Analysis of Resonator-Based Harmonic Estimation in Case of Data Loss

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***Abstract*—**The paper investigates the behavior of the resonator-based spectral observer (RBO) for lossy measurements. The RBO is an efficient real-time method designed for measuring the harmonic components of periodic signals. Recently some experiments have shown that RBO can tolerate some data loss, but no exact criteria of the convergence have been available. The main contribution of this paper is a set of conditions which enable to predict whether the observer is able to find the harmonic components or not. The necessary condition of convergence helps to recognize the situations when the error-free estimation of the harmonic components is not possible. The sufficient conditions of convergence give safe regions of data loss parameters (e.g., data loss ratio) where the unbiased harmonic estimation can always be ensured. Simulations are also presented to demonstrate the effect data loss in different situations.

***Index Terms*—resonator-based observer, recursive spectrum analysis, data loss, missing observations, convergence**

I. Introduction

Resonator-based observer (RBO) is a recursive algorithm which can be used for the calculation of the harmonic components of periodic signals. One of the advantages of the recursive spectrum estimation algorithms is that they have better tracking property than block-based methods (e.g., DFT: Discrete Fourier Transform). This is particularly important when the spectrum estimation is used in real-time systems. Application examples of the RBO are, e.g., active noise and distortion control [1], [2], [3], analysis of power line signals [4], [5], electronic instruments [6], system identification [7], etc.

If the frequency of a periodic signal is known, the RBO can estimate its Fourier components without leakage and picket fence even in the case of non-coherent sampling.

The RBO has proven to be robust in different real-life applications [1], [2], [5], [6], where the measurement is corrupted by noise, and the analog part of the system shows weak nonlinear behavior. Recently some experiments have shown that RBO can also tolerate some data loss [3]. However, no limits or rules are available for data loss that allow unbiased measurements of harmonic components.

In fact, the RBO is a state observer, and its periodic input signal is modeled as the output of a linear system whose observed state variables represent the harmonic components. The Kalman filter is a potential competitor of the RBO in the field of observer-based, recursive spectrum estimation [8], however, data loss analysis of Kalman filter [9] has not been applied to this particular application to reveal practically important properties. Several other real-time algorithms exist, e.g., [10], [11], [12], [13] and references therein, but the effect of data loss on these algorithms is not known yet. Analysis of data loss is found in the case of offline, batch methods [14], [15], [16], but these papers do not discuss the case of recursive spectrum estimation. Data loss can be regarded as a special case of uneven sampling, and the observer-based spectrum estimation has also been extended to unevenly sampled data [17]. However, it has not been explicitly investigated, how the sampling pattern influences the convergence of the observer.

The aim of this paper is to clarify the influence of data loss on the operation of the RBO. To achieve this, first an appropriate ”lossy” periodic signal model and corresponding observer is introduced. Then necessary and sufficient conditions of the unbiased harmonic estimation are developed. The conditions enable to calculate safe operating regions of parameters of data loss which ensure that harmonic decomposition can be solved for any pattern of data loss. Such bounds can be given for data loss probability and for the minimal length of consecutively processed samples. It is shown that random data loss allows unbiased harmonic estimation. Examples are also presented which demonstrate that both the signal parameters and the data loss pattern influence the convergence of the algorithm.

The paper is structured as follows. Section II shortly summarizes the operation of the RBO. The method of the handling of data loss is discussed in Section III. In Section IV, a set of conditions are derived for the convergence of the RBO. In Section V, simulation examples are presented, where the theorems are applied to different data loss patterns.

II. Review of the Resonator-Based Structure

The RBO is based on the so-called conceptual signal model [18], which means that a periodic signal can be represented as the output of a linear system whose state variables correspond to the harmonic components of the signal [18], [19], [20]. Hence, the harmonic components can be estimated by a properly designed state observer. Different standard methods, like Luenberger observer and Kalman filter, have already been proposed for the state estimation. The RBO uses the Luenberger concept. The advantage of the RBO over the Kalman-filter-based algorithm is that its design does not require the exact knowledge of the covariance matrix of noise [8].

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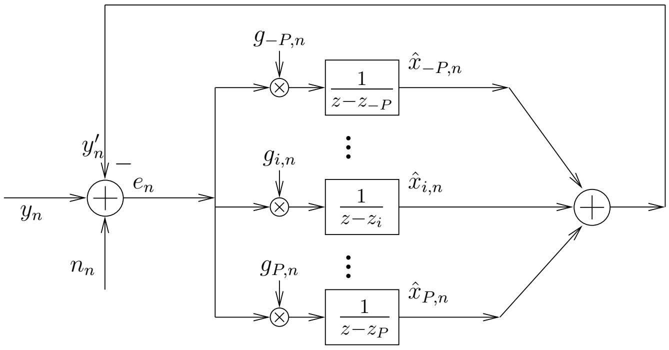

Fig. 1. Block diagram of the resonator-based observer

The conceptual signal model [18] is described as follows:

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n, \quad \mathbf{x}_n = [x_{i,n}]^{\mathrm{T}} \in \mathbb{C}^{N\times 1},\ i = -P \dots P, \quad (1)$$

$$y_n = \sum_{i=-P}^{P} x_{i,n} = \mathbf{c}^{\mathrm{T}}\mathbf{x}_n, \quad (2)$$

$$\mathbf{c} = [1\ 1 \dots 1]^{\mathrm{T}} \in \mathbb{R}^{N\times 1}, \quad (3)$$

$$\mathbf{A} = \mathrm{diag}(z_i) = \langle z_i \rangle \in \mathbb{C}^{N\times N}, \quad i = -P \dots P, \quad (4)$$

$$z_i = e^{\jmath 2\pi i f_{\mathrm{r}}}, \quad (5)$$

where y_n is the output of the signal model (the input of the observer). The signal y_n is the sum of P real harmonic components so it is composed of $N = 2P + 1$ complex exponential functions including DC as well. The harmonic exponential components are collected into the state vector $\mathbf{x}_n$. Let us note that $\mathbf{x}_0$ consists of the complex Fourier coefficients of y_n. As (1) shows, a complex harmonic component, $x_{i,n+1}$, is obtained by multiplying its previous value by z_i, which rotates it at every sampling instant by a phase corresponding to its frequency. The state variables form complex conjugate pairs for real signals: $x_{i,n} = x^*_{-i,n}$. The relative fundamental frequency of y_n is denoted by f_{r}; it is the ratio of the fundamental and the sampling frequencies: $f_{\mathrm{r}} = \frac{f_1}{f_{\mathrm{s}}}$. The frequency of the i-th harmonic component is: $\{if_{\mathrm{r}};\ i = -P \dots P\}$. Similarly to other algorithms [10], the RBO can be extended to measure the frequency, f_{r}, if it is unknown or time varying [5].

The conceptual signal model can be considered as the summed output of resonators which can generate any periodic signal with components up to the half of the sampling frequency. The corresponding observer can be seen in Fig. 1, and the system equations are as follows [18], [5]:

$$\hat{\mathbf{x}}_{n+1} = \mathbf{A}\hat{\mathbf{x}}_n + \mathbf{g}e_n = \mathbf{A}\hat{\mathbf{x}}_n + \mathbf{g}(y_n - y'_n + n_n), \quad (6)$$

$$y'_n = \mathbf{c}^{\mathrm{T}}\hat{\mathbf{x}}_n, \quad (7)$$

$$\mathbf{g} = [g_i]^{\mathrm{T}}, \{g_i;\ i = -P \dots P\}, \quad (8)$$

$$g_i = \frac{\alpha}{N} z_i \ \longleftrightarrow\ \mathbf{g} = \frac{\alpha}{N}\mathbf{A}\mathbf{c}, \quad (9)$$

where y'_n and e_n denote the estimated signal and the estimation error, respectively, n_n denotes a zero-mean noise that is independent of all variables, $\{\hat{\mathbf{x}}_n = [\hat{x}_{i,n}]^{\mathrm{T}}; i = -P \dots P\}$ is the estimated state vector, g_i are free parameters to set the poles of the system. The vector $\mathbf{g}$ is often called observer gain. Due to the complex exponentials, the channels of the observer can be considered as time-invariant systems with a pole on the unit circle. This is why they are called resonators. Usually three cases are distinguished according to the parameter setups:

a) $f_{\mathrm{r}} = \frac{1}{N}$, $\alpha = 1$: the sampling frequency is the integral multiple of the fundamental frequency, which means that the resonator poles are arranged uniformly on the unit circle. The observer has deadbeat settling, i.e., it finds the unknown state $\mathbf{x}_n$ within at most N steps. The RBO performs a set of orthogonal transforms, and it corresponds to the recursive discrete Fourier transform [18], [19]. The advantage of this setup is the speed of convergence, but its noise rejection is poor.
   In this special case, N could be even, which involves that a resonator is also found at $f_{\mathrm{s}}/2$. This means that the indices in (1)-(9) are in the range $i = -P \dots P+1$, and $P = \frac{N}{2} - 1$.
b) $f_{\mathrm{r}} = \frac{1}{N}$, $0 < \alpha < 1$: the observer still performs orthogonal transforms, but the settling is not deadbeat. The RBO performs exponential averaging at the frequency of each harmonic component [7]. The poles of the observer are $p_i = (1-\alpha)^{1/N} \cdot e^{\jmath \frac{2\pi}{N} i}$.
c) f_{r} = arbitrary, $0 < \alpha \le 1$: the observer has infinite impulse response. The settling is not deadbeat even if $\alpha = 1$, but the observer is still fairly fast if all harmonic components are observed up to $\frac{f_{\mathrm{s}}}{2}$ [5]. Analytical results regarding noise suppression and settling time are found in [7]. A design method exists for setting arbitrary poles or even deadbeat settling [18], but usually it is not used in real-time systems due to its computational complexity.

The resonators operate as bandpass filters around their center frequencies. The resonators can be tuned with α: small α results in good noise rejection but in slow settling. Design considerations are given in [7].

Compared to non-iterative spectrum estimation methods, e.g., notch filters [12] where frequency selectivity is achieved by zero-pole cancellation of serially connected filters, the RBO is numerically less sensitive due to its recursive structure [20].

Fundamental relations between different spectrum estimation algorithms have already been reported [19], [21]. The RBO and its variants [18], [5] also show some common features with least mean square (LMS)-based algorithms [10], [11]. The main advantage of the RBO is that it offers a method which enables to set arbitrary poles of the observer; even deadbeat settling can be achieved for an arbitrary frequency set [18]. The form of the RBO investigated in this paper coincides with the transformed form of an LMS-based algorithm presented in [11] so they are compatible regarding their performances. However, the theory of the RBO offers an observer-based framework which constitutes the basis for some of our theorems.

III. Data Loss in the Resonator-Based Observer

In order to model the data loss, a so called data availability indicator function, K_n, is used:

$$K_n = \begin{cases} 1, & \text{if the sample is processed at } n \\ 0, & \text{if the sample is lost at } n \end{cases}, \quad (10)$$

where n is the time index. Samples which are not lost will be termed as processed samples. This kind of indicator function is often used to describe data loss [9].

The data loss rate can also be defined with K_n as:

$$\gamma = \mathbb{P}\mathrm{rob}\left\{K_n = 0\right\}. \quad (11)$$

The proposed state estimation rule for lossy measurements has been derived from (6) by incorporating the indicator function, K_n:

$$\hat{\mathbf{x}}_{n+1} = \mathbf{A}\hat{\mathbf{x}}_n + \mathbf{g}K_n e_n = \mathbf{A}\hat{\mathbf{x}}_n + \mathbf{g}K_n(y_n - y'_n + n_n). \quad (12)$$

The observer defined by (12) assumes the signal model

$$\mathbf{x}_{n+1} = \mathbf{A}\mathbf{x}_n, \quad (13)$$

$$y_n = \mathbf{c}^{\mathrm{T}} K_n \mathbf{x}_n. \quad (14)$$

It can be verified by comparing (6) and (12): y_n has an extra multiplicative term, K_n, which can be included into (2).

Equations (14) and (12) mean that if a sample is lost, i.e., $K_n = 0$, no information appears about $\mathbf{x}_n$ on the output. In this case, the observer's state variables are updated solely using the state transition matrix of the signal model, but no innovation term is used.

The following equation defines the error of the parameter estimation:

$$\tilde{\mathbf{x}}_n = \hat{\mathbf{x}}_n - \mathbf{x}_n. \quad (15)$$

The update algorithm can be obtained for $\tilde{\mathbf{x}}_n$ by subtracting (1) from (12):

$$\tilde{\mathbf{x}}_{n+1} = \mathbf{A}\tilde{\mathbf{x}}_n + \mathbf{g}K_n e_n. \quad (16)$$

Incorporating (2), (7), (15), the error signal in Fig. 1 can be expressed as follows:

$$e_n = y_n - y'_n + n_n = \mathbf{c}^{\mathrm{T}}\mathbf{x}_n - \mathbf{c}^{\mathrm{T}}\hat{\mathbf{x}}_n + n_n = -\mathbf{c}^{\mathrm{T}}\tilde{\mathbf{x}}_n + n_n. \quad (17)$$

This new formula of the error allows us to rewrite (16) as:

$$\tilde{\mathbf{x}}_{n+1} = \left(\mathbf{A} - \mathbf{g}K_n\mathbf{c}^{\mathrm{T}}\right)\tilde{\mathbf{x}}_n + \mathbf{g}K_n n_n. \quad (18)$$

Depending on whether a sample is processed or lost, the state transition matrix of the RBO takes the value $(\mathbf{A} - \mathbf{g}\mathbf{c}^{\mathrm{T}})$ or $\mathbf{A}$, respectively. In noiseless case (18) reduces to:

$$\tilde{\mathbf{x}}_{n+1} = \left(\mathbf{A} - \mathbf{g}K_n\mathbf{c}^{\mathrm{T}}\right)\tilde{\mathbf{x}}_n. \quad (19)$$

IV. Conditions of the Convergence

This section presents the main theoretical results. The necessary and the sufficient conditions of the convergence are formulated, completed by useful and important corollaries. First, the noiseless case is investigated, when $n_n = 0$; then the results are generalized for noisy case. Convergence means that, in the noiseless case, the estimation error, i.e. $\tilde{\mathbf{x}}_n$, while in noisy case, its expected value tends to zero as $n \to \infty$ even in the presence of data loss. In other words, the RBO yields the asymptotically unbiased estimate of harmonic components.

A. Necessary Condition of the Convergence

The necessary condition of convergence is established within the framework of the observer theory. It is investigated when the signal model extended with the data loss is observable.

To achieve this, it is useful to define the vector

$$\mathbf{c}_n = \mathbf{A}^n\mathbf{c} = [c_{i,n}]^{\mathrm{T}} \in \mathbb{C}^{N\times 1}, \quad (20)$$

$$c_{i,n} = e^{\jmath 2\pi i f_{\mathrm{r}} n}, \; i = -P \dots P. \quad (21)$$

The vector $\mathbf{c}_n$ consists of the complex exponential functions belonging to the i-th harmonic frequency at time instant n. The elements $c_{i,n}$ are obtained from (20) using equations (3), (4), and (5). Note that the n-th power of the diagonal elements of $\mathbf{A}$ are $z_i^n = \left(e^{\jmath 2\pi i f_{\mathrm{r}}}\right)^n = e^{\jmath 2\pi i f_{\mathrm{r}} n}$.

The necessary condition of the unbiased harmonic estimation is a straightforward application of the observability condition of linear systems, which is formulated in the following theorem.

Theorem 1: The unbiased estimate of harmonic components of a periodic signal exists only if the series of vectors $\{\mathbf{c}_i K_i\}$ span an N dimensional space.

Proof: In order to be able to estimate the harmonic components of a signal, the state variables of the signal model should be observable. The observability matrix [22] of system described by (13) and (14) is:

$$\begin{aligned}\mathcal{O}_n^{\mathrm{T}} &= \left[\mathbf{c}K_0 \; (\mathbf{c}^{\mathrm{T}}K_1\mathbf{A})^{\mathrm{T}} \; (\mathbf{c}^{\mathrm{T}}K_2\mathbf{A}^2)^{\mathrm{T}} \; \dots \; (\mathbf{c}^{\mathrm{T}}K_n\mathbf{A}^n)^{\mathrm{T}}\right] = \\ &\left[K_0\mathbf{c} \; K_1\mathbf{A}\mathbf{c} \; K_2\mathbf{A}^2\mathbf{c} \; \dots \; K_n\mathbf{A}^n\mathbf{c}\right] = \\ &\left[K_0\mathbf{c}_0 \; K_1\mathbf{c}_1 \; K_2\mathbf{c}_2 \; \dots \; K_n\mathbf{c}_n\right]. \end{aligned} \quad (22)$$

A system is observable if the observability matrix, $\mathcal{O}_n$, has full rank, i.e. $\mathrm{rank}(\mathcal{O}_n) = N$ [22]. To ensure this condition, the rows of $\mathcal{O}_n$ must have at least N linearly independent vectors, i.e., they should span the N dimensional space. ■

It is shown in Appendix A that the observability condition is also *sufficient* for the convergence of a deadbeat observer:

Theorem 2: A deadbeat RBO yields the unbiased estimate of harmonic components of a periodic signal *if and only if* the series of vectors $\{\mathbf{c}_i K_i\}$ span an N dimensional space.

B. Discussion of Special Cases

A special case is when the resonator frequencies are arranged uniformly up to the sampling frequency, which implies that $f_{\mathrm{r}} = \frac{1}{N}$, and the period of y_n is N. On the other hand, this is the case of coherent sampling. Furthermore, the vectors $\mathbf{c}_n$ are orthogonal [18] and have period of length N:

$$\mathbf{c}_n = \mathbf{c}_{n+N} \quad (23)$$

which can be verified by substituting $f_{\mathrm{r}} = \frac{1}{N}$ into (21).

As (23) shows, N different orthogonal $\mathbf{c}_n$ vectors exist, all of which should be present in the observability matrix to span an N dimensional space, which ensures the observability. A particular vector, $\mathbf{c}_m$, is present in the observability matrix if any of the samples $\{y_{m+lN};\; l = 0, 1, 2, \dots\}$ is processed, i.e., K_{m+lN}=1 at least for one value of l. Hence, an interpretation of the necessary condition of observability is that at least one sample should be processed at all of the N different positions

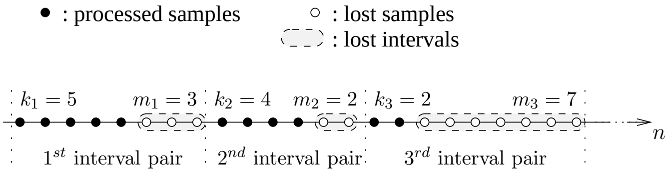

Fig. 2. Explanation of notations. Number of lost intervals: $\mathcal{P} = 3$. Number of processed samples: k=k_1+k_2+k_3=11

of the signal period. Note that processed samples do not need to belong to the same period. A well-known application example which satisfies this condition is the equivalent sampling, when apparently consecutive values of one period of the signal are sampled from different periods. This technique ensures that each sample of the period of the signal is processed.

However, if a sample is missing from the same position of each period, there are less than N different $\mathbf{c}_n$ vectors in the observability matrix which can not span an N dimensional space. Hence, the harmonic decomposition is impossible. An example for the systematic loss of data from the same position of each period is when the signal is saturated, and the distorted samples are discarded.

C. Sufficient Condition of the Convergence

The necessary condition is often difficult to test since it may require intuition and/or analytical considerations, and does not guarantee the convergence. The sufficient condition will be found by investigating whether the Euclidean norm of the observation error, $\|\tilde{\mathbf{x}}_n\|$, converges to zero. If $\|\tilde{\mathbf{x}}_n\| \to 0$ as $n \to \infty$, then the final parameter error is zero, i.e., the observer yields the unbiased estimate of harmonic components.

Before the theorem is formulated, some quantities are to be defined. Let us suppose that the eigenvalues of the state transition matrix of the observer belonging to processed samples have single multiplicity. In this case, it can be expressed as

$$\mathbf{A} - \mathbf{g}\mathbf{c}^{\mathrm{T}} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{-1}, \tag{24}$$

where the eigenvalues are the diagonal elements of $\mathbf{\Lambda}$, and the matrix $\mathbf{U}$ contains the eigenvectors. The case of multiple eigenvalues will be discussed later.

Let us define some more variables:

- η is a scalar variable so that

$$\eta = \|\mathbf{U}\| \left\|\mathbf{U}^{-1}\right\| \tag{25}$$

- $\bar{\lambda}$ is the magnitude of the maximum eigenvalue of $\left(\mathbf{A} - \mathbf{g}\mathbf{c}^{\mathrm{T}}\right)$. Equivalently, it is the Euclidean norm of $\mathbf{\Lambda}$:

$$\bar{\lambda} = ||\mathbf{\Lambda}|| \tag{26}$$

- k is the total number of samples processed by the RBO
- $\mathcal{P}$ is the number of such time intervals where samples are not processed, as illustrated in Fig. 2.

Parameters $\bar{\lambda}$ and η can be calculated from (25) and (26) off-line. In the case of uniform resonator alignment: $\bar{\lambda} = (1-\alpha)^{1/N}$ [7]. Note that k counts samples while $\mathcal{P}$ counts intervals.

Now, the theorem on the sufficient condition of the convergence can be formulated.

Theorem 3: Suppose the eigenvalues of the state transition matrix have single multiplicity. If the ratio of the intervals composed of lost samples to the number of processed samples is less than a critical value defined in (28), then the RBO yields the asymptotically unbiased estimate of harmonic components of a periodic signal:

$$\text{if} \quad \frac{\mathcal{P}}{k} < \pi_{\mathrm{cr}} \Rightarrow \lim_{\substack{n\to\infty \\ k\to\infty}} \tilde{\mathbf{x}}_n = 0, \tag{27}$$

$$\pi_{\mathrm{cr}} = -\frac{\log \bar{\lambda}}{\log \eta}. \tag{28}$$

Proof: The state transition matrix from the initial state of $\tilde{\mathbf{x}}_n$ can be found by the recursive expansion of (19):

$$\tilde{\mathbf{x}}_{n+1} = \prod_{i=0}^{n} \left(\mathbf{A} - \mathbf{g}K_i\mathbf{c}^{\mathrm{T}}\right) \ \tilde{\mathbf{x}}_0. \tag{29}$$

If the decomposition of the form (24) exists, its k-th power can be expressed in the following way:

$$\left(\mathbf{A} - \mathbf{g}\mathbf{c}^{\mathrm{T}}\right)^k = \mathbf{U}\mathbf{\Lambda}^k\mathbf{U}^{-1}, \tag{30}$$

as $f(\mathbf{X}) = \mathbf{U}f(\mathbf{\Lambda}_{\mathbf{X}})\mathbf{U}^{-1}$ for any $\mathbf{X}$ whose eigenvalues have single multiplicity. This formula can be substituted into (29) by forming groups of consecutive interval pairs from the products as Fig. 2 shows. The first interval in the i-th interval pair is composed of $k_i > 0$ processed samples, and it is followed by the second interval which contains $m_i > 0$ lost samples. Equation (29) can be rewritten in this context as

$$\tilde{\mathbf{x}}_{n+1} = \prod_{i=1}^{\mathcal{P}} (\mathbf{A})^{m_i} \left(\mathbf{A} - \mathbf{g}\mathbf{c}^{\mathrm{T}}\right)^{k_i} \ \cdot \tilde{\mathbf{x}}_0 \tag{31}$$

$$= \prod_{i=1}^{\mathcal{P}} (\mathbf{A})^{m_i}\, \mathbf{U}\mathbf{\Lambda}^{k_i}\mathbf{U}^{-1} \ \cdot \tilde{\mathbf{x}}_0, \tag{32}$$

where $\mathcal{P}$ stands for the number of interval pairs up to the time instant n. Note that $\mathcal{P}$ also equals the number of intervals where data are lost, since all interval pairs contain exactly one such an interval. This form of (29) can be used without loss of generality, since if the signal stream begins with lost sample(s), then $k_1 = 0$, and if it is terminated with processed sample(s), then $m_{\mathcal{P}} = 0$. An upper bound on the norm of $\tilde{\mathbf{x}}_{n+1}$ can be found by taking the norm of (32) and applying the inequality $\|\mathbf{AB}\| \leq \|\mathbf{A}\|\,\|\mathbf{B}\|$ [24]:

$$\left\|\tilde{\mathbf{x}}_{n+1}\right\| \leq \prod_{i=1}^{\mathcal{P}} \|(\mathbf{A})^{m_i}\| \left\|\mathbf{U}\mathbf{\Lambda}^{k_i}\mathbf{U}^{-1}\right\| \ \|\tilde{\mathbf{x}}_0\|. \tag{33}$$

The norms can be calculated as

$$\left\|\mathbf{U}\mathbf{\Lambda}^{k_i}\mathbf{U}^{-1}\right\| \leq \|\mathbf{U}\|\,\|\mathbf{\Lambda}\|^{k_i} \left\|\mathbf{U}^{-1}\right\| = \eta \cdot \bar{\lambda}^{k_i}, \tag{34}$$

where (25) and (26) were applied.

Furthermore the following inequality is valid:

$$\|(\mathbf{A})^{m_i}\| \leq \|\mathbf{A}\|\,\|\mathbf{A}\| \ldots \|\mathbf{A}\|\,\|\mathbf{A}\| = 1^{m_i} = 1, \tag{35}$$

where (B-1) in Appendix B is used for the case of lost samples. Note that the norm is independent of m_i for an interval. This is the reason why the sufficient condition contains the number of lost intervals, not the number of lost samples.

Substituting (34) and (35) into (33), one obtains

$$\left\|\tilde{\mathbf{x}}_{n+1}\right\| \leq \prod_{i=1}^{\mathcal{P}} \eta \bar{\lambda}^{k_i} \;\; \|\tilde{\mathbf{x}}_0\| = \eta^{\mathcal{P}} \bar{\lambda}^{\Sigma k_i} \;\; \|\tilde{\mathbf{x}}_0\|$$

$$\left\|\tilde{\mathbf{x}}_{n+1}\right\| \leq \eta^{\mathcal{P}} \bar{\lambda}^{k} \;\; \|\tilde{\mathbf{x}}_0\| = \left(\eta^{\frac{\mathcal{P}}{k}} \bar{\lambda}\right)^{k} \;\; \|\tilde{\mathbf{x}}_0\|, \tag{36}$$

where $k = \sum_{i=1}^{\mathcal{P}} k_i$ is the total number of processed samples. Let us search for a constant, ν, for which $\left(\eta^{\nu}\bar{\lambda}\right) < 1$:

$$\eta^{\nu}\bar{\lambda} < 1$$

$$\nu \log \eta + \log \bar{\lambda} < 0$$

$$\nu < -\frac{\log \bar{\lambda}}{\log \eta} = \pi_{\mathrm{cr}}. \tag{37}$$

In this case, $\lim_{k\to\infty}\left(\eta^{\frac{\mathcal{P}}{k}}\bar{\lambda}\right)^{k} = 0$ for any $\frac{\mathcal{P}}{k} \leq \nu$, so $\lim_{k\to\infty}\|\tilde{\mathbf{x}}_n\| = 0$ due to (36). Since $\frac{\mathcal{P}}{k} \leq \nu < \pi_{\mathrm{cr}}$, the theorem is proven. ■

The eigenvalues of $(\mathbf{A} - \mathbf{g}\mathbf{c}^{\mathrm{T}})$ have generally single multiplicity, so the decomposition (24) exists. A practically important exception is the deadbeat observer which has multiple eigenvalues in the origin. Nevertheless, the sufficient condition for this case is given by *Theorem 2* (see Appendix A).

Now some important corollaries are formulated.

Perhaps the most commonly used parameter of data loss is the data loss rate (γ) that is the ratio of lost samples to the total number of samples. Noting that the number of lost samples is not lower than the number of lost intervals (see Fig. 2), an upper bound can be put on $\frac{\mathcal{P}}{k}$ with γ as

$$\frac{\mathcal{P}}{k} \leq \frac{1}{1/\gamma - 1}. \tag{38}$$

Combining (38) and (27) yields the following corollary.

Corollary 1: If the data loss rate γ is less than a critical value given by (39), the RBO yields the asymptotically unbiased estimate of the harmonic components of a periodic signal:

$$\text{if} \quad \gamma < \frac{1}{1 + 1/\pi_{\mathrm{cr}}} \Rightarrow \lim_{n\to\infty} \tilde{\mathbf{x}}_n = 0. \tag{39}$$

Based on *Theorem 3*, an alternative sufficient condition of convergence can also be formulated.

Theorem 4: If a signal stream contains infinite number of intervals composed of at least L_{cr} consecutive processed samples, then the RBO yields the asymptotically unbiased estimate of the harmonic components of a periodic signal.

$$L_{\mathrm{cr}} = \left\lfloor \frac{1}{\pi_{\mathrm{cr}}} + 1 \right\rfloor \tag{40}$$

Proof: Theorem 3 holds even in the case if only those processed samples are taken into account at the calculation of k which belong to the intervals where at least L_{cr} consecutive samples are processed. The other processed samples should be assigned to those intervals which contained originally only lost samples. Any samples can be assigned to these intervals since the only condition is that the norm of the state transition matrix of the observer should be one, which ensures that the final result of (35) remains valid. However, this condition is true for processed and lost samples as proved in Appendix B.

Suppose that there are infinite number of intervals where at least L_{cr} consecutive samples are processed. These groups of samples are separated by such intervals which also contain lost samples. Since $\mathcal{P}$ is the number of intervals which contain lost samples, and these intervals are followed by at least L_{cr} processed samples, so the number of processed samples is $k \geq \mathcal{P}L_{\mathrm{cr}}$, hence

$$\frac{\mathcal{P}}{k} \leq \frac{\mathcal{P}}{\mathcal{P}L_{\mathrm{cr}}} = \frac{1}{L_{\mathrm{cr}}} = \frac{1}{\lfloor 1 + 1/\pi_{\mathrm{cr}} \rfloor} < \pi_{\mathrm{cr}}, \tag{41}$$

so the condition of *Theorem 3* is fulfilled, and the parameter error tends to zero. ■

There are some applications that demonstrate the significance of this theorem, e.g., packet-based data transmission over a lossy link, real-time processing of a signal in bursts when the continuous signal processing is not possible due to computational limitations. In these cases, one can achieve the unbiased harmonic estimation by choosing the packet or burst length not shorter than L_{cr}.

Theorem 4 can also be used to consider the convergence in the case of random data loss as follows.

Corollary 2: The RBO yields the asymptotically unbiased estimate of the harmonic components of a periodic signal if the data loss can be modeled with a random, Bernoulli process.

Proof: If the data loss is modeled as a Bernoulli process, each sample is processed or lost with a probability q and $1-q$, respectively. Hence, the probability that L_{cr} consecutive samples are processed is $q^{L_{\mathrm{cr}}}$ which is greater than zero under realistic assumptions. The law of large numbers ensures that after a sufficiently long observation, there are enough number of intervals which satisfy the hypothesis of *Theorem 4*, so the harmonic estimation error converges to zero. ■

D. Convergence in Case of Noisy Observations

In this subsection it is shown that the above presented theorems are also valid when observations are noisy.

The extension of the necessary condition of unbiased estimation is straightforward since the observability condition of linear systems also holds in the presence of noise [23]. Hence, *Theorem 1* is valid in noisy case.

In the case of sufficient conditions, the convergence of the *expected value* of $\tilde{\mathbf{x}}_n$ can be proven. The proof is based on the concept that (18) is linear in $\tilde{\mathbf{x}}_n$ and in n_n, so the final parameter error is obtained as the superposition of the system's response to the noise and signal inputs. It will be shown that the noise does not influence the convergence of the mean, so it causes no bias in the estimation.

To enable the investigation of convergence in the mean, the expected value of (18) should be calculated [13]:

$$\mathbb{E}_K\left\{\tilde{\mathbf{x}}_{n+1}\right\} = \mathbb{E}_K\left\{\left(\mathbf{A} - \mathbf{g}K_n\mathbf{c}^{\mathrm{T}}\right)\tilde{\mathbf{x}}_n + \mathbf{g}K_n n_n\right\}, \tag{42}$$

where $\mathbb{E}_K$ denotes the conditional expected value with respect to the sequence of indicators $\{K_0, K_1, \ldots\}$. In the case if K_n is a random variable, the application of the usual expected value instead of $\mathbb{E}_K$ would require the evaluation of probabilities of all terms containing K_n. However, $\mathbb{E}_K$ preserves the sequence of K_n which enables the investigation of data loss similarly to the noiseless case. The terms $\mathbf{A}$, $\mathbf{g}$, $\mathbf{c}^{\mathrm{T}}$, K_n are constant with respect to $\mathbb{E}_K$, so (42) can be rewritten as:

$$\mathbb{E}_K\{\tilde{\mathbf{x}}_{n+1}\} = \left(\mathbf{A} - \mathbf{g}K_n\mathbf{c}^{\mathrm{T}}\right)\mathbb{E}_K\{\tilde{\mathbf{x}}_n\} \quad (43)$$
$$= \prod_{i=0}^{n}\left(\mathbf{A} - \mathbf{g}K_i\mathbf{c}^{\mathrm{T}}\right)\mathbb{E}_K\{\tilde{\mathbf{x}}_0\}. \quad (44)$$

The noise term of (42) vanishes in (43) since (42) is linear in n_n, furthermore n_n is independent of K_n, and $\mathbb{E}_K\{n_n\}$=0.

The proof of *Theorem 3* (eq. (29)-(37)) has shown that the norm of the matrix product $\prod_{i=0}^{n}\left(\mathbf{A}–\mathbf{g}K_i\mathbf{c}^{\mathrm{T}}\right)$ in (44) tends to zero if any of the sufficient conditions hold, i.e., $\lim_{n\to\infty}\mathbb{E}_K\{\tilde{\mathbf{x}}_n\}$=0. The proofs of other sufficient conditions refer to the proof of *Theorem 3*, so they are valid, as well. In the special case of *Theorem 2*, the RBO is deadbeat, which involves that the matrix product $\prod_{i=0}^{n}\left(\mathbf{A}–\mathbf{g}K_i\mathbf{c}^{\mathrm{T}}\right)$ is zero if at least N samples are present. Hence, $\lim_{n\to\infty}\mathbb{E}_K\{\tilde{\mathbf{x}}_n\}$=0 in this case, as well.

It can be concluded that the RBO yields the unbiased estimate of harmonic components in noisy case, as well, if any of the hypotheses of *Theorem2,3,4* or *Corollary1,2* is true.

V. Simulation Results

This section contains some examples which illustrate the application of the theorems. Three examples will be presented which model a common application where the Fourier coefficients of the mains are measured. The sampling frequency is f_{s}=10 kHz, the amplitudes of the harmonics are inversely proportional to their order, i.e., $\left\{|x_{i,n}| = \frac{1}{i}:\ i = 1\ldots P,\ n = \ldots\right\}$, and initial phases are set randomly. The SNR is 20 dB.

In the simulations, $\|\mathbb{E}_K\{\tilde{\mathbf{x}}_n\}\|$ is displayed, where $\mathbb{E}_K\{\tilde{\mathbf{x}}_n\}$ is obtained from the ensemble average of 500 simulations. If $\|\mathbb{E}_K\{\tilde{\mathbf{x}}_n\}\|$ converges toward zero, it means that the observer can perform the harmonic decomposition without bias.

In the first two examples, the fundamental frequency is set to f_1=50 Hz. Two different observer setups and two data loss patterns are tested. The patterns (*pattern 1* and *pattern 2*) are periodic with period 200 and 199 samples, respectively, and they contain one lost sample at the last position of each period. *Pattern 1* is described as

$$K_1 = 1, \ldots, K_{199} = 1, K_{200} = 0 \quad K_n = K_{n+200}, \quad (45)$$

while in the case of *pattern 2*:

$$K_1 = 1, \ldots, K_{198} = 1, K_{199} = 0 \quad K_n = K_{n+199}. \quad (46)$$

Example 1: f_1=50 Hz and α=1. This parameter setup results in a deadbeat observer with uniform resonator alignment. Necessary and sufficient conditions for the convergence are given in *Theorem 2*, and practical considerations are found in subsection IV-B. The simulation results are plotted in Fig. 3.

Note that $f_{\mathrm{r}} = \frac{50\ \mathrm{Hz}}{f_{\mathrm{s}}} = \frac{1}{200}$, so the period of the signal is $N = 200$, and according to (23): $\mathbf{c}_n = \mathbf{c}_{n+200}$.

In the case of *pattern 1*, the period of the data loss equals to that of the signal due to (45), so the last sample is missing from each period. As explained in subsection IV-B, the systematic loss of samples from the same position of the periods causes that the RBO can not perform the harmonic decomposition.

In the case of *pattern 2*, the period of K_n is 199, but that of $\mathbf{c}_n$ is 200, so they differ from each other by one sample. This means that the position of the lost sample "drifts" one position

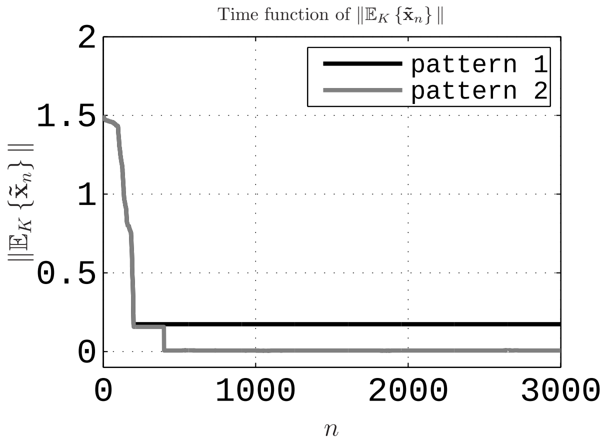

Fig. 3. Convergence of the RBO for f_1=50 Hz, deadbeat settling, and data loss with (black) *pattern 1* and (gray) *pattern 2*.

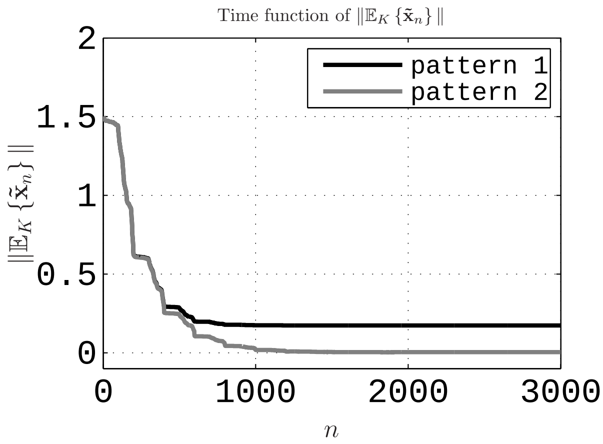

Fig. 4. Convergence of the RBO for f_1=50 Hz, $\alpha = 0.6$, and data loss with (black) *pattern 1* and (gray) *pattern 2*.

to the left in each consecutive period of y_n, hence all of the N different samples within the period of y_n are processed. As presented in subsection IV-B, this fact means that the RBO finds the harmonic decomposition, i.e., $\mathbb{E}_K\{\tilde{\mathbf{x}}_n\}$ →0.

Example 2: f_1=50 Hz and α=0.6. The resonator alignment is still uniform, but the settling is exponential. The necessary condition of the convergence for this case is found in *Theorem 1*, and sufficient conditions are given in *Theorem 3* and *4*. Since the data stream contains continuous bursts of processed samples that are of equal length, *Theorem 4* is more convenient to use. Simulation results are plotted in Fig. 4.

The graphs show that the observation error does not tend to zero in the case of *pattern 1*. The reason is the same as in the first example: the last sample is systematically missing from each period, which violates the necessary condition.

Although the observation error tends to zero for *pattern 2*, the convergence can not be proven by the sufficient conditions. The critical length of continuously processed samples is L_{cr}=200 according to *Theorem 4*, which is achieved by neither patterns. This example illustrates an important pro-

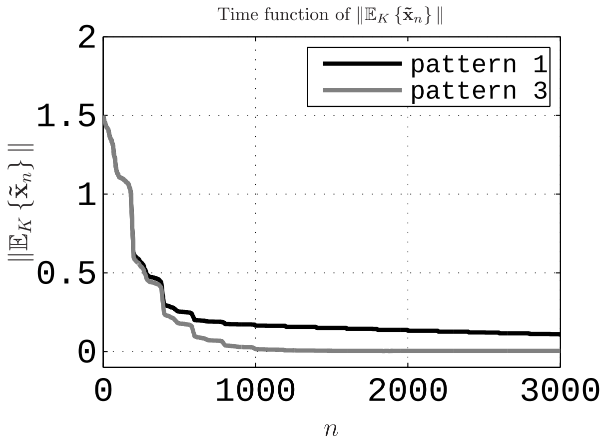

Fig. 5. Convergence of the RBO for f_1=50.025 Hz, $\alpha = 0.6$, and data loss with (black) *pattern 1* and (gray) *pattern 3*.

perty of sufficient conditions. The critical values of data loss parameters given by sufficient conditions should ensure that no data loss pattern exist which violate the necessary condition of observability. Indeed, in the case of *pattern 1* the length of continuously processed samples is 199, which is less only by one sample than the critical length, L_{cr}, but causes that the signal is not observable. Hence, if a sufficient condition is not met, then the convergence can not be guaranteed.

Example 3: f_1=50.025 Hz and α=0.6. The settling is exponential, and the resonator alignment is *not* uniform. The small deviation of the frequency from 50 Hz simulates the fluctuation of power line frequency. The necessary and the sufficient conditions are given in *Theorem 1* and in *Theorem 4*, respectively, as explained in example 2. The time diagram of the convergence is plotted in Fig. 5.

Although the estimation error tends toward zero in the case of *pattern 1*, the convergence is noticeably slow. In practical situations, when the harmonic components change fast in time, such a system cannot be used. This example demonstrates that if the observer and signal parameters are close to a critical parameter setup where convergence is not possible, then data loss can seriously degrade the performance of the RBO.

In this example, a new pattern (*pattern 3*) is constructed instead of *pattern 2,* which ensures that sufficient conditions are met. The critical burst length is L_{cr}=207 according to (40), so the new pattern is chosen as

$$K_1 = 1, \ldots, K_{207} = 1, K_{208} = 0 \quad K_n = K_{n+208}. \tag{47}$$

Fig. 5 shows that the convergence of RBO has became faster for *pattern 3,* and the convergence can also be proven.

VI. Conclusions

This paper discussed the effect of data loss on the resonator-based harmonic estimation. The main contribution of the paper is a set of conditions which enable to predict whether the RBO is able to find the unbiased estimate of harmonic components.

The necessary condition of convergence pointed out that in certain situations the error-free estimation of the harmonic components is not possible. Simulations and theoretical considerations showed that the most critical case is the coherent sampling of the signals together with synchronously repeated data loss pattern.

The sufficient conditions of unbiased harmonic estimation help to check the convergence of the RBO even without the knowledge of the exact pattern of data loss. Furthermore they show that unbiased estimation of harmonic components can always be ensured if one of the following conditions is met:

- the data loss rate is lower than a critical threshold
- the signal contains long enough intervals without lost data
- samples are lost randomly.

Possible subjects of future research are how the data loss influences the speed of convergence and the noise sensitivity.

Appendix A

Proof of *Theorem 2*.

To prove the statement we use an alternative form of the observer (19) with a transformed state vector [18]:

$$\begin{aligned} \tilde{\mathbf{x}}^{\mathrm{t}}_{n+1} &= \left(\mathbf{I} - \tfrac{1}{N}\mathbf{c}^*_n K_n \mathbf{c}^{\mathrm{T}}_n\right)\tilde{\mathbf{x}}^{\mathrm{t}}_n & \text{(A-1)} \\ &= \textstyle\prod_{i=0}^{n}\left(\mathbf{I} - \tfrac{1}{N}\mathbf{c}^*_i K_i \mathbf{c}^{\mathrm{T}}_i\right)\tilde{\mathbf{x}}^{\mathrm{t}}_0 & \text{(A-2)} \end{aligned}$$

where $\mathbf{I}$ is the identity matrix, and

$$\tilde{\mathbf{x}}^{\mathrm{t}}_n = \mathbf{A}^{-n}\cdot\tilde{\mathbf{x}}_n, \tag{A-3}$$

$$\mathbf{c}_n = [c_{i,n}]^{\mathrm{T}},\ c_{i,n} = e^{\jmath\frac{2\pi}{N}in},\ i = 0\ldots N-1. \tag{A-4}$$

If $f_{\mathrm{r}} = \frac{1}{N}$, the vectors $\mathbf{c}_n$ are periodic with N, and $\{\mathbf{c}_l;\ l = 0\ldots N-1\}$ form an orthogonal basis [18]:

$$\mathbf{c}_n = \mathbf{c}_{n+N}, \tag{A-5}$$

$$\mathbf{c}^{\mathrm{H}}_l\mathbf{c}_q = \sum_{i=0}^{N-1} e^{\jmath\frac{2\pi}{N}(q-l)i} = \begin{cases} N, & \text{if } q = l \\ 0, & \text{if } q \neq l \end{cases}. \tag{A-6}$$

The advantage of the transformed from (A-1) is that the state transition matrix is an identity matrix in the case, when $K_n = 0$. Hence, the state variable $\tilde{\mathbf{x}}^{\mathrm{t}}_n$, is transformed only if a sample is not lost, so only these state transitions must be considered during the analysis.

The orthogonality of $\mathbf{c}_n$ ensures that the multiplication of the state transition matrices is commutative:

$$\begin{aligned} &\left(\mathbf{I} - \tfrac{1}{N}\mathbf{c}^*_n\mathbf{c}^{\mathrm{T}}_n\right)\left(\mathbf{I} - \tfrac{1}{N}\mathbf{c}^*_k\mathbf{c}^{\mathrm{T}}_k\right) = \\ &\quad\left(\mathbf{I} - \tfrac{1}{N}\mathbf{c}^*_k\mathbf{c}^{\mathrm{T}}_k\right)\left(\mathbf{I} - \tfrac{1}{N}\mathbf{c}^*_n\mathbf{c}^{\mathrm{T}}_n\right), \end{aligned} \tag{A-7}$$

and these matrices are projectors, i.e.

$$\left(\mathbf{I} - \tfrac{1}{N}\mathbf{c}^*_k\mathbf{c}^{\mathrm{T}}_k\right)\left(\mathbf{I} - \tfrac{1}{N}\mathbf{c}^*_k\mathbf{c}^{\mathrm{T}}_k\right) = \left(\mathbf{I} - \tfrac{1}{N}\mathbf{c}^*_k\mathbf{c}^{\mathrm{T}}_k\right). \tag{A-8}$$

The equalities can be verified by performing the matrix and vector multiplications on both sides and by using (A-6).

Due to the periodicity of $\mathbf{c}_n$, there are at most N different state transition matrices, $\left(\mathbf{I} - \frac{1}{N}\mathbf{c}^*_n\mathbf{c}^{\mathrm{T}}_n\right)$, which must be present in the product (A-2), otherwise there would not exist N different $K_n\mathbf{c}_n$ vectors, thus the necessary condition of *Theorem 1* would not be satisfied. Equations (A-7) and (A-8) show that the state transition matrices in the product (A-2) can be arbitrarily rearranged without changing the final state. If we rearrange these products in such a way that their indices are in ascending order, i.e. $n = [0 \ldots N-1]$, we obtain the form of the observer without data loss, the convergence of which has already been proven [18]. This means that the necessary condition for the convergence is sufficient, as well.

APPENDIX B

Lemma B-I: If $\alpha \in [0 \ldots 2]$ then

$$\left\| \mathbf{A} - \mathbf{g} K_i \mathbf{c}^{\mathrm{T}} \right\| = 1; \quad K_i = \{0, 1\}. \tag{B-1}$$

Proof: The norm of an arbitrary matrix, $\mathbf{Z}_i$, is computed as follows [24]:

$$\|\mathbf{Z}_i\|^2 = \max_{\|\mathbf{x}\|=1} \|\mathbf{Z}_i \mathbf{x}\| = \max_{\|\mathbf{x}\|=1} \mathbf{x}^{\mathrm{H}} \mathbf{Z}_i^{\mathrm{H}} \mathbf{Z}_i \mathbf{x}. \tag{B-2}$$

Let us use the notation $\mathbf{Z}_i = \left(\mathbf{A} - \mathbf{g} K_i \mathbf{c}^{\mathrm{T}}\right)$. We will use the property of $\mathbf{c}$ that $\mathbf{c}^{\mathrm{H}} \mathbf{c} = \mathbf{c}^{\mathrm{T}} \mathbf{c} = N$ which follows from (3), furthermore (9) can be substituted for $\mathbf{g}$, thus:

$$\begin{aligned}
\mathbf{Z}_i^{\mathrm{H}} \mathbf{Z}_i &= \left(\mathbf{A} - \tfrac{\alpha}{N} \mathbf{A} \mathbf{c} K_i \mathbf{c}^{\mathrm{T}}\right)^{\mathrm{H}} \left(\mathbf{A} - \tfrac{\alpha}{N} \mathbf{A} \mathbf{c} K_i \mathbf{c}^{\mathrm{T}}\right) \\
&= \left(\mathbf{I} - \tfrac{\alpha}{N} \mathbf{c} K_i \mathbf{c}^{\mathrm{T}}\right)^{\mathrm{H}} \mathbf{A}^{\mathrm{H}} \mathbf{A} \left(\mathbf{I} - \tfrac{\alpha}{N} \mathbf{c} K_i \mathbf{c}^{\mathrm{T}}\right) \\
&= \mathbf{I} - 2 \tfrac{\alpha}{N} \mathbf{c} K_i \mathbf{c}^{\mathrm{T}} + \left(\tfrac{\alpha}{N}\right)^2 \mathbf{c} K_i \mathbf{c}^{\mathrm{T}} \mathbf{c} K_i \mathbf{c}^{\mathrm{T}} \\
&= \mathbf{I} - \tfrac{1}{N}(2\alpha - \alpha^2) \mathbf{c} K_i \mathbf{c}^{\mathrm{T}} = \mathbf{I} - f(\alpha) \mathbf{c} K_i \mathbf{c}^{\mathrm{T}},
\end{aligned} \tag{B-3}$$

where $f(\alpha) = \frac{1}{N}(2\alpha - \alpha^2)$. $\mathbf{A}^{\mathrm{H}} \mathbf{A} = \mathbf{I}$, since $\mathbf{A}$ is a diagonal matrix (4), and multiplying the diagonal elements yields one: $z_i^* z_i = e^{-\jmath 2\pi i f_r} e^{\jmath 2\pi i f_r} = 1$. The analysis of $f(\alpha)$ shows that:

$$f(\alpha) \geq 0 \text{ if } 0 \leq \alpha \leq 2 \tag{B-4}$$

The norm is obtained form (B-2) and (B-3):

$$\begin{aligned}
\|\mathbf{Z}_i\|^2 &= \max_{\|\mathbf{x}\|=1} \left[\mathbf{x}^{\mathrm{H}} \left(\mathbf{I} - f(\alpha) \mathbf{c} K_i \mathbf{c}^{\mathrm{T}}\right) \mathbf{x}\right] \\
&= \max_{\|\mathbf{x}\|=1} \left[\mathbf{x}^{\mathrm{H}} \mathbf{x} - f(\alpha)(\mathbf{x}^{\mathrm{H}} \mathbf{c}) K_i (\mathbf{c}^{\mathrm{T}} \mathbf{x})\right] \\
&= \max_{\|\mathbf{x}\|=1} \left[1 - f(\alpha) |\mathbf{c}^{\mathrm{T}} \mathbf{x}|^2 K_i\right].
\end{aligned} \tag{B-5}$$

$\|\mathbf{Z}_i\|^2 = 1$ since $K_i = \{1, 0\}$, $|\mathbf{c}^{\mathrm{T}} \mathbf{x}|^2 \geq 0$ and $f(\alpha) \geq 0$ if $\alpha \in [0 \ldots 2]$ due to (B-4). Hence, the lemma is proven. ■

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REFERENCES

[1] L. Sujbert, G. Péceli, “Periodic noise cancellation using resonator based controller,” *Active’97,* Aug. 1997., Budapest, Hungary, pp. 905–916.

[2] L. Sujbert, B. Vargha, “Active distortion cancellation of sinusoidal sources,” *Proc. of the IEEE Instrumentation and Measurement Technology Conference,* Como, Italy, May 18-20., 2004., pp. 322–325.

[3] L. Sujbert, K. Molnár, Gy. Orosz, L. Lajkó, “Wireless sensing for active noise control,” *IEEE Instrumentation and Measurement Technology Conference,* Sorrento, Italy, April 24-27., 2006., pp. 123–128.

[4] J. J. Tomic, M. D. Kusljevic, D. P. Marcetic, “An adaptive resonator-based method for power measurements according to the IEEE trial use standard 1459-2000,” *IEEE Trans. on Instrumentation and Measurement,* vol. 59, no. 2, pp. 250–258, Feb. 2010.

[5] F. Nagy, “Measurement of signal parameters using nonlinear observers,” *IEEE Trans. on Instrumentation and Measurement,* vol. IM-41 no. 1, pp. 152–155, Feb. 1992.

[6] A. Görgényi *et al.,* “DSP-based electromagnetic flowmeter with sinusoidal excitation,” *IEEE Instrumentation and Measurement Technology Conference,* Ottawa, Canada, May 16-19., 2005., pp. 1023-1026.

[7] L. Sujbert, G. Péceli, Gy. Simon, “Resonator based non-parametric identification of linear systems,” *IEEE Trans. on Instrumentation and Measurement,* vol. 54, no. 1, pp. 386–390, Feb. 2005.

[8] R. R. Bitmead, A. C. Tsoi, P. J. Parker, “A Kalman filtering approach to short-time Fourier analysis,” *IEEE Trans. on Acoustics, Speech and Signal Processing,* vol. 34, no. 6, pp. 1493–1501, Dec. 1986.

[9] B. Sinopoli, L. Schenato, M. Franceschetti , K. Poolla, M. I. Jordan, S. S. Sastry, “Kalman filtering with intermittent observations,” *IEEE Trans. on Autom. Control,* vol. 49, no. 9, pp. 1453–1464, Sept. 2004.

[10] Y. Xiao, L. Ma, and R. K. Ward, “Fast RLS Fourier analyzers capable of accommodating frequency mismatch,” *Signal Processing,* vol. 87, no. 9, pp. 2197–2212, Sept. 2007.

[11] C. A. Vaz, X. Kong, N. V. Thakor, “An adaptive estimation of periodic signals using a Fourier linear combiner,” *IEEE Trans. Signal Process.,* vol. 42, no. 1, pp. 1–10, Jan. 1994.

[12] M. T. Kilani, J. F. Chicharo, “A constrained notch Fourier transform,” *IEEE Trans. Signal Process.,* vol. 43, no. 9, pp. 2058–2067, 1995.

[13] H. C. So, “Adaptive algorithm for sinusoidal interference cancellation,” *Electron. Lett.,* vol. 33, no. 22, pp. 1910–1912, Oct. 1997.

[14] N. R. Lomb, “Least squares frequency analysis of unequally spaced data,” *Astrophys. & Space Sci.,* vol. 39, no. 2, pp. 447–462, 1976.

[15] J. D. Scargle, “Studies in astronomical time series analysis. III - Fourier transforms, autocorrelation functions, and cross-correlation functions of unevenly spaced data,” *Astrophysical Journal,* vol. 343, part 1., pp. 874–887, Aug. 15, 1989.

[16] P. M. T. Broersen, S. de Waele, R. Bos, “Estimation of autoregressive spectra with randomly missing data,” *IEEE Instrumentation and Measurement Technology Conference,* Vail, CO, USA, May 20-22. 2003., pp. 1154–1159.

[17] G. H. Hostetter, “Recursive discrete Fourier transformation with unevenly spaced data,” *IEEE Trans. on Acoustic, Speech, and Signal Processing,* vol. ASSP-31, no. 1, pp. 206–209, Feb. 1983.

[18] G. Péceli,“A common structure for recursive discrete transforms,” *IEEE Trans. Circuits Syst.,* vol. CAS-33, no. 10, pp.1035–1036, 1986.

[19] R. R. Bitmead, “On recursive discrete Fourier transformation,” *IEEE Trans. on Acoustic, Speech, and Signal Processing,* vol. ASSP-30, no. 2, pp. 319–322, Apr. 1982.

[20] G. H. Hostetter, “Recursive discrete Fourier transformation,” *IEEE Trans. on Acoustic, Speech, and Signal Processing,* vol. ASSP-28, no. 2, pp. 184–190, Apr. 1980.

[21] B. Widrow, P. Baudrenghien, M. Vetterli, P. Titchener, “Fundamental relations between the LMS algorithm and the DFT,” *IEEE Trans. on Circuits and Syst.,* vol. 34, pp. 814–820, Jul. 1987.

[22] K. J. Åström, B. Wittenmark, *Computer Controlled Systems, Theory and Design,* 2nd ed., Prentice Hall, Englewood Cliffs, 1990.

[23] H. F. Chen, “Stochastic observability,” *Scientia Sinica,* vol. 20, no. 3, pp. 305–324, 1977.

[24] G. H. Golub, C. F. Van Loan, *Matrix Computations,* The Johns Hopkins University Press, Baltimore, 1996.

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