# Determination of Emission Limits of UWB Impulse Radio Systems

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**Abstract**—The emission limits of UWB impulse radio is regulated by FCC (USA). However exact mathematical description of the prescribed peak and average power limits are not provided in the literature. This contribution gives closed form unified expressions for the FCC power limits exploiting Jacobi theta functions and based on models elaborated for the interpretation of FCC Regulations and UWB IR carriers. Different UWB IR carriers appearing in the literature are compared in terms of UWB system design parameters. The relationship between the design parameters and FCC Regulations is provided.

## 1. Introduction

Carriers used in ultra-wideband (UWB) systems are not exactly defined however spectral emission limits are strictly regulated and interoperability among UWB nodes from different vendors in a network must be assured, therefore the prescriptions of FCC Regulations [1] and IEEE 802.15.4a Standard [2], respectively, must be obeyed. The determination of FCC emission limits are based on mainly heuristic approach and exact mathematical models and formulas are hardly provided in the literature. This contribution provides mathematical models for the interpretation of FCC Regulations and also for the UWB impulse radio (IR) carriers furthermore determines the peak and average power limits introducing Jacobi theta functions. The model introduced is unified so not only deterministic but even chaotic signals, e.g. FM-DCSK modulated UWB IR carriers can be applied to the model. The paper compares the different type of UWB IR carriers in terms of the parameters circuit and system designers rely on, namely, peak voltage and pulse energy, respectively and provides the relationship between these parameters and FCC Regulations.

## 2. Restrictions on UWB IR carriers

Exact shapes of UWB IR carriers are not defined. Instead, the properties of UWB IR signals are regulated both in the time and frequency domains by the (i) FCC Regulations [1] and (ii) IEEE 802.15.4a Standard [2].

### 2.1. FCC Limits on Radiated UWB IR Signals

The restrictions on UWB IR signals were published by FCC in 2002 [1]. To limit the interference caused in other radio systems both the peak and average values of the Equivalent Isotropically Radiated Power (EIRP) transmitted by a UWB device are limited:

1. "There is a limit on the *peak* level of the emissions contained within a 50-MHz bandwidth centered on the frequency at which the highest radiated emission occurs $\cdots$ That limit is 0 dBm EIRP."
2. The *average* "radiated emissions $\cdots$ shall not exceed" -41.3 dBm EIRP "when measured using a resolution bandwidth of 1 MHz" over the frequency band of 3.1 GHz to 10.6 GHz.
   "The RMS average measurement is based on the use of a spectrum analyzer with a resolution bandwidth of 1 MHz, an RMS detector, and a 1 ms or less averaging time."

The 50-MHz and 1-MHz RF bandpass filters are referred to as FCC bandpass filters in the rest of the paper.

### 2.2. Requirements of IEEE Standard

The IEEE Standard gives the specification for the physical layer of the Low-Rate Wireless Personal Area Networks [2].

The implementation of the UWB pulse generator is the key issue in the design of UWB transceivers. To give a high level of freedom, the IEEE Standard does not specify the exact shape of UWB carrier. Large variation of UWB IR carriers has been proposed in the literature [3]–[4]. Any kind of UWB pulses can be used provided that they satisfy the FCC Regulations and obey the transmit spectrum mask of IEEE Standard.

However, UWB transceivers manufactured by different vendors have to operate in the same network. To assure interoperability among the different UWB transceivers, the IEEE Standard defines a *square root raised cosine (RRC)* reference pulse and specifies the properties of cross-correlation of the envelope of UWB pulse used by an IEEE 802.15.4a-compliant UWB transceiver and the reference pulse. For details, please refer to [2].

## 3. Mathematical Model

The UWB circuit and system designers need two basic parameters, namely, (i) $V_{peak}$, i.e., the peak voltage of the envelope of UWB pulse generated at the transmitter and (ii) $E_p$, i.e., the energy carried by one UWB pulse. $V_{peak}$ gives the required output voltage swing to be assured at the UWB transmitter while $E_p$ determines the attainable radio coverage.

This paper provides the relationship among these UWB parameters and the FCC Regulations. Only those UWB pulses are considered here which satisfy the IEEE Standard. To derive the analytical expression, first the FCC Regulations have to be interpreted and a mathematical model has to be elaborated.

### 3.1. Interpretation of FCC Regulations

Let $P_{peak}^{FCC}$ and $P_{avg}^{FCC}$ denote the FCC peak and average power limits, respectively. The mathematical model constructed from the FCC Regulations is depicted in Fig. 1 where $g^T(t)$ denotes the train of UWB IR carrier pulses to be measured, $h^{FCC}(t)$ is the impulse response of the FCC bandpass filter, $\omega_{CF} = 2\pi f_{CF}$ and $RBW$ give the center frequency and resolution bandwidth, respectively, of that filter. Note, the FCC peak power limit is determined from the peak value of FCC filter output $y^T(t)$ while the measurement of the FCC average power limit needs an RMS detector (RMS DET) and averaging (AVG).

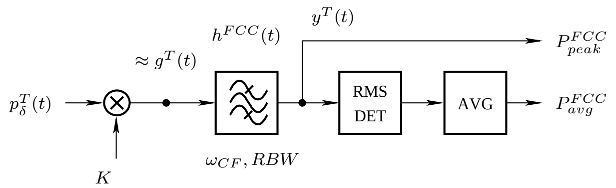


Figure 1: Mathematical model for the interpretation of FCC Regulations.

One bit information can be carried by either a single UWB IR carrier pulse or a burst of them. The latter solution has been used in [2] to improve the coverage of UWB radio. The periodicity of UWB IR signals is reflected by the upper $T$ index in both Fig. 1 and the rest of this contribution.

### 3.2. IEEE 802.15.4a-compliant UWB IR Pulses

The use of the RRC function as UWB IR carrier gives the trivial solution and it assures the implementation of an ISI free Nyquist channel however it cannot be generated by simple CMOS analog microwave circuits.

The most popular UWB envelope is the *Gaussian pulse* [3]

$$p^{[GAUSS]}(t) = V_{peak} \exp\left(\frac{-t^2}{2u_B^2}\right) \tag{1}$$

where the upper index [GAUSS] identifies the type of UWB envelope but in general upper index is dropped, and $u_B = 1/(2\pi f_B \sqrt{\lg(e)})$ is determined by the 10-dB RF bandwidth $2f_B$ of the UWB carrier.

Researchers at the Massachusetts Institute of Technology (USA) approximated the Gaussian pulse in their built UWB transmitter by generating a *Tanh pulse* ([TANH]) as UWB envelope. As alternative they proposed a square pulse filtered by a second order low-pass filter ([FILT]) as UWB envelope [5].

To get the UWB IR carrier, $g^T(t)$, the UWB envelopes given above are upconverted into the microwave frequency region

$$g^T(t) = p^T(t)\cos(2\pi f_{CF} t) \tag{2}$$

### 3.3. Unified Model for UWB Impulses

To get a single unified mathematical model for the different UWB IR carriers, they are approximated as the product of a Dirac delta function $p^T_\delta(t)$ and a weighting constant $K$ in Fig. 1. $K$ establish connection between the unified model and the parameters of different UWB IR carriers that meet the FCC Regulations.

To verify the approximation recall that, according to the FCC Regulations, the RF bandwidth of a UWB signal has to be at least 500 MHz, consequently, the bandwidth of a UWB IR signal is always much greater than that of the FCC bandpass filter. If so then, over the frequency band where the frequency response of FCC filter is not negligible, the spectrum of $g^T(t)$ can be substituted by the spectrum of weighted Dirac delta functions, i.e., the UWB pulse can be considered as an impulse excitation to the FCC filter.

To get a unified model, the UWB IR carrier given by (2) is decomposed into a product

$$g^T(t) \approx K p^T_\delta(t) \tag{3}$$

as shown in Fig. 1 where

$$p^T_\delta(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT) \tag{4}$$

Taking the Fourier transform of (3) exploiting (4) we get

$$\mathcal{F}\left\{g^T(t)\right\} \approx \frac{K}{T} \sum_{m=-\infty}^{+\infty} \delta\left(f - \frac{m}{T}\right) \tag{5}$$

Note, two conditions have to be met, namely (i) the FCC peak limit is read and (ii) spectral line should appear at the frequency of highest emission.

As an example let's apply the unified model to a UWB IR carrier with Gaussian envelope of (1) to get

$$K = V_{peak}\sqrt{2\pi}u_B/2 \quad \text{and} \quad E_p = \sqrt{\pi}u_B V_{peak}^2/(2Z_0)$$

where $Z_0$ is the impedance of termination.

## 4. Derivation of Peak and Average Power Limits

Section 3.3 has shown that a UWB IR signal can be considered as an impulse excitation to the FCC bandpass filter. FCC limits are derived from $y^T(t)$ of Fig. 1, i.e., from the output of FCC filter.

According to the FCC Regulations let both $P^{FCC}_{avg}$ and $P^{FCC}_{peak}$ be measured by a spectrum analyzer.

The most common type of IF filters used in spectrum analysis is a Gaussian filter [6], its impulse response is

$$h^{FCC}(t) = \frac{2}{\sqrt{2\pi}\,\lambda} \exp\left[-\frac{(t-\tau_{CF})^2}{2\lambda^2}\right] \cos\left[\omega_{CF}(t-\tau_{CF})\right] \tag{6}$$

where $\tau_{CF}$ denotes the total delay of the spectrum analyzer and $\lambda = \sqrt{2\ln\sqrt{2}}/(\pi RBW)$. Note, the Resolution BandWidth, $RBW$, is equal to the 3-dB bandwidth of the FCC bandpass filter.

The mathematical model of Fig. 1 shows that the response of FCC filter to *a single isolated UWB pulse* $g(t)$ is

$$y(t) = \frac{2K}{\sqrt{2\pi}\,\lambda} \exp\left[-\frac{(t-\tau_{CF})^2}{2\lambda^2}\right] \cos\left[\omega_{CF}(t-\tau_{CF})\right] \tag{7}$$

Let us apply a UWB pulse train to the input of FCC filter. Since the FCC filter is a LTI system, the superposition theorem can be applied and $y^T(t)$ can be expressed as a sum of delayed weighted-by-$K$ impulse responses $h^{FCC}(t)$ of the FCC filter.

### 4.1. FCC Peak Power Limit

To derive an analytic expression for the FCC peak power limit three conditions have to be met, namely, (i) center frequency $f_{CF}$ of FCC filter shown in Fig.1 has to be set to the UWB frequency at which the highest radiated emission occurs, (ii) $RBW$ has to be set according to FCC Regulations, and (iii) peak voltage of FCC filter output has to be determined.

To get the peak power only the *peak voltage* has to be expressed, consequently, the calculation can be simplified considerably by substituting the output voltage of FCC filter by its *envelope* in (7)

$$y(t) \leq y_{env}(t) = \frac{2K}{\sqrt{2\pi}\,\lambda} \exp\left[-\frac{(t-\tau_{CF})^2}{2\lambda^2}\right] \tag{8}$$

When the pulse repetition frequency, $PRF \ll RBW$, the responses of FCC filter to the individual UWB pulses are completely separated and overlapping does not occurs. This situation is shown in Fig. 2, where the Gaussian waveform depicted is the envelope of bandpass RF signal measured at the FCC filter output. Note, the envelope, $y^T_{env}(t)$ is a periodic waveform with the period time $T = 1/PRF$ and its peaks appear at $\tau_{CF} \pm nT,\ n = 0, 1, 2, \ldots$.

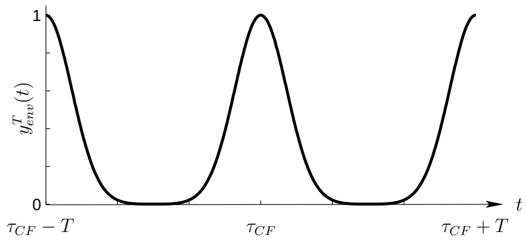

Figure 2: Envelope of FCC filter output when $PRF \ll RBW$.

In the general case overlapping occurs. The dashed, dotted and dash-dotted curves gives the responses of the FCC filter to three consecutive individual UWB pulses in Fig. 3, while the solid one is the envelope of FCC filter output. Only three individual responses are plotted in the figure, the presence of the further ones is marked by three dots on both sides. A few important conclusions can be drawn: (i) since the worst-case situation has to be considered and the superposition theorem holds to the FCC filter, the individual responses have to be summed and the peaks increase in size with increasing overlappin, (ii) the peaks of FCC filter output appears at $\tau_{CF} \pm nT, n = 0, 1, 2, \ldots$.

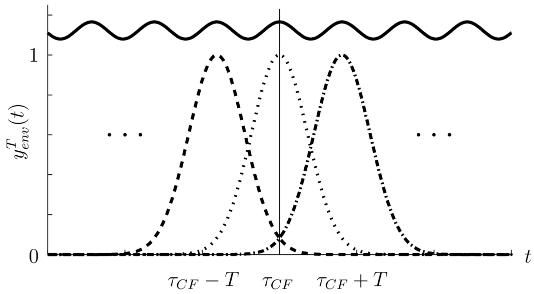

Figure 3: Envelope of FCC filter output when $PRF > RBW$. Note, the FCC filter outputs generated by the individual UWB pulses overlap one another.

Let the peak power be calculated at $t = \tau_{CF}$. At this time instant the peak value of FCC filter output is

$$y^T_{env}(\tau_{CF}) = \frac{2K}{\sqrt{2\pi}\lambda}\left(\cdots + \exp\left[-\frac{(T)^2}{2\lambda^2}\right] + 1 + \exp\left[-\frac{(-T)^2}{2\lambda^2}\right] + \cdots\right) \quad (9)$$

where the three terms in the bracket give, from the left to the right, respectively, the contributions of the dashed, dotted and dash-dotted curves shown in Fig. 3.

Exploiting the symmetry of the exponential terms we can put (9) into the form

$$y^T_{env}(\tau_{CF}) = \frac{2K}{\sqrt{2\pi}\lambda}\left(1 + 2\sum_{n=1}^{\infty}\exp\left[-\frac{(nT)^2}{2\lambda^2}\right]\right) \quad (10)$$

To get a closed-form expression for the FCC peak power limit, the sum of exponentials has to be expressed.

The Jacobi theta functions are defined in mathematics. One of them takes the form in the notation of Whittaker and Watson [7] as

$$\vartheta_3(z, q) = 1 + 2\sum_{n=1}^{\infty} q^{n^2}\cos(2nz) \quad (11)$$

where $q$ and $z$ denote the nome and argument, respectively.

A comparison of (10) and (11) shows that if we substitute $z = 0$ and $q = \exp\left(-\frac{T^2}{2\lambda^2}\right) = \exp\left[-\frac{1}{2(\lambda PRF)^2}\right]$ in (11) then the peak voltage of FCC filter output can be expressed by the Jacobi theta function as

$$y^T_{env}(\tau_{CF}) = \frac{2K}{\sqrt{2\pi}\lambda}\,\vartheta_3\left(0, \exp\left[-\frac{1}{2(\lambda PRF)^2}\right]\right) \quad (12)$$

Although an analytic expression is not available for the calculation of $\vartheta_3(z, q)$, its value can be determined numerically [8].

Jacobi theta function with argument $z = 0$ and nome $q = \exp\left(-\frac{T^2}{2\lambda^2}\right) = \exp\left[-\frac{1}{2(\lambda PRF)^2}\right]$ can be approximated by a piece-wise linear function with a negligible error

$$\vartheta_3(0, \lambda PRF) = \begin{cases} 1, & \text{if } \lambda PRF \le 1/\sqrt{2\pi} \\ \sqrt{2\pi}\,\lambda PRF, & \text{otherwise} \end{cases} \quad (13)$$

Now all equations are available to express the peak power at the FCC filter output.

Let the FCC bandpass filter be terminated by $Z_0$. As shown by Figs. 2 and 3, the peak voltage at the FCC filter output appears at $t = \tau_{CF} \pm nT,\ n = 0, 1, 2, \ldots$ and the peak power is obtained as

$$P_{peak} = \left[y^T_{env}(\tau_{CF})\right]^2 / Z_0 \quad (14)$$

Substituting (12) into (14), taking into account the piece-wise linear approximation of (13) and substituting $\lambda$ the peak power at the FCC bandpass filter output is obtained as a functions of the parameters of UWB pulse and FCC Regulations

$$P_{peak} = \frac{\pi}{\ln\sqrt{2}}(RBW)^2\frac{K^2}{Z_0}\begin{cases} 1, & \text{if } \lambda PRF \le 1/\sqrt{2\pi} \\ 2\pi(\lambda PRF)^2, & \text{otherwise} \end{cases} \quad (15)$$

### 4.2. FCC Average Power Limit

Figures 2 and 3 show that the output of FCC bandpass filter is a periodic signal so it can be represented by its Fourier series. According to Parseval's Relation to get the average power at the FCC filter output, the Fourier coefficients of $y^T(t)$ has to be calculated. The steps of investigation can be determined from Fig. 1: (i) first the Fourier coefficients of $Kp^T_\delta(t)$ are calculated, (ii) then those of FCC filter output are derived and (iii) finally, the average power measured at the FCC filter output is determined.

The FCC filter is driven by a sequence of Dirac delta functions $Kp_\delta(t)$. The Fourier coefficients of this periodic excitation $Kp^T_\delta(t)$ can be calculated from the Fourier transform of one period as

$$a_k = \frac{1}{T}\mathcal{F}\left\{Kp_\delta(t)\right\}\Big|_{f=\frac{k}{T}} = \frac{K}{T}\,\delta_{ik} \quad (16)$$

where $\delta_{ik}$ is the Kronecker delta function, $i = fT$ and $k = 0, \pm1, \pm2, \ldots$.

The Fourier coefficients of the periodic FCC filter output $y^T(t)$ are obtained as

$$b_k = H^{FCC}(f)\Big|_{f=\frac{k}{T}} a_k = \frac{K}{T}\,H^{FCC}\left(\frac{k}{T}\right) \quad (17)$$

where $H^{FCC}(f) = \mathcal{F}\left\{h^{FCC}(t)\right\}$.

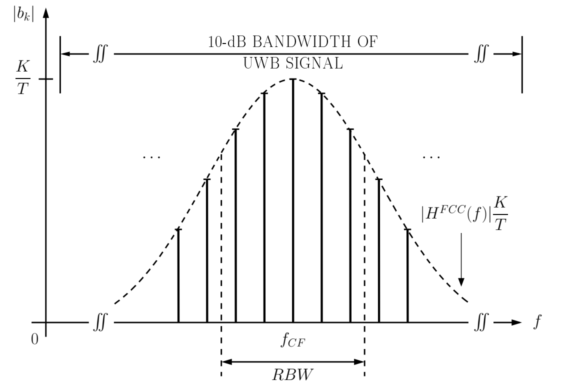


Figure 4: The spectrum of the response of Gaussian filter used in spectrum analyzers, i.e. the FCC filter to the UWB IR carrier train.

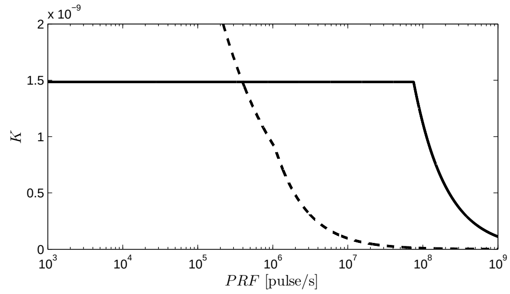


Figure 5: Maximum allowable values of parameter $K$ versus pulse repetition frequency. The solid and dashed curves show the values of parameter $K$ that are compliant to the FCC peak and average, respectively, power limits. The RBW values are 1 MHz and 50 MHz for the FCC peak and average power limits, respectively.

The absolute value of Fourier coefficients $b_k$ are plotted for $f > 0$ in Fig. 4. Note, their values reflects the frequency response of Gaussian FCC bandpass filter.

The average power at the FCC filter output can be determined by the Parseval's Relation

$$P_{avg} = \frac{1}{Z_0}\sum_{k=-\infty}^{\infty}|b_k|^2 = \frac{1}{Z_0}\left(\frac{K}{T}\right)^2\sum_{k=-\infty}^{\infty}\left|H^{FCC}\left(\frac{k}{T}\right)\right|^2 \quad (18)$$

The next step is the determination of the sum of $H^{FCC}(f)$.

Substituting $H^{FCC}(f)$ into the right hand side of (18), a compact equation is obtained

$$\sum_{k=-\infty}^{\infty}\left|H^{FCC}\left(\frac{k}{T}\right)\right|^2 = 4\sum_{k=1}^{\infty}\exp\left(-4\left[\pi\lambda(\frac{k}{T} - f_{CF})\right]^2\right) \quad (19)$$

Assume that $f_{CF}$ is an entire multiple of $PRF$, then substituting (19) into (18) and applying the approximation procedure with the Jacobi theta function as done in Sec. 4.1 we get the average power as

$$P_{avg} = 4\,(PRF)^2\,\frac{K^2}{Z_0}\begin{cases} 1/(\sqrt{4\pi}\,\lambda PRF), & \text{if } \lambda PRF \leq 1/\sqrt{4\pi} \\ 1, & \text{otherwise} \end{cases} \quad (20)$$

## 5. Interpretation of results

As mentioned in Sec. 3 the peak voltage and pulse energy of the UWB pulse generated at the transmitter are important parameters for circuit designers. Eqs. (15) and (20) establish the relationship between the FCC Regulations and $V_{peak}$ and also $E_p$ through the parameter $K$. The graphical interpretation of the FCC peak and average power via parameter $K$ when the FCC limits are just met is shown in Fig. 5 as a function of PRF. Observe, the low-rate UWB IR systems are *peak power limited* while high-rate ones are *average power limited*. The crossing point of the low-rate and high-rate systems is at the PRF of 396 kpulse/s.

Table 1 shows the calculated values of $V_{peak}$ and $E_p$ for different UWB pulse candidates, having 500 MHz 10-dB RF bandwidths, at $10^4$ pulse/s and $10^6$ pulse/s pulse repetition frequencies where the peak and average, respectively, FCC limits take effect. The greater the value of $E_p$ the larger the coverage of the UWB transmitter.

The values of $V_{peak}$ and $E_p$ for different pulse shapes only slightly varies, therefore those pulse shapes are reasonable to prefer whose implementation is simple.

| | $PRF = 10^4$ [pulse/s] | | $PRF = 10^6$ [pulse/s] | |
|---|---|---|---|---|
| | $V_{peak}$ [V] | $E_p$ [pJ] | $V_{peak}$ [V] | $E_p$ [pJ] |
| [RRC] | 1.28 | 32.59 | 0.80 | 12.86 |
| [GAUSS] | 1.22 | 25.77 | 0.77 | 10.17 |
| [TANH] | 1.25 | 25.57 | 0.79 | 10.09 |
| [FILT] | 1.2 | 26.53 | 0.76 | 10.47 |

Table 1: Allowable peak voltages and pulse energies for different UWB pulse candidates having 500 MHz 10-dB RF bandwidth at pulse repetition frequencies of $10^4$ [pulse/s] and $10^6$ [pulse/s].

## 6. Conclusions

Starting from the FCC Regulations and IEEE 802.15.4a Standard this contribution elaborated unified mathematical models for (i) the interpretation of FCC Regulations and (ii) the UWB IR carriers. Based on the models unified formulas for the determination of peak and average power limits have been elaborated. We have shown that Jacobi theta functions describe the nonlinear behavior of peak and average power limits as a function of pulse repetition frequency. Power limits of low-rate and high-rate systems determines a special value of pulse repetition frequency at 396 kpulse/s. Before and after that point, the peak and average power limit, respectively, takes effect.

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