal values: J_1=4.9, J_2=5.1 and J_3=8.9 resulted in two different N_{opt} values. In the case of J_1 and J_2 the variance of the estimator is higher (see table I.) since the Blackman-Harris windows on the sine and on the harmonic distortion cover the same bins in the frequency domain which corrupts the estimation algorithm [5].The increased values of the variances result in two different solutions (of which only one is correct) because in some cases a higher number of periods (and number of samples) meets the Carbone-Chiorboli condition, while in other cases (due to the change of the value of the frequency estimator) the value of ΔJ will be outside the range defined by (5). In the case of J_3 the optimal number of periods (the number of periods corresponding to the optimal number of samples) was 8 (this is the correct value) in 9919 cases and 5 in 81 cases. The variance of the frequency estimator is similar to the values of other frequencies (apart from the J_1 and J_2 cases) so a closer look was taken on the 81 cases. When the signal was truncated to 8 periods the value of the ΔJ deviation was larger than the Carbone-Chiorboli condition by only 10^{-8} (the value of the bound is 10^{-6}). So in these cases N_{opt} was determined incorrectly due to the error of the frequency estimator.

Based on these simulations it can be stated that if the number of measured periods is higher than five, the determination of the exact ideal number of samples based on the frequency estimator is very likely to be a unique value. In the next chapter the algorithm will be tested on real measurements.

B. Testing with real measurements

In the measurements the 16-bit ADC of an NI myDAQ device was used which has a sampling frequency of f_s=200 kHz. The excitation signal was generated by a Brüel & Kjaer Sine Generator Type 1051. The number of samples and the signal frequency were chosen to be $N=2^{20}$ and f_x=3333 Hz, so in the case of accurate f_x and f_s the number of periods would be J=16665 which is relative prime to N. Then the number of periods was estimated using the frequency domain estimator and the optimal number of samples was determined which resulted in N_{opt}= 625183. Then two histogram tests were performed: in the first one the whole record was used as input, while in the second one only the first N_{opt} samples. This latter result is considered errorless. The results can be seen in Fig. 6.

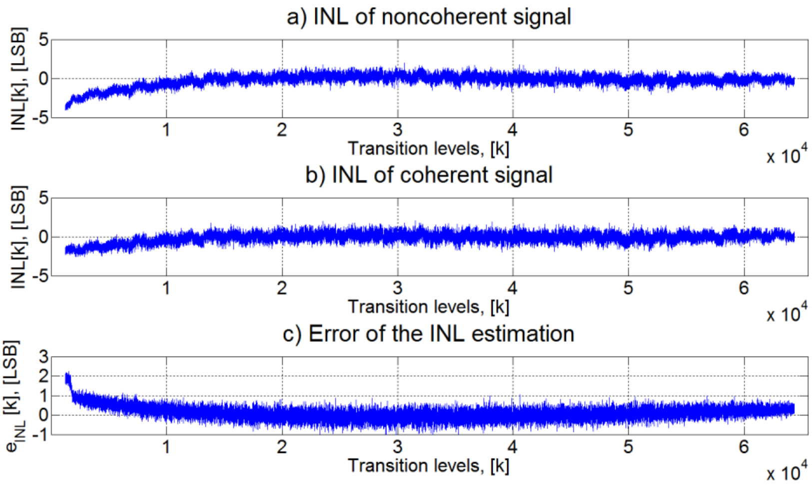

Fig. 6. Comparison of the histogram test results.

In Fig. 6c it can be seen that due to the presence of the additional samples the result of the histogram test contained a 2 LSB error of the INL estimation.

IV. Conlusions

In this paper an estimation method was presented which can help the users decide if a measured sine wave meets the conditions to perform an accurate histogram test thus it can be a supplement to standard ADC testing pogram packages like [6]. The method was tested with simulations and real measurements. These tests showed that determining the optimal number of samples based on the estimation can significantly increase the accuracy of the result of the histogram test.

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